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Analog of a Compact Calabi-Yau Manifold based on the Algebra of Signatures

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Abstract
A multidimensional Ricci-flat space is proposed, in which additional dimensions harmoniously compensate each other's manifestations in accordance with the internal topological structure of a given manifold, i.e., without additional conditions. At the same time, the geometric and topological parameters of such a space, developed within the framework of Algebra the Signature, turned out to be sufficient for the creation of metric-dynamic models of all elementary particles included in the Standard Model [1-10]. In particular, it is possible to geometrize such concepts as charge, spin, inertial mass, colors and confinement of the quarks, and also propose ways of metric-dynamically substantiating the nature of gravity, dark matter and energy, etc. In connection with these possibilities of the multidimensional geometry based on the Algebra of signature, the multidimensional Ricci-flat space generated by it can be proposed as an alternative to the Calabi-Yau manifold used in superstring theory.

Keywords: Multidimensional Theory, Multidimensional Ricci-Flat Space, Calabi-Yau Mani Fold, Fabric of Space

Introduction
For the geometric and topological description of many elementary particles comprising the Standard Model, the properties of the 4-dimensional Minkowski spacetime proved insufficient. One approach to introducing additional spatial parameters involves increasing the number of dimensions.

In particular, within superstring theory, each point in 3-dimensional space is associated with a compact, Ricci-flat Calabi-Yau manifold, with three complex (or six spatial) extra dimensions curled up at very small scales (~ the Planck length). Similarly, *M*-theory adds seven spatial coordinates.

As a result, the properties of the elementary particles of the Standard Model (their types, masses, and interactions) are "encoded" in the geometric and topological structure of the Calabi-Yau manifold with the special holonomy SU(*n*). The Ricci tensor of a Calabi-Yau manifold is zero ($R_{ij} = 0$) this is the zero-energy-momentum condition of the background geometry.

In superstring theory, force-carrying particles (photons, gluons, and W/Z bosons) "live" on so-called bundles over a Calabi-Yau manifold. Quarks and leptons correspond to the zero modes of the Dirac operator on the Calabi-Yau manifold. Their number (the number of particle generations) is determined by the difference in the Euler numbers of its two natural submanifolds. The values of the Yukawa couplings (which determine the fermion masses) and gauge interaction constants are calculated using integrals of differential forms over cycles in the Calabi-Yau manifold.

Despite the fact that superstring theory has brought us significantly closer to understanding the structural organization of the microworld and led to major breakthroughs in the development of mathematics, it has significant shortcomings: the inability to directly experimentally verify the phenomena it predicts. This is due to the extremely high energies required to observe string behavior, a billion billion times greater than the energy available in modern experiments:
- the complexity of abstract mathematics, with some key elements of string and superstring theories not yet having a rigorous mathematical foundation. For example, to "fix" the shape (the size of the "holes") of Calabi-Yau manifolds, additional mechanisms (flux compactifications, instanton corrections), etc. are needed;
- approximately 10^{500} different Calabi-Yau manifolds are known (the landscape problem). Each Calabi-Yau manifold generates its own low-energy physics. This makes it extremely difficult to choose a single solution that would correspond to our observations and raises doubts about the ability of this theory to unambiguously describe the real world, etc.

This article considers an alternative method for adding extra dimensions while maintaining the condition $R_{ij} = 0$, which was proposed in the Algebra of signature [1-4]. Such a harmonious expansion of the possibilities of geometry and topology makes it possible to construct metric-dynamic models of all elementary particles included in the Standard Model and to develop a hierarchical cosmological model, i.e. to form model ideas about the entire Universe as a whole [5-13].

Materials and Method Superposition of Metric Spaces with Different Signatures (i.e., Topologies)

The Algebra of signature developed in suggests considering each point in space, for example, point *O*, as the intersection place of 16 metric spaces [1,3,4].

$$
\begin{array}{ll}
ds^{(++++)2} = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 & ds^{(----)2} = -dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \\
ds^{(---+)2} = -dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(+++-)2} = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(+--+)2} = dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(-++-)2} = -dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 & ds^{(-+++)2} = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \\
ds^{(--+-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(++-+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
ds^{(-+--)2} = -dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 & ds^{(+-++)2} = dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 \\
ds^{(+-+-)2} = dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(-+-+)2} = -dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
ds^{(++--)2} = dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 & ds^{(--++)2} = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2
\end{array}
\tag{1}
$$

with the corresponding 16 possible signatures forming an antisymmetric matrix

$$
sign\left(ds^{(a,b)2}\right) = \begin{pmatrix}
(++++) & (+++-) & (-++-) & (++-+) \\
(---+) & (-+++) & (--++) & (-+-+) \\
(+--+) & (++--) & (+---) & (+-++) \\
(--+-) & (+-+-) & (-+--) & (----)
\end{pmatrix}.
\tag{2}
$$

According to Felix Klein's classification [14], metric spaces with metrics (1) can be divided into three topological types:

Type 1: Metric 4-spaces whose signatures consist of four identical signs [14]:

$$
\begin{array}{ll}
x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 & (++++) \\
-x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 & (----)
\end{array}
\tag{3}
$$

these are the so-called null metric 4-spaces. These "spaces" have only one real point, located at the origin of the light cone. All other points in these extensions are imaginary. Essentially, the first Expression (3) describes not an "extension," but a single point (or "white" point), and the second de-scribes a single antipoint (or "black" point).

Type 2: Metric 4-spaces whose signatures consist of two positive and two negative signs [14]:

$$
\begin{array}{ll}
x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 & (+--+) \\
x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0 & (++--) \\
x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0 & (+-+-) \\
-x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0 & (-++-) \\
-x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0 & (--++) \\
-x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0 & (-+-+)
\end{array}
\tag{4}
$$

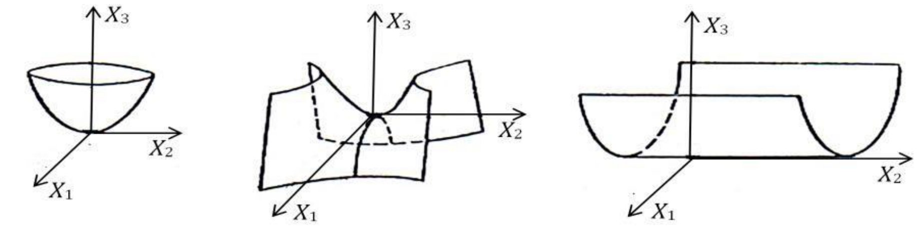
these are different variants of 4-dimensional tori.

Type 3: Metric 4-spaces, whose signatures consist of three identical signs and one opposite sign [14]:

$$\begin{array}{ll} -x_0^2-x_1^2-x_2^2+x_3^2=0 & (---+) \\ -x_0^2-x_1^2+x_2^2-x_3^2=0 & (--+-) \\ -x_0^2+x_1^2-x_2^2-x_3^2=0 & (-+--) \\ x_0^2-x_1^2-x_2^2-x_3^2=0 & (+---) \\ x_0^2+x_1^2+x_2^2-x_3^2=0 & (+++-) \\ x_0^2+x_1^2-x_2^2+x_3^2=0 & (++-+) \\ x_0^2-x_1^2+x_2^2+x_3^2=0 & (+-++) \\ -x_0^2+x_1^2+x_2^2+x_3^2=0 & (-+++) \end{array} \quad (5)$$

these are oval 4-surfaces: ellipsoids, elliptical paraboloids, and two-sheeted hyperboloids.

