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A Complete Analysis of Study-State-Error in Non-Unity Feedback Gain System

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Abstract
Steady-state error is a critical performance parameter in control system analysis, representing the difference between the desired output and the actual output as time approaches infinity. While steady-state error in unity feedback systems is well established, practical control systems often employ non-unity feedback due to sensor scaling, feedback conditioning, and system constraints. This paper presents a comprehensive analytical study of steady-state error in non-unity feedback control systems. Mathematical derivations for standard test inputs—step, ramp, and parabolic—are provided using the Final Value Theorem. System type classification is extended to non-unity feedback configurations, and numerical examples are included to illustrate the impact of feedback transfer functions on steady-state performance. The results demonstrate that feedback elements significantly influence error constants and must be carefully designed to meet performance specifications.

Introduction
In modern engineering applications, control systems play a vital role in ensuring that physical processes behave in a desired and predictable manner. From industrial automation and robotics to power systems and aerospace engineering, the accuracy of a control system directly influences its reliability and performance. One of the most important measures of this accuracy is the steady-state error, which represents the difference between the desired input and the actual output of the system after all transient effects have disappeared.

In classical control analysis, steady-state error is often evaluated under the assumption of a unity feedback configuration. While this assumption simplifies mathematical analysis, it does not always reflect real-world systems. In practice, the feedback path frequently contains sensors, transducers, amplifiers, or filtering elements, resulting in what is known as a non-unity feedback control system. Such feedback elements are essential for measurement, scaling, and noise reduction, but they a
alter the overall system behaviour and error characteristics.

The presence of a non-unity feedback path significantly affects the steady-state performance of a control system. The feedback transfer function can modify the effective system type, influence static error constants, and, in some cases, introduce additional dynamics that degrade accuracy. As a result, methods developed exclusively for unity feedback systems cannot be directly applied without modification. This makes steady-3state error analysis in non-unity feedback systems both practically important and theoretically relevant.

This paper focuses on a systematic and intuitive analysis of steady-state error in non-unity feedback control systems. By relating the feedback dynamics to equivalent unity feedback representations, the paper develops clear expressions for error constants corresponding to standard test inputs. The aim is to provide a human-cantered and application-oriented understanding that bridges classical control theory with practical system design considerations.

Gap in the Literature
Despite the extensive body of research on steady-state error analysis in classical control systems, several important gaps remain when non-unity feedback configurations are considered. Most foundational textbooks and academic studies primarily focus on unity feedback systems, presenting steady-state error formulations that assume ideal

feedback conditions. As a result, non-unity feedback systems are often treated as special cases rather than as practical configurations that dominate real-world applications.

A significant gap in the existing literature is the lack of a unified and systematic framework for steady-state error analysis that explicitly incorporates feedback dynamics. While some studies suggest converting non-unity feedback systems into equivalent unity feedback models, the underlying assumptions, limitations, and physical interpretations of this transformation are not always clearly discussed. This can lead to confusion among students and practicing engineers when applying theoretical results to practical systems involving sensors and measurement elements.

Another limitation is the insufficient attention given to the impact of feedback element dynamics on system type and static error constants. Many existing works consider feedback paths as pure gains, overlooking cases where feedback transfer functions contain poles or zeros. Such dynamics can significantly alter steady-state performance, yet they are rarely analysed in detail or supported with illustrative examples in the literature.

Furthermore, there is a noticeable scarcity of application-oriented studies that connect steady-state error theory in non-unity feedback systems with real industrial implementations. Numerical examples, simulation-based validation, and design guidelines tailored to non-unity feedback configurations are often limited or absent. This restricts the practical usefulness of existing research for control system designers.
The present work addresses these gaps by providing a clear analytical treatment of steady-state error in non-unity feedback control systems, emphasizing the role of feedback dynamics, system type modification, and practical design implications.

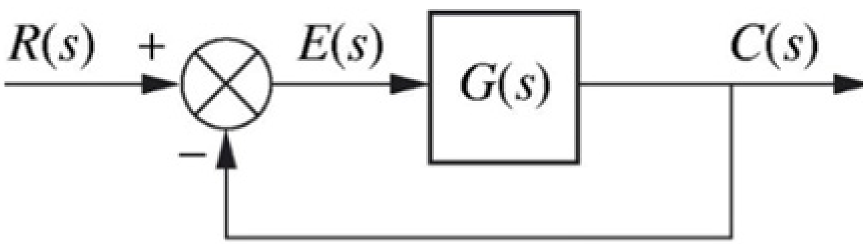
Mathematical Modelling and Theoretical Development

Basics of Unity Feedback Control System

A unity feedback control system is a closed-loop control system in which the feedback signal is directly equal to the system output. In such a system, the feedback transfer function is unity, that is,
$H(s)=1$.

This configuration is widely used in classical control analysis due to its mathematical simplicity and its ability to clearly illustrate fundamental control concepts such as stability, transient response, and steady-state error.