A simplified illustration of the relationship between the signature of a 2-dimensional space and its topology is shown in Figure 1. This figure shows that the signature of a quadratic form is uniquely related to the topology of a 2-dimensional extension, but not vice versa. The topology of an extension is a much more comprehensive concept than the signature of its metric.



a) signature (+ +) $x_3 = x_1^2 + x_2^2$ parabolic surface; *b*) signature (− +) $x_3 = x_2^2 - x_1^2$ saddle surface; *c*) signature (+ 0) $x_3 = x_1^2$ U-shaped surface

Figure 1: Illustration of the Connection between the Signature of a 2-Dimensional Space and its Topology [14]

Splitting zero

The sum of all 16 metrics (1) is zero:

$$\begin{aligned} ds_\Sigma^2 = {} & ds^{(+---)2} + ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + \\ & + ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2} + \\ & + ds^{(-+++)2} + ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + \\ & + ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2} = 0. \end{aligned} \quad (6)$$

Indeed, adding up the metrics (1), we obtain

$$\begin{aligned} ds_\Sigma^2 = {} & (dx_0dx_0 - dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 + dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + \\ & + (-dx_0dx_0 - dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 - dx_1dx_1 - dx_2dx_2 + dx_3dx_3) + \\ & + (-dx_0dx_0 - dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 + dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + \\ & + (-dx_0dx_0 + dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 - dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + \\ & + (-dx_0dx_0 + dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + (-dx_0dx_0 - dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + \\ & + (dx_0dx_0 + dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + (-dx_0dx_0 + dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + \\ & + (dx_0dx_0 + dx_1dx_1 - dx_2dx_2 + dx_3dx_3) + (-dx_0dx_0 - dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + \\ & + (dx_0dx_0 - dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + (-dx_0dx_0 + dx_1dx_1 - dx_2dx_2 + dx_3dx_3) = 0. \end{aligned} \quad (7)$$

Instead of summing the homogeneous terms in Ex. (7), we can sum only the signs in front of these terms:

$$
\begin{array}{lcccr}
0 = & \underline{(0\ \ 0\ \ 0\ \ 0)} & + & \underline{(0\ \ 0\ \ 0\ \ 0)} & = 0 \\
0 = & (+\ \ +\ \ +\ \ +) & + & (-\ \ -\ \ -\ \ -) & = 0 \\
0 = & (-\ \ -\ \ -\ \ +) & + & (+\ \ +\ \ +\ \ -) & = 0 \\
0 = & (+\ \ -\ \ -\ \ +) & + & (-\ \ +\ \ +\ \ -) & = 0 \\
0 = & (-\ \ -\ \ +\ \ -) & + & (+\ \ +\ \ -\ \ +) & = 0 \\
0 = & (+\ \ +\ \ -\ \ -) & + & (-\ \ -\ \ +\ \ +) & = 0 \\
0 = & (-\ \ +\ \ -\ \ -) & + & (+\ \ -\ \ +\ \ +) & = 0 \\
0 = & (+\ \ -\ \ +\ \ -) & + & (-\ \ +\ \ -\ \ +) & = 0 \\
0 = & \underline{(-\ \ +\ \ +\ \ +)} & + & \underline{(+\ \ -\ \ -\ \ -)} & = 0 \\
0 = & (0\ \ 0\ \ 0\ \ 0)_+ & + & (0\ \ 0\ \ 0\ \ 0)_+ & = 0.
\end{array}
\tag{8}
$$

Operations on signatures of type (8) will be called topological rankings, where the summation (or subtraction) of the signs (+) and (–) is performed both by columns and rows according to the rules of arithmetic:

$$(+) + (+) = 2(+), \quad (-) + (+) = (0), \qquad (+) - (+) = (0), \quad (-) - (+) = 2(-), \tag{9}$$

$$(+) + (-) = (0), \quad (-) + (-) = 2(-), \qquad (+) - (-) = 2(+), \quad (-) - (-) = (0). \tag{10}$$

The sum of the signs in the topological rankings (8), both across columns and across rows between rankings, is zero. Therefore, we will call this ranking identity "splitting zero." At the same time, the topological Ex. (8) is a mathematical expression of "zero vacuum balance."

Any row from the numerators of rankings (8) can be transferred to their denominators, inverting the signs and maintaining row-by-row zero vacuum balance. For example, let's transfer the fourth row from the top of the numerator of rankings (8) to their denominator:

$$
\begin{array}{lcccr}
0 = & (+\ \ +\ \ +\ \ +) & + & (-\ \ -\ \ -\ \ -) & = 0 \\
0 = & (-\ \ -\ \ -\ \ +) & + & (+\ \ +\ \ +\ \ -) & = 0 \\
0 = & (+\ \ -\ \ -\ \ +) & + & (-\ \ +\ \ +\ \ -) & = 0 \\
0 = & (+\ \ +\ \ -\ \ -) & + & (-\ \ -\ \ +\ \ +) & = 0 \\
0 = & (-\ \ +\ \ -\ \ -) & + & (+\ \ -\ \ +\ \ +) & = 0 \\
0 = & (+\ \ -\ \ +\ \ -) & + & (-\ \ +\ \ -\ \ +) & = 0 \\
0 = & \underline{(-\ \ +\ \ +\ \ +)} & + & \underline{(+\ \ -\ \ -\ \ -)} & = 0 \\
0 = & (+\ \ +\ \ -\ \ +)_+ & + & (-\ \ -\ \ +\ \ -)_+ & = 0.
\end{array}
\tag{11}
$$

The topological ranking expression (11) shows that each pair of mutually opposite 4-metrics (i.e., quadratic forms), for example, $ds^{(+\,+\,-\,+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2$ and $ds^{(-\,-\,+\,-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2$, can be expressed by the additive superposition (i.e., addition) of 7 + 7 = 14 4-submetrics with signatures from the numerators of topological rankings of the form (11). All other mutually opposite pairs of 4-metrics in rankings (8) can also be expressed through the sums of 7 + 7 = 14 4-submetrics with the corresponding signatures.

In turn, any mutually opposite pairs of 4-submetrics with signatures (8) can also be expressed as sums of 7 + 7 = 14 4-sub-submetrics with corresponding signatures. This "deepening" can continue ad infinitum.

Thus, at each point in the void (including the point O under study), an additive superposition of an infinite number of metric 4-spaces with 16 types of signatures (i.e., topologies) occurs, including two types of points (3), six types of tori (4), and eight types of oval surfaces (5), which, on average, cancel each other out (i.e., the "zero vacuum balance" condition is satisfied).

The set of all such points leads to the formation of a 3-dimensional zero Ricci-flat space (manifold), since the zero topological node (8) at each point of such a space leads to the total metric (6) $ds_\Sigma^2 = 0 + 0 + 0 + 0 = 0$, which is the trivial solution of the Einstein vacuum equation $R_{ij} = 0$.

Other Possible Operations with Topological Rankings

The topological "splitting of zero" (i.e., ranking expression) (8) allows certain operations to be performed in the neighborhood of each point O under study without violating the "zero vacuum balance."

Such operations include, for example, symmetrically transferring the first and fifth columns in Ex. (8) to the other side of the equation with sign inversion, while maintaining row-by-row and ordinal equality to zero:

(12)

$$
\begin{array}{lccccl}
0 = & \underline{(0\ \ 0\ \ 0)} & + & \underline{(0\ \ 0\ \ 0)} & = 0 & \\
- = & (+\ +\ +) & + & (-\ -\ -) & = + & \to 0 \\
+ = & (-\ -\ +) & + & (+\ +\ -) & = - & \to 0 \\
- = & (-\ -\ +) & + & (+\ +\ -) & = + & \to 0 \\
+ = & (-\ +\ -) & + & (+\ -\ +) & = - & \to 0 \\
- = & (+\ -\ -) & + & (-\ +\ +) & = + & \to 0 \\
+ = & (+\ -\ -) & + & (-\ +\ +) & = - & \to 0 \\
- = & (-\ +\ -) & + & (+\ -\ +) & = + & \to 0 \\
+ = & \underline{(+\ +\ +)} & + & \underline{(-\ -\ -)} & = - & \to 0 \\
0 = & (0\ \ 0\ \ 0)_+ & + & (0\ \ 0\ \ 0)_+ & = 0. &
\end{array}
$$

Similarly, any corresponding (i.e., mutually opposite) columns of the ranking expression (8) can be symmetrically transferred to the other side of the equation.

Mixed row and column transfer operations are also possible in topological rankings (8), which do not violate the row-by-row equality to zero, for example:

(13)

$$
\begin{array}{lccccl}
- = & (+\ +\ +) & + & (-\ -\ -) & = + & \to 0 \\
- = & (-\ -\ -) & + & (+\ +\ +) & = + & \to 0 \\
- = & (+\ -\ -) & + & (-\ +\ +) & = + & \to 0 \\
+ = & (+\ +\ -) & + & (-\ -\ +) & = - & \to 0 \\
+ = & (-\ +\ -) & + & (+\ -\ +) & = - & \to 0 \\
+ = & (+\ -\ +) & + & (-\ +\ -) & = - & \to 0 \\
\underline{- =} & \underline{(-\ +\ +)} & + & \underline{(+\ -\ -)} & \underline{= +} & \underline{\to 0} \\
- = & (+\ +\ -)_+ & + & (-\ -\ +)_+ & = + & \to 0.
\end{array}
$$

Such operations correspond to different vacuum symmetries.

Four-Strand Fabric of Space

Let's transfer the signatures (− + + +) and (+ − − −) from the numerators of the topological rankings (8) to their denominators.

(14)

$$
\begin{array}{lcll}
(+\ +\ +\ +) & + & (-\ -\ -\ -) & = 0 \\
(-\ -\ -\ +) & + & (+\ +\ +\ -) & = 0 \\
(+\ -\ -\ +) & + & (-\ +\ +\ -) & = 0 \\
(-\ -\ +\ -) & + & (+\ +\ -\ +) & = 0 \\
(+\ +\ -\ -) & + & (-\ -\ +\ +) & = 0 \\
(-\ +\ -\ -) & + & (+\ -\ +\ +) & = 0 \\
\underline{(+\ -\ +\ -)} & + & \underline{(-\ +\ -\ +)} & = 0 \\
(+\ -\ -\ -)_+ & + & (-\ +\ +\ +)_+ & = 0.
\end{array}
$$

In expanded shape, the rankings (14) have the following form

(15)

$$
\begin{array}{ll}
ds^{(++++)2} = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 & ds^{(----)2} = -dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \\
ds^{(---+)2} = -dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(+++-)2} = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(+--+)2} = dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(-++-)2} = -dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(--+-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(++-+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
ds^{(-+--)2} = -dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 & ds^{(+-++)2} = dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 \\
ds^{(+-+-)2} = dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(-+-+)2} = -dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
\underline{ds^{(++--)2} = dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2} & \underline{ds^{(--++)2} = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2} \\
ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 & ds^{(-+++)2} = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2
\end{array}
$$

The ranking Ex. (14) is equivalent to the fact that the addition (i.e. additive superposition) of 7 metric spaces with signatures (topologies) specified in the numerator of the left ranking (14) (or (15)), forms a metric Minkowski 4-space with the metric

$$ds^{(+---)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = dx_0{}^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2 \quad \text{with signature } (+---), \qquad (16)$$

where $ds^{(+---)2} = ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2}$.

Similarly, the additive superposition of 7-metric spaces with signatures specified in the numerator of the right rank (14) (or (15)) forms a metric Minkowski 4-antispace with the metric

$$ds^{(-+++)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = -dx_0{}^2 + dx_1{}^2 + dx_2{}^2 + dx_3{}^2 \quad \text{with signature } (-+++), \qquad (17)$$

where $ds^{(-+++)2} = ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2}$.

In this case, the topological zero vacuum balance is not disturbed: (+ − − −) + (− + + +) = (0 0 0 0), or in transposed form

$$\begin{array}{c} (+\,-\,-\,-) \\ \underline{(-\,+\,+\,+)} \\ (0\;0\;\;0\;0)_+ \end{array} \qquad (18)$$

Let's recall that in Einstein's general theory of relativity (GTR), there is only one metric 4-space with a signature, for example, (+ − − −). However, in the theory of physical space developed here, it is the intersection (i.e., intertwining) of at least two 4-dimensional manifolds with mutually opposite signatures (+ − − −) and (− + + +). Moreover, at the next (second) level of scattering, it is necessary to consider the intersection (i.e., intertwining) of 16 metric spaces (1) with signatures (topologies) (2).

The intertwining of these two mutually opposite spaces is due to the fact that the result of averaging metrics (16) and (17)

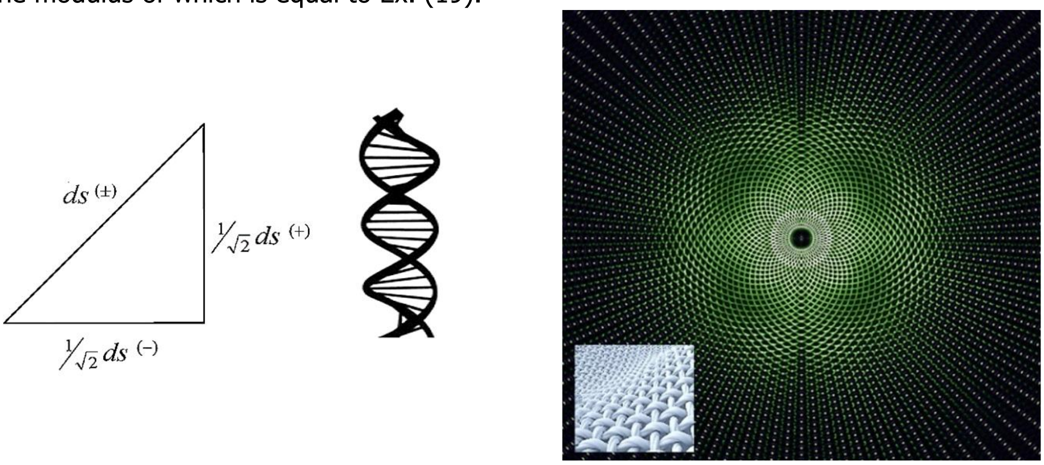
$$ds^{(\pm)2} = \tfrac{1}{2}\left(ds^{(+---)\,2} + ds^{(-+++)\,2}\right) = \tfrac{1}{2}\left(ds^{(+)2} + ds^{(-)2}\right), \qquad (19)$$

$$ds^{(\pm)2} = \tfrac{1}{2}\left(ds^{(+---)\,2} + ds^{(-+++)\,2}\right) = \tfrac{1}{2}\left(ds^{(+)2} + ds^{(-)2}\right),$$

resembles the Pythagorean theorem $c^2 = a^2 + b^2$. This means that the line segments $(\tfrac{1}{2})^{1/2}ds^{(+)}$ and $(\tfrac{1}{2})^{1/2}ds^{(-)}$ are always mutually perpendicular to each other $ds^{(+)} \perp ds^{(-)}$ (see Figure 2a), i.e. they form a double helix (see Figure 2b)

$$ds^{(\pm)} = \frac{1}{\sqrt{2}}\left(ds^{(+)} + i\,ds^{(-)}\right), \qquad (20)$$

the square of the modulus of which is equal to Ex. (19).



***a*)** ***b*)** ***c*)**

Figure 2: *a*) Mutually perpendicular segments $(\tfrac{1}{2})^{1/2}ds^{(+)}$ and $(\tfrac{1}{2})^{1/2}ds^{(-)}$ are the sides of a right triangle; *b*) The lines of a regular double helix are mutually perpendicular $ds^{(+)} \perp ds^{(-)}$; *c*) Illustration of the fabric of space

We represent metrics (16) and (17) in the following form:

$$\begin{cases} ds_0^{(+)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = ds^{(+)\prime}ds^{(+)\prime\prime} = cdt'cdt'' - dx'dx'' - dy'dy'' - dz'dz''; \\ ds_0^{(-)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = ds^{(-)\prime}ds^{(-)\prime\prime} = -cdt'cdt'' + dx'dx'' + dy'dy'' + dz'dz''. \end{cases} \quad (21)$$

where the segments of the four lines (i.e., "threads") $ds^{(+)\prime}, ds^{(+)\prime\prime}, ds^{(-)\prime}, ds^{(-)\prime\prime}$ are mutually perpendicular

$$ds^{(+)\prime} \perp ds^{(+)\prime\prime} \perp ds^{(-)\prime} \perp ds^{(-)\prime\prime} \quad (22)$$

and form spirals, described by complex numbers

$$ds^{(+,+)} = \frac{1}{\sqrt{2}}(ds^{(+)\prime} + ids^{(+)\prime\prime}), \quad (23)$$
$$ds^{(-,-)} = \frac{1}{\sqrt{2}}(ds^{(-)\prime} + jds^{(-)\prime\prime}),$$
$$ds^{(+,-)} = \frac{1}{\sqrt{2}}(ds^{(+)\prime} + kds^{(-)\prime}),$$
$$ds^{(-,+)} = \frac{1}{\sqrt{2}}(ds^{(+)\prime\prime} + lds^{(-)\prime\prime}),$$

where i, j, k, l are imaginary units.

The interweaving of these spirals forms a four-thread fabric of space (see Figure 2*c*).

On the one hand, such a 4-thread fabric of space is absent, since metric spaces with metrics (16) and (17) completely compensate for each other's manifestations

$$ds^{(+---)2} + ds^{(-+++)2} = (c^2dt^2 - dx^2 - dy^2 - dz^2) + (-c^2dt^2 + dx^2 + dy^2 + dz^2) = 0c^2dt^2 + 0dx^2 + 0dy^2 + 0dz^2 = 0, \quad (23)$$

on the other hand, the segments of their geodesic lines $ds^{(+)\prime}, ds^{(+)\prime\prime}, ds^{(-)\prime}, ds^{(-)\prime\prime}$ are mutually perpendicular, which on the other hand, the segments of their geodesic lines creates a stable illusion of their constant coexistence in the form of a 3-meter grid (Figure 2c), each cell of which changes (rotates) in a whimsical manner with a frequency

$$f = 1/dt.$$

That is, the 4-thread fabric of space under consideration is not a static 3-dimensional network, but a ubiquitously constantly seething extension, each local cubic cell of which constantly rotates bizarrely with a frequency that determines the unit of its characteristic time $dt = 1/f$. Moreover, the rhythmic fluctuations of all cubic cells of the seething fabric of space are so bizarre that all 8 of their triple angles rotate in mutually opposite directions with respect to the adjacent triple angles of neighboring cubic cells (see Figure 3) (see §3 in [1]), so as not to disturb the complete "vacuum balance".