In a unity feedback system, the error signal is defined as the difference between the reference input and



the output:

$$E(s)=R(s)-C(s)$$

The output is related to the error through the forward path:

$$C(s)=G(s)E(s)$$

Substituting C(s)into the error equation:

$$E(s)=R(s)-G(s)E(s)$$
$$E(s)(1+G(s))=R(s)$$
$$E(s)=R(s)/(1+G(s)) \quad \text{.............(i)}$$

Steady state error is the error at time at infinite so applying final value theorem we get :

$$e_{ss}=\lim_{s\Rightarrow 0} s\left(R(s)/(1+G(s)\right)$$

Steady-State Error for Standard Test Inputs

Step Input

For a unit step input:

$$R(s)=1/s$$
$$e_{ss}=\lim_{s\Rightarrow 0} s\left(R(s)/(1+G(s)\right)$$

Define the position error constant:

$$K_p=\lim\ G(s)$$

$s \to 0$

Substituting the value R(S) we get the value of Steady state er as:

$$e_{ss} = 1/1+K_p$$

Ramp Input

For a unit ramp input:

$$R(s)=1/(s2)$$

Substituting the value in the equation of Steady State error

$$e_{ss} = \lim_{s \to 0} 1/(1/ s) (1 + G(s))$$

Define the velocity error constant:

$$K_v = \lim_{s \to 0} sG(s)$$

Thus $e_{ss} = 1/K_v$

Parabolic Input

For a unit parabolic input:

$$R(s)=1/(s3)$$

Substituting the value in the equation of Steady State error

$$e_{ss} = \lim_{s \to 0} 1/(s2)*(1+G(s))$$

Define the acceleration error constant:

$$K_a = \lim_{s \to 0} (s2)G(s)$$

$$e_{ss} = 1/K_a$$

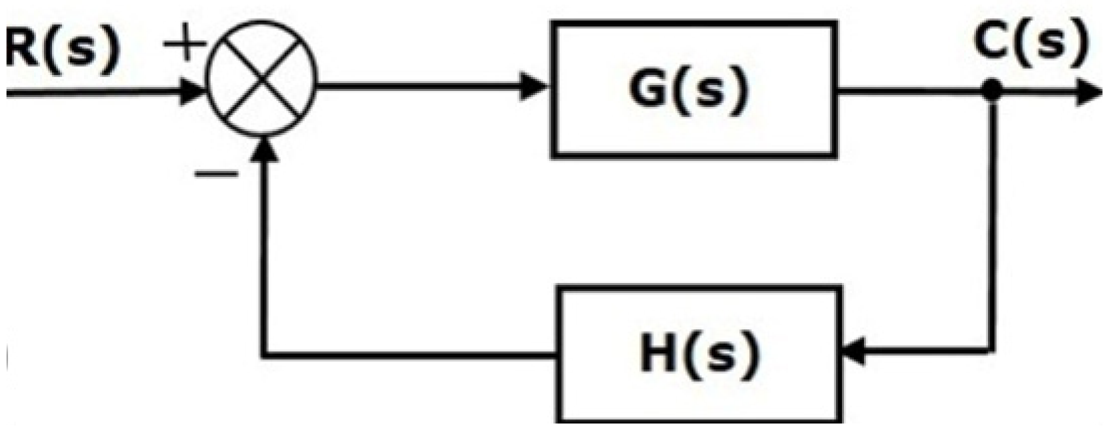
Non-Unity Feedback Control System

A. System Description

A general non-unity feedback control system is shown conceptually as:

- Forward path transfer function: G(s)
- Feedback path transfer function: H(s)

The error signal is given by:



$$E(s)=R(s)-B(s)=R(s)-H(s)C(s)$$

Relationship between Steady-State Error and Closed-Loop Transfer

The steady-state error of a closed-loop system was related to the forward path transfer function G(s) of the system, which is usually known. Often, the closed-loo transfer function is derived in the analysis process, and it would be ot interest to establish the relationships between the steady-state error and the coefficients ot the closed-loog transfer function. As it turns out, the closed-loop transfer function can be used to find the steady-state error of systems with unity as well as non unity feedback. For the present discussion, let us impose the following condition:

$$\lim_{s \to 0} H(s) = H(0) = K_h = \text{Constant}$$

It is only possible when H(s) doesn’t have type zero system.

When the feedback signal compared with the input is:

feedback=Kh*(steady-state output)

To make steady-state error zero when feedback equals input, they redefine the reference signal as:

$$R(t)/Kh$$

Therefore error in time domain is:

$$e(t)=(R(t)/Kh)-Y(t) \quad ....(ii)$$

- The feedback path has gain Kh.
- The signal compared at the summing junction is Kh*y(t).
- To normalize the comparison (so we can use the standard form), they divide the reference by Kh.

This makes the analysis simpler and includes the unity-feedback case (Kh=1)as a special case.

Now in frequency domain

$$E(s) = R(s) / Kh - Y(s) \quad ....(iii)$$

We know that close loop transfer function is given by ,

$$T(s)=Y(s)/R(s)$$

Substituting the value of Y(s) in terms of input and transfer function we in eqn.(iii) we get

$$E(s) = (1/Kh)*(1-Kh*T(s))*R(s)$$

Therfore

$$ess = \lim_{s \to 0} (1/Kh)(1-Kh*T(s))*R(s)$$

This is the final equation of the steady-state error of system which has non unity gain.

Conclusion

In this paper, we carefully derived the steady-state error for a general feedback control system with a feedback gain KHK_HKH. By redefining the reference signal and expressing the error in a normalized form, we obtained a general expression that also includes the unity-feedback case as a special situation. Using Laplace transform techniques and the Final Value Theorem, we showed that the steady-state error depends mainly on the low-frequency behavior of the closed-loop transfer function M(s).

The analysis also highlighted the importance of system stability and the requirement that no poles exist at the origin for the steady-state error calculation to be valid. Overall, this derivation provides a clear and systematic way to understand how feedback structure and system dynamics influence steady-state accuracy. These results form an important foundation for evaluating system performance and designing controllers that meet desired accuracy requirements.

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