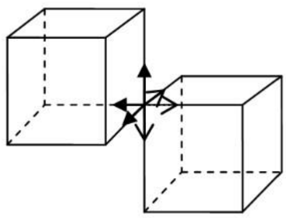


Fig. 3: The triangular angles of adjacent cubic cells rotate in mutually opposite directions (i.e., clockwise and counter-clockwise)

Sixteen-Thread Fabric of Space

At the next deeper level of consideration, the number of dimensions is 16 × 4 = 64 = 2^6. The metric-dynamic properties of a local vacuum region are characterized by the superposition (i.e., additive superposition or averaging) of sixteen metrics (1) with all 16 possible signatures (2).

$$
\begin{aligned}
ds_\Sigma{}^2 = 1/16\,(& ds^{(+---)2} + ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + \\
& + ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2} + \\
& + ds^{(-+++)2} + ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + \\
& + ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2}).
\end{aligned} \tag{24}
$$

In this case, the fabric of space is woven from 16 threads

$$
\begin{aligned}
ds'_{(16)} = 1/\sqrt{16}\,(& \eta_1\, ds^{(+---)'} + \eta_2\, ds^{(++++)'} + \eta_3\, ds^{(---+)'} + \eta_4\, ds^{(+--+)'} + \\
& + \eta_5\, ds^{(--+-)'} + \eta_6\, ds^{(++--)'} + \eta_7\, ds^{(-+--)'} + \eta_8\, ds^{(+-+-)'} + \\
& + \eta_9\, ds^{(-+++)'} + \eta_{10}\, ds^{(----)'} + \eta_{11}\, ds^{(+++-)'} + \eta_{12}\, ds^{(-++-)'} + \\
& + \eta_{13}\, ds^{(++-+)'} + \eta_{14}\, ds^{(--++)'} + \eta_{15}\, ds^{(+-++)'} + \eta_{16}\, ds^{(-+-+)'}),
\end{aligned} \tag{25}
$$

and 16 antistrands

$$
\begin{aligned}
ds''_{(16)} = 1/\sqrt{16}\,(& \eta_1\, ds^{(+---)''} + \eta_2\, ds^{(++++)''} + \eta_3\, ds^{(---+)''} + \eta_4\, ds^{(+--+)''} + \\
& + \eta_5\, ds^{(--+-)''} + \eta_6\, ds^{(++--)''} + \eta_7\, ds^{(-+--)''} + \eta_8\, ds^{(+-+-)''} + \\
& + \eta_9\, ds^{(-+++)''} + \eta_{10}\, ds^{(----)''} + \eta_{11}\, ds^{(+++-)''} + \eta_{12}\, ds^{(-++-)'} + \\
& + \eta_{13}\, ds^{(++-+)''} + \eta_{14}\, ds^{(--++)''} + \eta_{15}\, ds^{(+-++)''} + \eta_{16}\, ds^{(-+-+)''}).
\end{aligned} \tag{26}
$$

where η_m is an orthonormal basis of 16 unit objects (m = 1,2,3,…,16) of the imaginary unit type, satisfying the anticommutation relation of the Clifford algebra.

$$
\eta_m\eta_n + \eta_n\eta_m = 2\delta_{mn}\,, \tag{27}
$$

where δ_{nm} is the 16×16 identity matrix.

Thus, at this level of consideration, the nodes and bundles of the fabric of space consist of 16 × 2 = 32 intertwined lines (threads) (25) and antilines (i.e., antithreads) (26), which are conventionally assigned "colors" according to their signatures:

(28)

Red	(+ + + +)	+	(– – – –)	Anti-red
Yellow	(– – – +)	+	(+ + + –)	Anti-yellow
Orange	(+ – – +)	+	(– + + –)	Anti-orange
Green	(– – + –)	+	(+ + – +)	Anti-green
Blue	(+ + – –)	+	(– – + +)	Anti-blue
Dark blue	(– + – –)	+	(+ – + +)	Anti-dark blue
Violet	(+ – + –)	+	(– + – +)	Anti-violet
White	$(+---)_+$	+	$(-+++)_+$	Black

Formal coloring of the 32 threads and antithreads $ds^{(+---)}$, $ds^{(-+++)}$, $ds^{(++++)}$, $ds^{(++-+)}$, … , $ds^{(-+-+)}$ allows us to represent the fabric of space woven from them in colored form (see Figure 4).

In a flat multidimensional space (including when $R_{ij} = 0$), such a "colored" fabric, on the one hand, can be considered present, and, on the other hand, it is absent, since the sum of all 16 "colored" axial lines (threads) is zero

$$
\begin{aligned}
ds_\Sigma = & (-dx_0 - dx_1 - dx_2 - dx_3) + (dx_0 + dx_1 + dx_2 + dx_3) + \\
& + (dx_0 + dx_1 + dx_2 - dx_3) + (-dx_0 - dx_1 - dx_2 + dx_3) + \\
& + (-dx_0 + dx_1 + dx_2 - dx_3) + (dx_0 - dx_1 - dx_2 + dx_3) + \\
& + (dx_0 + dx_1 - dx_2 + dx_3) + (-dx_0 - dx_1 + dx_2 - dx_3) + \\
& + (-dx_0 - dx_1 + dx_2 + dx_3) + (dx_0 + dx_1 - dx_2 - dx_3) + \\
& + (dx_0 - dx_1 + dx_2 + dx_3) + (-dx_0 + dx_1 - dx_2 - dx_3) + \\
& + (-dx_0 + dx_1 - dx_2 + dx_3) + (dx_0 - dx_1 + dx_2 - dx_3) + \\
& + (dx_0 - dx_1 - dx_2 - dx_3) + (-dx_0 + dx_1 + dx_2 + dx_3) = 0.
\end{aligned} \tag{29}
$$

In other words, in this case there is no need to impose any additional conditions on the folding of extra dimensions, since they are rolled up naturally due to their mutual compensation.

"Colored" Affinors

An orthonormal basis of 16 unit objects η_m satisfying the requirements of the Clifford algebra (27) can be obtained as follows.

Consider a metric with signature $(+---)$

$$ds^{(+---)2} = dx_0{}^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2.$$

For brevity, we omit the signs of the differentials in this metric.

$$s^{(+---)2} = x_0{}^2 - x_1{}^2 - x_2{}^2 - x_3{}^2. \quad (30)$$

The quadratic form (30), as is known, is the determinant of a Hermitian 2×2-matrix. The quadratic form (30), as is known, is the determinant of a Hermitian 2×2-matrix

$$\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}_{det} = \begin{vmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{vmatrix} = x_0^2 - x_1^2 - x_2^2 - x_3^2 = s^{(+---)2} \text{ with signature } (+---). \quad (31)$$

That this matrix is Hermitian can easily be verified by direct calculation:

$$\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}^+ = \begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}. \quad (32)$$

In spinor theory, matrices of the form (26) are called mixed Hermitian spin tensors of second rank [6].

Let's represent the 2×2-matrix (26) in expanded form

$$A_4 = \begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix} = x_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1 \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (33)$$

where $\sigma_0^{(+---)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $\sigma_1^{(+---)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$; $\sigma_2^{(+---)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$; $\sigma_3^{(+---)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

is set of Pauli-Cayley matrices.

In spinor theory, A_4-matrices (i.e., affinors) of the form (27) are assigned one-to-one correspondence to quaternions of the type

$$q = x_0 + \vec{e}_1 x_1 + \vec{e}_2 x_2 + \vec{e}_3 x_3, \quad (34)$$

under isomorphism

$$\vec{e}_1 \to \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \quad \vec{e}_2 \to \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \vec{e}_3 \to \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (35)$$

Above we present only a special case of the spintensor representation of the quadratic form (25); upon closer examination, $s^{(+---)2} = x_0^2 - x_1^2 - x_2^2 - x_3^2$ is the determinant of all the 2×2-matrices (Hermitian spintensors) given below:

$$\begin{pmatrix} x_0+x_3 & x_1+ix_2 \\ x_1-ix_2 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_1-ix_2 \\ x_1+ix_2 & x_0+x_3 \end{pmatrix} \begin{pmatrix} x_0+x_3 & x_1-ix_2 \\ x_1+ix_2 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_1+ix_2 \\ x_1-ix_2 & x_0+x_3 \end{pmatrix} \begin{pmatrix} ix_1-x_2 & -x_0+x_3 \\ x_0+x_3 & ix_1+x_2 \end{pmatrix} \quad (36)$$

$$\begin{pmatrix} x_0+x_1 & x_3+ix_2 \\ x_3-ix_2 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_3-ix_2 \\ x_3+ix_2 & x_0+x_1 \end{pmatrix} \begin{pmatrix} x_0+x_1 & x_3-ix_2 \\ x_3+ix_2 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_3+ix_2 \\ x_3-ix_2 & x_0+x_1 \end{pmatrix} \begin{pmatrix} ix_2-x_1 & -x_0+x_3 \\ x_0+x_3 & ix_2+x_1 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_2 & x_1+ix_3 \\ x_1-ix_3 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_1-ix_3 \\ x_1+ix_3 & x_0+x_2 \end{pmatrix} \begin{pmatrix} x_0+x_2 & x_1-ix_3 \\ x_1+ix_3 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_1+ix_3 \\ x_1-ix_3 & x_0+x_2 \end{pmatrix} \begin{pmatrix} ix_1-x_3 & -x_0+x_2 \\ x_0+x_2 & ix_1+x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_3 & x_2+ix_1 \\ x_2-ix_1 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_2-ix_1 \\ x_2+ix_1 & x_0+x_3 \end{pmatrix} \begin{pmatrix} x_0+x_3 & x_2-ix_1 \\ x_2+ix_1 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_2+ix_1 \\ x_2-ix_1 & x_0+x_3 \end{pmatrix} \begin{pmatrix} ix_3-x_2 & -x_0+x_1 \\ x_0+x_1 & ix_3+x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_1 & x_2+ix_3 \\ x_2-ix_3 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_2-ix_3 \\ x_2+ix_3 & x_0+x_1 \end{pmatrix} \begin{pmatrix} x_0+x_1 & x_2-ix_3 \\ x_2+ix_3 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_2+ix_3 \\ x_2-ix_3 & x_0+x_1 \end{pmatrix} \begin{pmatrix} ix_2-x_3 & -x_0+x_1 \\ x_0+x_1 & ix_2+x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_2 & x_3+ix_1 \\ x_3-ix_1 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_3-ix_1 \\ x_3+ix_1 & x_0+x_2 \end{pmatrix} \begin{pmatrix} x_0+x_2 & x_3-ix_1 \\ x_3+ix_1 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_3+ix_1 \\ x_3-ix_1 & x_0+x_2 \end{pmatrix} \begin{pmatrix} ix_3-x_1 & -x_0+x_2 \\ x_0+x_2 & ix_3+x_1 \end{pmatrix}$$

$$\begin{pmatrix} ix_2-x_1 & -x_0+x_3 \\ x_0+x_3 & ix_2+x_1 \end{pmatrix} \begin{pmatrix} ix_2-x_1 & x_0+x_3 \\ -x_0+x_3 & ix_2+x_1 \end{pmatrix} \begin{pmatrix} ix_1-x_3 & x_0+x_2 \\ -x_0+x_2 & ix_1+x_3 \end{pmatrix} \begin{pmatrix} ix_2-x_3 & x_0+x_1 \\ -x_0+x_1 & ix_2+x_3 \end{pmatrix} \begin{pmatrix} ix_3-x_1 & x_0+x_2 \\ -x_0+x_2 & ix_3+x_1 \end{pmatrix}$$

Each of the 2×2-matrices (30) can be realized with probability 1/(their total number). Therefore, affinor (27) is statistical in nature; that is, this affinor constantly changes chaotically during a random transition from one Hermitian spintensor (30) to another. Moreover, the complete set of spintensors (30) forms a group.

Similarly, each quadratic form with the corresponding signature (1):

(37)

$$\begin{array}{ll}
ds^{(++++)2} = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 & ds^{(----)2} = -dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \\
ds^{(---+)2} = -dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(+++-)2} = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(+--+)2} = dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(-++-)2} = -dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(--+-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(++-+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
ds^{(-+--)2} = -dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 & ds^{(+-++)2} = dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 \\
ds^{(+-+-)2} = dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(-+-+)2} = -dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
ds^{(++--)2} = dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 & ds^{(--++)2} = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 \\
ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 & ds^{(-+++)2} = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2
\end{array}$$

can be represented as a spin tensor or an A_4-matrix (i.e., affinor), which are shown in Table 1:

Table 1: Spin-tensors and A_4-matrices with different signatures

1	$\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-ix_3 \end{pmatrix}_{det} = x_0^2+x_1^2+x_2^2+x_3^2 = 0$, signature $(++++)$. $\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$; where $\sigma_0^{(++++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $\sigma_1^{(++++)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$; $\sigma_2^{(++++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\sigma_3^{(++++)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$.
2	$\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-x_3 \end{pmatrix}_{det} = x_0^2+x_1^2+x_2^2-x_3^2 = 0$, signature $(+++-)$. $\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; where $\sigma_0^{(+++-)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $\sigma_1^{(+++-)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$; $\sigma_2^{(+++-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\sigma_3^{(+++-)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.
3	$\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+ix_3 \end{pmatrix}_{det} = -x_0^2+x_1^2+x_2^2-x_3^2 = 0$, signature $(-++-)$. $\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$; where $\sigma_0^{(-++-)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; $\sigma_1^{(-++-)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$; $\sigma_2^{(-++-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\sigma_3^{(-++-)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$.
4	$\begin{pmatrix} x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0-ix_3 \end{pmatrix}_{det} = x_0^2+x_1^2-x_2^2+x_3^2 = 0$, signature $(++-+)$. $\begin{pmatrix} x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0-ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$; where $\sigma_0^{(++-+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $\sigma_1^{(++-+)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\sigma_2^{(++-+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$; $\sigma_3^{(++-+)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$.

5	$\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ -ix_1+x_2 & -x_0+x_3 \end{pmatrix}_{det} = -x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0,$ signature $(- - - +)$. $\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ -ix_1+x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ where $\sigma_0^{(---+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(---+)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(---+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(---+)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$
6	$\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+x_3 \end{pmatrix}_{det} = -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0,$ signature $(- + + +)$. $\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ where $\sigma_0^{(-+++)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(-+++)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \sigma_2^{(-+++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(-+++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ are the Cayley matrices.
7	$\begin{pmatrix} x_0+x_3 & x_1+x_2 \\ x_1-x_2 & -x_0+x_3 \end{pmatrix}_{det} = -x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0,$ signature $(- - + +)$. $\begin{pmatrix} x_0+x_3 & x_1+x_2 \\ x_1-x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ where $\sigma_0^{(--++)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \sigma_1^{(--++)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \sigma_2^{(--++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \sigma_3^{(--++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
8	$\begin{pmatrix} x_0+x_3 & -x_1+x_2 \\ x_1+x_2 & -x_0+x_3 \end{pmatrix}_{det} = -x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0,$ signature $(- + - +)$. $\begin{pmatrix} x_0+x_3 & -x_1+x_2 \\ x_1+x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ where $\sigma_0^{(-+-+)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(-+-+)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \sigma_2^{(-+-+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(-+-+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$
9	$\begin{pmatrix} x_0-ix_3 & x_1-ix_2 \\ x_1+ix_2 & x_0+ix_3 \end{pmatrix}_{det} = x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0,$ signature $(+ - - +)$. $\begin{pmatrix} x_0-ix_3 & x_1-ix_2 \\ x_1+ix_2 & x_0+ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + x_3\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix};$ where $\sigma_0^{(+--+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(+--+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(+--+)} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}; \sigma_3^{(+--+)} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}.$
10	$\begin{pmatrix} x_0-x_3 & x_1+x_2 \\ -x_1+x_2 & x_0+x_3 \end{pmatrix}_{det} = x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0,$ signature $(+ + - -)$. $\begin{pmatrix} x_0-x_3 & x_1+x_2 \\ -x_1+x_2 & x_0+x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$ where $\sigma_0^{(++--)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(++--)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(++--)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(++--)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$
11	$\begin{pmatrix} x_0+x_3 & x_1+ix_2 \\ x_1-ix_2 & x_0-x_3 \end{pmatrix}_{det} x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0,$ signature $(+ - - -)$. $\begin{pmatrix} x_0+x_3 & x_1+ix_2 \\ x_1-ix_2 & x_0-x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix};$ where $\sigma_0^{(+---)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(+---)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(+---)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3^{(+---)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$

12	$\begin{pmatrix} x_0+ix_3 & x_1+x_2 \\ x_1-x_2 & x_0-ix_3 \end{pmatrix}_{det} = x_0^2-x_1^2+x_2^2+x_3^2=0,$ signature $(+-++)$. $\begin{pmatrix} x_0+ix_3 & x_1+x_2 \\ x_1-x_2 & x_0-ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix};$ where $\sigma_0^{(+-++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(+-++)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(+-++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(+-++)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$
13	$\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ x_1-x_2 & x_0+ix_3 \end{pmatrix}_{det} = -x_0^2-x_1^2+x_2^2-x_3^2=0,$ signature $(--+-)$. $\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ x_1-x_2 & x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ where $\sigma_0^{(--+-)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_1^{(--+-)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(--+-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(--+-)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$
14	$\begin{pmatrix} x_0-x_3 & x_1+x_2 \\ x_1-x_2 & x_0+x_3 \end{pmatrix}_{det} = x_0^2-x_1^2+x_2^2-x_3^2=0,$ signature $(+-+-)$. $\begin{pmatrix} x_0-x_3 & x_1+x_2 \\ x_1-x_2 & x_0+x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$ where $\sigma_0^{(+-+-)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1^{(+-+-)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(+-+-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(+-+-)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$
15	$\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0+ix_3 \end{pmatrix}_{det} = -x_0^2+x_1^2-x_2^2-x_3^2=0,$ signature $(-+--)$. $\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ where $\sigma_0^{(-+--)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_1^{(-+--)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(-+--)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_3^{(-+--)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$
16	$\begin{pmatrix} -x_0+ix_3 & x_1+ix_2 \\ x_1-ix_2 & x_0+ix_3 \end{pmatrix}_{det} = -x_0^2-x_1^2-x_2^2-x_3^2=0,$ signature $(----)$. $\begin{pmatrix} -x_0+ix_3 & x_1+ix_2 \\ x_1-ix_2 & x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ where $\sigma_0^{(----)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_1^{(----)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \sigma_2^{(----)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3^{(----)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$

Each A_4-matrix from the Table 1 is associated with a "colored" quaternion with the corresponding signature (Table 2), where following objects are used as imaginary units

$$\vec{e}_1 \to \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \vec{e}_2 \to \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{e}_3 \to \sigma_3 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \quad \vec{e}_4 \to \sigma_4 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{38}$$

$$\vec{e}_5 \to \sigma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \vec{e}_6 \to \sigma_6 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \vec{e}_7 \to \sigma_7 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \quad \vec{e}_8 \to \sigma_8 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\vec{e}_9 \to \sigma_9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{e}_{10} \to \sigma_{10} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \vec{e}_{11} \to \sigma_{11} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad \vec{e}_{12} \to \sigma_{12} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$\vec{e}_{13} \to \sigma_{13} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{e}_{14} \to \sigma_{14} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \vec{e}_{15} \to \sigma_{15} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \quad \vec{e}_{16} \to \sigma_{16} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix},$$

where σ_{ij} are the Pauli-Cayley spin-matrices, which are generators of the Clifford algebra and satisfy the conditions

$$\sigma_i\sigma_j + \sigma_j\sigma_i = \begin{cases} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ при } i \neq j, \\ 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ при } i = j. \end{cases} \tag{39}$$

We note again that Table 1 contains only special cases of spintensor representations of quadratic forms (31), similar to the complete set of 2×2-matrices (36) for a single quadratic form $s^{(+---)2} = x_0^2 - x_1^2 - x_2^2 - x_3^2$ with signature (+– – –).

For clarity, all types of A_4-matrices (i.e., affinors) and all varieties of "colored" quaternions are summarized in Table 2.

Table 2: Quadratic forms, A_4-matrices and "colored" quaternions

Quadratic form	A_4-matrix	"Colored" quaternion	Stignatur
$ds_1^2 = x_0^2 + x_1^2 + x_2^2 + x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$	$z_1 = x_0 + ix_1 + jx_2 + kx_3$	{+ + + +}
$ds_2^2 = x_0^2 - x_1^2 - x_2^2 + x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + x_3\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$	$z_2 = x_0 - ix_1 - jx_2 + kx_3$	{+ – – +}
$ds_3^2 = x_0^2 + x_1^2 + x_2^2 - x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$z_3 = x_0 + ix_1 + jx_2 - kx_3$	{+ + + –}
$ds_4^2 = x_0^2 + x_1^2 - x_2^2 - x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$z_4 = x_0 + ix_1 - jx_2 - kx_3$	{+ + – –}
$ds_5^2 = -x_0^2 + x_1^2 + x_2^2 - x_3^2$	$-x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$	$z_5 = -x_0 + ix_1 + jx_2 - kx_3$	{– + + –}
$ds_6^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$z_6 = x_0 - ix_1 - jx_2 - kx_3$	{+ – – –}
$ds_7^2 = x_0^2 + x_1^2 - x_2^2 + x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$	$z_7 = x_0 + ix_1 - jx_2 + kx_3$	{+ + – +}
$ds_8^2 = x_0^2 - x_1^2 + x_2^2 + x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$	$z_8 = x_0 - ix_1 + jx_2 + kx_3$	{+ – + +}
$ds_9^2 = -x_0^2 - x_1^2 - x_2^2 + x_3^2$	$-x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$z_9 = -x_0 - ix_1 - jx_2 + kx_3$	{– – – +}
$ds_{10}^2 = -x_0^2 - x_1^2 + x_2^2 - x_3^2$	$-x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$	$z_{10} = -x_0 - ix_1 + jx_2 - kx_3$	{– – + –}
$ds_{11}^2 = -x_0^2 + x_1^2 + x_2^2 + x_3^2$	$-x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$z_{11} = -x_0 + ix_1 + jx_2 + kx_3$	{– + + +}
$ds_{12}^2 = x_0^2 - x_1^2 + x_2^2 - x_3^2$	$x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$z_{12} = x_0 - ix_1 + jx_2 - kx_3$	{+ – + –}
$ds_{13}^2 = -x_0^2 - x_1^2 + x_2^2 + x_3^2$	$-x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$z_{13} = -x_0 - ix_1 + jx_2 + kx_3$	{– – + +}
$ds_{14}^2 = x_0^2 - x_1^2 + x_2^2 + x_3^2$	$-x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$	$z_{14} = -x_0 + ix_1 + jx_2 + kx_3$	{– + – +}
$ds_{15}^2 = -x_0^2 + x_1^2 - x_2^2 + x_3^2$	$-x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$z_{15} = -x_0 + ix_1 - jx_2 - kx_3$	{– + – –}
$ds_{16}^2 = -x_0^2 - x_1^2 - x_2^2 - x_3^2$	$-x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$	$z_{16} = -x_0 - ix_1 - jx_2 - kx_3$	{– – – –}

One of the variants of the state of a local section of the fabric of space is determined by the super-position (i.e., interweaving) of 16 "colored" threads specified by the affinors from Table 2, subject to the maintenance of vacuum balance (i.e., equality to zero):

$$\begin{aligned} & x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + x_1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \\ & + x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + x_3\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} + \\ & + x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \\ & + x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + x_3\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \end{aligned} \tag{40}$$

$$+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&i\end{pmatrix}+$$

$$+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&1\end{pmatrix}+$$

$$+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&-i\end{pmatrix}+$$

$$+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&-i\end{pmatrix}+$$

$$+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&-1\end{pmatrix}+$$

$$+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&i\end{pmatrix}+$$

$$+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&1\end{pmatrix}+$$

$$+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&-1\end{pmatrix}+$$

$$+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_2\begin{pmatrix}0&-i\\-i&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&1\end{pmatrix}+$$

$$+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&-i\end{pmatrix}+$$

$$+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&-1\end{pmatrix}+$$

$$+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&i\\i&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&i\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}.$$

A direct calculation easily shows that the sum of all 16 types of "colored" quaternions from Table 2 is also equal to zero

$$\sum_{k=1}^{16} z_k = 0. \tag{41}$$

As a result, the addition (or averaging) of all types of "colored" quaternions is balanced relative to zero, i.e., satisfies the "vacuum balance" condition.

Using Spin-Tensors with Different Signatures Let's consider two examples using spin-tensors.

Example 1: Let a column matrix and its Hermitian conjugate row matrix be given

$$\begin{pmatrix}s_1\\s_2\end{pmatrix}, (s_1^*, s_2^*), \tag{42}$$

which describe the state of the spinor.

The spin projections on the coordinate axis for the case when the metric 4-space has the signature $(+\,-\,-\,-)$ can be determined using spin-tensor (32) and A_4-matrices (33)

$$(s_1^*, s_2^*)\begin{pmatrix}x_0+x_3 & x_1+ix_2\\ x_1-ix_2 & x_0-x_3\end{pmatrix}\begin{pmatrix}s_1\\s_2\end{pmatrix}= \tag{43}$$

$$=x_0(s_1^*,s_2^*)\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}s_1\\s_2\end{pmatrix}-x_1(s_1^*,s_2^*)\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}s_1\\s_2\end{pmatrix}-x_2(s_1^*,s_2^*)\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}s_1\\s_2\end{pmatrix}-x_3(s_1^*,s_2^*)\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}s_1\\s_2\end{pmatrix}=$$

$$=(s_1^*s_1+s_2^*s_2)x_0-(-s_2^*s_1-s_2^*s_1)x_1-(is_2^*s_1-is_1^*s_2)x_2-(-s_1^*s_1+s_2^*s_2)x_3.$$

Example 2: Let the forward and reverse waves be described by expressions

$$\vec{E}_1^{(+)} = \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)}, \tag{44}$$

$$\vec{E}_2^{(-)} = \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)}, \tag{45}$$

where $a+$ and $a-$ are the amplitudes of the forward and reverse waves. In general, these are complex numbers:

$$\bar{a}_+ = a_+ e^{i\phi_+}, \quad \bar{a}_- = a_- e^{-i\phi_-}, \quad \bar{a}_+^* = a_+ e^{-i\phi_+}, \quad \bar{a}_-^* = a_- e^{i\phi_-}, \tag{46}$$

which contain information about the phases of the waves φ_+ and φ_-.

Mutually opposite waves (44) and (44) can be represented as a two-component spinor:

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = |\psi\rangle = \begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} \tag{47}$$

and its Hermitian conjugate spinor

$$(s_1^*, s_2^*) = |\psi\rangle^+ = \langle\psi| = \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)}, \quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right). \tag{48}$$

The normalization condition in this case is expressed by the equality

$$(s_1^*, s_2^*)\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \langle\psi|\psi\rangle = \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)} \quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right)\begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} = |\bar{a}_+|^2 + |\bar{a}_-|^2. \tag{49}$$

To find the projections of the spin (circular polarization) of a light beam on the coordinate axes, we use the spin-tensor

$$A_3 = \begin{pmatrix} x_3 & x_1 + ix_2 \\ x_1 - ix_2 & -x_3 \end{pmatrix} = x_1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{50}$$

which is related to the 3-dimensional metric

$$det(A_3) = \begin{pmatrix} x_3 & x_1 - ix_2 \\ x_1 - ix_2 & -x_3 \end{pmatrix}_{det} = \begin{vmatrix} x_3 & x_1 - ix_2 \\ x_1 + ix_2 & -x_3 \end{vmatrix} = -(x_1^2 + x_1^2 + x_1^2), \tag{51}$$

with signature $(- - -)$.
Assuming in Ex. (50) $x_1 = x_2 = x_3 = 1$, we consider the spin projections on the coordinate axes

$$(s_1^*, s_2^*)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + (s_1^*, s_2^*)\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + (s_1^*, s_2^*)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= (s_2^* s_1 + s_2^* s_1) + (-is_2^* s_1 + is_1^* s_2) + (s_1^* s_1 - s_2^* s_2). \tag{52}$$

Substituting spinors (47) and (48) into this expression, we obtain the following three spin projections on the corresponding coordinate axes $x_1 = x, \ x_2 = y, \ x_3 = z$:

$$\langle s_x\rangle = \langle\psi|-\sigma_1|\psi\rangle = (s_1^*, s_2^*)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)}, \quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} = \bar{a}_-^* \bar{a}_+ e^{-i\frac{4\pi}{\lambda}(ct-r)} + \bar{a}_+^* \bar{a}_- e^{i\frac{4\pi}{\lambda}(ct-r)}; \tag{53}$$

$$\langle s_y\rangle = \langle\psi|-\sigma_2|\psi\rangle = (s_1^*, s_2^*)\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)}, \quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right)\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} =$$

$$= \bar{a}_-^* \bar{a}_+ e^{-i\frac{4\pi}{\lambda}(ct-r)} + \bar{a}_+^* \bar{a}_- e^{i\frac{4\pi}{\lambda}(ct-r)} = i\left[\bar{a}_+^* \bar{a}_- e^{i\frac{4\pi}{\lambda}(ct-r)} - \bar{a}_-^* \bar{a}_+ e^{-i\frac{4\pi}{\lambda}(ct-r)}\right]; \tag{54}$$

$$\langle s_z \rangle = \langle \psi | -\sigma_3 | \psi \rangle = (s_1^*, s_2^*) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)}, \quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)} \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} = |\bar{a}_+|^2 - |\bar{a}_-|^2. \qquad (55)$$

In the case of $\varphi_+ = \varphi_- = 0$, Formulas (53) – (55) take the following simplified form:

$$\langle s_x \rangle = 2a_+a_- \, cos\left[\frac{4\pi}{\lambda}(ct-r)\right] = 2a_+a_- \, cos[2(\omega t - kr)], \qquad (56)$$

$$\langle s_y \rangle = 2a_+a_- \, sin\left[\frac{4\pi}{\lambda}(ct-r)\right] = 2a_+a_- \, sin[2(\omega t - kr)],$$

$$\langle s_z \rangle = |a_+|^2 - |a_-|^2.$$

In the case of equality of the amplitudes of the direct and backward waves $a_+ = a_-$, instead of Eqs. (56), we obtain the following average spin projections

$$\langle s_x \rangle = 2a_+^2 \, cos[2(\omega t - kr)], \qquad (57)$$

$$\langle s_y \rangle = 2a_+^2 \, sin[2(\omega t - kr)],$$

$$\langle s_z \rangle = 0.$$

.

The projection of the spin (the rotating vector of the electric field strength) on the direction of propagation of the light beam *Z* is unchanged and equal to zero. At the same time, its projection onto the *XY* plane, perpendicular to the direction of propagation of this beam, rotates around the *Z* axis with an angular velocity $\omega = 4\pi c/\lambda$. Thus, the spinor representation of the propagation of a conjugated pair of waves leads to a description of circular polarization without resorting to additional hypotheses.

Similarly, can be performed an analysis of wave propagation in a 3-dimensional metric extent with signatures: (− − −), (+ − −), (− + −), (− − +), (+ + +), (− + +), (+ − +), (+ + −).

On Average, a Flat "Colored" Spatial Fabric

We assume that the state of a local vacuum region is a manifestation of the averaged superposition of 16 curved metric spaces with different signatures (2)

$$\begin{aligned} ds_\Sigma{}^2 = 1/16 \, (& ds^{(+---)2} + ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + \\ & + ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2} + \\ & + ds^{(-+++)2} + ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + \\ & + ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2}), \end{aligned} \qquad (58)$$

where

$$\begin{aligned}
ds^{(++++)2} &= g_{00}^{(1)} dx_0{}^2 + g_{11}^{(1)} dx_1{}^2 + g_{22}^{(1)} dx_2{}^2 + g_{33}^{(1)} dx_3{}^2 \\
ds^{(---+)2} &= -g_{00}^{(2)} dx_0{}^2 - g_{11}^{(2)} dx_1{}^2 - g_{22}^{(2)} dx_2{}^2 + g_{33}^{(2)} dx_3{}^2 \\
ds^{(+--+)2} &= g_{00}^{(3)} dx_0{}^2 - g_{11}^{(3)} dx_1{}^2 - g_{22}^{(3)} dx_2{}^2 + g_{33}^{(3)} dx_3{}^2 \\
ds^{(--+-)2} &= -g_{00}^{(4)} dx_0{}^2 - g_{11}^{(4)} dx_1{}^2 + g_{22}^{(4)} dx_2{}^2 - g_{33}^{(4)} dx_3{}^2 \\
ds^{(-+--)2} &= -g_{00}^{(5)} dx_0{}^2 + g_{11}^{(5)} dx_1{}^2 - g_{22}^{(5)} dx_2{}^2 - g_{33}^{(5)} dx_3{}^2 \\
ds^{(+-+-)2} &= g_{00}^{(6)} dx_0{}^2 - g_{11}^{(6)} dx_1{}^2 + g_{22}^{(6)} dx_2{}^2 - g_{33}^{(6)} dx_3{}^2 \\
ds^{(++--)2} &= g_{00}^{(7)} dx_0{}^2 + g_{11}^{(7)} dx_1{}^2 - g_{22}^{(7)} dx_2{}^2 - g_{33}^{(7)} dx_3{}^2 \\
ds^{(+---)2} &= g_{00}^{(8)} dx_0{}^2 - g_{11}^{(8)} dx_1{}^2 - g_{22}^{(8)} dx_2{}^2 - g_{33}^{(8)} dx_3{}^2
\end{aligned}$$

$$\begin{aligned}
ds^{(----)2} &= -g_{00}^{(9)} dx_0{}^2 - g_{11}^{(9)} dx_1{}^2 - g_{22}^{(9)} dx_2{}^2 - g_{33}^{(9)} dx_3{}^2 \\
ds^{(+++-)2} &= g_{00}^{(10)} dx_0{}^2 + g_{11}^{(10)} dx_1{}^2 + g_{22}^{(10)} dx_2{}^2 - g_{33}^{(10)} dx_3{}^2 \\
ds^{(-++-)2} &= -g_{00}^{(11)} dx_0{}^2 + g_{11}^{(11)} dx_1{}^2 + g_{22}^{(11)} dx_2{}^2 - g_{33}^{(11)} dx_3{}^2 \\
ds^{(++-+)2} &= g_{00}^{(12)} dx_0{}^2 + g_{11}^{(12)} dx_1{}^2 - g_{22}^{(12)} dx_2{}^2 + g_{33}^{(12)} dx_3{}^2 \\
ds^{(+-++)2} &= g_{00}^{(13)} dx_0{}^2 - g_{11}^{(13)} dx_1{}^2 + g_{22}^{(13)} dx_2{}^2 + g_{33}^{(13)} dx_3{}^2 \\
ds^{(-+-+)2} &= -g_{00}^{(14)} dx_0{}^2 + g_{11}^{(14)} dx_1{}^2 - g_{22}^{(14)} dx_2{}^2 + g_{33}^{(14)} dx_3{}^2 \\
ds^{(--++)2} &= -g_{00}^{(15)} dx_0{}^2 - g_{11}^{(15)} dx_1{}^2 + g_{22}^{(15)} dx_2{}^2 + g_{33}^{(15)} dx_3{}^2 \\
ds^{(-+++)2} &= -g_{00}^{(16)} dx_0{}^2 + g_{11}^{(16)} d\, x_1{}^2 + g_{22}^{(16)} dx_2{}^2 + g_{33}^{(16)} dx_3{}^2
\end{aligned}$$

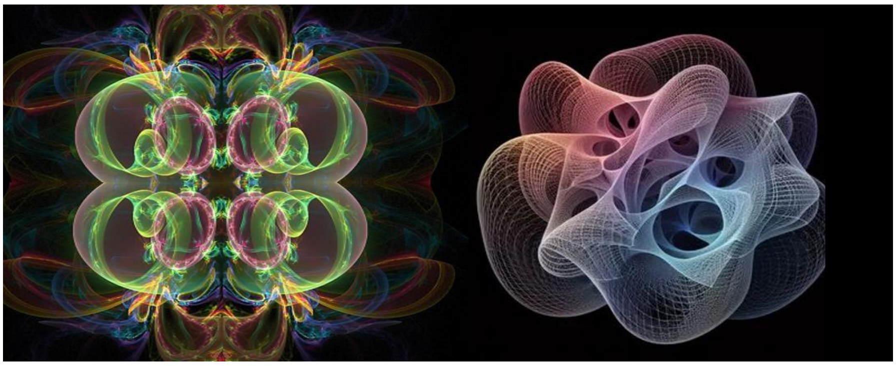
here $g_{ij}^{(n)} = g_{ij}^{(n)}(t)$ is diagonal components of the metric tensor of the n-th metric space with the corresponding signature (i.e., topology). These components determine the curvature of the n-th metric space and are random functions of time *t*.

In this case, a chaotically fluctuating, but on average vanishing the "colored" fabric of space, is woven from curved threads that vary randomly. A particular case of the interweaving of these "colored" threads (lines) can be described by an averaged superposition of curved affinors.

$$
\begin{aligned}
ds_{(16)} = \frac{1}{\sqrt{16}} \Bigg[& \sqrt{g_{00}^{(1)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(1)}(t)}dx_1 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \sqrt{g_{22}^{(1)}(t)}dx_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(1)}(t)}dx_3 \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \qquad (59) \\
& + \sqrt{g_{00}^{(2)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(2)}(t)}dx_1 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(2)}(t)}dx_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \sqrt{g_{33}^{(2)}(t)}dx_3 \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} + \\
& + \sqrt{g_{00}^{(3)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(3)}(t)}dx_1 \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + \sqrt{g_{22}^{(3)}(t)}dx_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(3)}(t)}dx_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(4)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(4)}(t)}x_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(4)}(t)}x_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(4)}(t)}x_3 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(5)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(5)}(t)}dx_1 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \sqrt{g_{22}^{(5)}(t)}d\,x_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(5)}(t)}dx_3 \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} + \\
& + \sqrt{g_{00}^{(6)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(6)}(t)}dx_1 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(6)}(t)}dx_2 \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + \sqrt{g_{33}^{(6)}(t)}dx_3 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(7)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(7)}(t)}dx_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(7)}(t)}dx_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(7)}(t)}dx_3 \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} + \\
& + \sqrt{g_{00}^{(8)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(8)}(t)}dx_1 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(8)}(t)}dx_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(8)}(t)}dx_3 \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \\
& + \sqrt{g_{00}^{(9)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(9)}(t)}dx_1 \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + \sqrt{g_{22}^{(9)}(t)}dx_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(9)}(t)}dx_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(10)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(10)}(t)}dx_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(10)}(t)}dx_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(10)}(t)}dx_3 \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} + \\
& + \sqrt{g_{00}^{(11)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(11)}(t)}dx_1 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \sqrt{g_{22}^{(11)}(t)}dx_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(11)}(t)}dx_3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(12)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(12)}(t)}dx_1 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(12)}(t)}dx_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(12)}(t)}dx_3 \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(13)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{11}^{(13)}(t)}dx_1 \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + \sqrt{g_{22}^{(13)}(t)}dx_2 \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} + \sqrt{g_{33}^{(13)}(t)}dx_3 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(14)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(14)}(t)}dx_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(14)}(t)}dx_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{33}^{(14)}(t)}dx_3 \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} + \\
& + \sqrt{g_{00}^{(15)}(t)}dx_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sqrt{g_{00}^{(15)}(t)}dx_1 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \sqrt{g_{00}^{(15)}(t)}dx_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{00}^{(15)}(t)}dx_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \\
& + \sqrt{g_{00}^{(16)}(t)}dx_0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \sqrt{g_{11}^{(16)}(t)}dx_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \sqrt{g_{22}^{(16)}(t)}dx_2 \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + \sqrt{g_{33}^{(16)}(t)}dx_3 \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \Bigg] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.
\end{aligned}
$$

This on-average missing "colored" spatial fabric (see Figure 4) is on-average Ricci-flat ($R_{ij} = 0$). In the neighborhood of each point of such a fabric, there is a chaotically changing topological knot (58) with an averaged signature (8), which largely corresponds to a constantly chaotically changing Calabi-Yau manifold (see Figure 5).

Figure 4: Illustration of a Fabric of Space Woven from 16 Types of "Colored" Threads (i.e., Interwoven "Colored" lines from 16 Intersecting Metric spaces (58) with Signatures (2)



a) *b*)

Figure 5: Illustration of a Comparison of the Instantaneous State of a local Topological Knot Woven from the "Colored" Threads of the Algebra of Signature and One of the Realizations of a Compact Calabi-Yau Manifold of Superstring Theory

Conclusion And Findings

"ALMIGHTY Created the world in 32 miraculous ways..."
"Sefer ha-Yetzirah" (Book of Creation)

This article proposes a multidimensional space (manifold) resulting from the additive superposition of 16-metric spaces (1) with signatures (i.e., topologies) (2). This multidimensional space is dis-cussed in more detail in the Algebra of signature [1 - 4].

The compactification of the extra dimensions in such a multidimensional space occurs automatically (i.e., without additional conditions), since the sum of all signs (determining the direction of the coordinate axes) is equal to split zero (8), in which 32(+) – 32(–) = 0.

Moreover, from such a multidimensional manifold, Minkowski space with the signature (+ – – –) and Minkowski antispace with the opposite signature (– + + +) can be simultaneously identified (see expressions (14) and (15)). Therefore, one of the fundamental conclusions of Algebra of signature [1,2,3,4,5,6] is the need to consider the space around us as a 3-dimensional fabric woven from at least 4 mutually perpendicular lines ("threads") (see Figure 2*c*).

Within the Algebra of signature, at a deeper level of consideration, the fabric of space is the result of the interweaving of 32 threads (16 "colored" + 16 "anticolored" = 32).

In the vicinity of any local region of the fabric of space, 32 “colored” threads can intertwine into such iridescent topological knots that, on average, they form a local region of flat space with a Ricci tensor equal to zero ($R_{ij} = 0$). In this case, the local variable topological knots of the Algebra of sig-nature are analogous to the constantly changing compact Calabi-Yau manifold in superstring theory (see Figure 5).

The multidimensional geometry and topology of the Algebra of signature are so rich in additional capabilities that, based on this elegant mathematics, it is possible to construct metric-dynamical (i.e., fully geometrized) models of all particles (bosons, fermions, hadrons, and mesons) that comprise the Standard Model of elementary particles [5-8]. It is also possible to propose metric-dynamic (i.e. geometric) ways to solve such problems of modern science as the baryon asymmetry of the Universe an explanation of the nature of the charge of particles and their inertia (i.e. mass) to identify ways to overcome gravity etc. [6-8,11].

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