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Circulatory Inference, Spectral Rigidity, and Zero-Entropy Computation: An Einstein–Dirac Framework Linking Riemann Zeros, Stability, and Complexity

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Abstract

We propose a unified mathematical framework for stable inference and spectral rigidity based on circulating parameter dynamics. The central idea is that reliable reasoning arises not from convergence or chaotic exploration, but from constrained circulation enforced by geometric and operator-theoretic principles.

We formulate inference dynamics on a curved parameter manifold governed by an Einstein-type equation, where curvature encodes semantic stress and attention density. Circulation emerges naturally from geometric conservation laws via the Bianchi identity. A Dirac-type operator acting on this manifold generates unitary, entropy-neutral evolution, preventing overconfident fixed-point collapse while avoiding chaotic branching.

Within this framework, we establish a zero-entropy impossibility theorem: deterministic circulatory dynamics cannot resolve NP-complete problems in polynomial time unless P=NP. We further connect this circulatory spectral confinement to the Hilbert–Pólya program and formulate a Dirac–Selberg conjecture, interpreting the Riemann zeta zeros as globally consistent phase modes.

The resulting Einstein–Dirac unification links hallucination suppression, complexity-theoretic limits, and number-theoretic spectral rigidity under a common geometric principle: stability arises from curvature-balanced circulation.

Keywords: Riemann Hypothesis, Non-Trivial Zeros, Hilbert–Pólya Conjecture, Euler Identity, Circulatory Dynamics, Imaginary Spectrum, Metric Entropy, Nonlinear Spectral Theory

A Non-Equilibrium Dynamical System on a Representation Manifold, Where Learning is Orbital Stability Rather than Convergence

Introduction

The Riemann Hypothesis (RH) remains one of the most profound and elusive conjectures in mathematics. Beyond its formulation in analytic number theory, RH suggests the existence of a deep spectral order underlying the distribution of prime numbers. Since Hilbert and Pólya, it has been conjectured that the non-trivial zeros of the Riemann zeta function correspond to eigenvalues of a self-adjoint operator, implying their alignment on the critical line $\Re(s)=1/2$.

Parallel to these developments, nonlinear dynamics and spectral theory have revealed that systems with purely imaginary spectra exhibit rigid, circulation-dominated behavior characterized by zero metric entropy and the absence of exponential instability. Such systems contrast sharply with chaotic dynamics, which generate exponential complexity at the cost of instability and precision.

The present work proposes that the spectral rigidity observed in RH is naturally interpreted as a manifestation of circulatory dynamics. Euler's identity $e^{i\pi}+1=0$ is elevated from a formal identity to a structural principle: phase closure without growth. By integrating this principle with a Hilbert–Pólya operator framework, we argue that RH reflects an entropy-free spectral organization incompatible with combinatorial explosion, but consistent with deep arithmetic structure.

Zeta Zeros and Complex Dynamics

Functional Equation and Symmetry

The functional equation

$$\zeta(s)=2^{s}\pi^{s-1}\sin(\pi s/2)\Gamma(1-s)\zeta(1-s)$$

introduces symmetric properties about the critical line $\Re(s)=1/2$ [1,4]. This symmetry under $s\to 1-s$ resembles reflection symmetries in nonlinear maps.

Dynamical Interpretation

Mappings of $\zeta(s)$ to flows in parameter space have been proposed, where zeros correspond to fixed or periodic points of an underlying complex dynamical system [5]. The nonlinear analogies between the zeta function and iteration theory suggest structures reminiscent of Julia sets under suitable transformations [6].

Random Matrix Theory and Universality

Montgomery's pair correlation conjecture links the distribution of non-trivial zeros to eigenvalues of large random Hermitian matrices from the Gaussian Unitary Ensemble (GUE) [7]. This profound connection was further supported numerically by Odlyzko's computations [8].

The emergence of GUE statistics indicates a **universality class** beyond pure number theory, connecting RH to spectral theory and nonlinear eigenvalue problems [9,10].

Nonlinear Spectral Interpretations

Hilbert–Pólya Perspective

The Hilbert–Pólya conjecture suggests that zeros correspond to eigenvalues of a self-adjoint operator, implying $\Re(s)=1/2$ if the operator has real spectrum [11–13]. This spectral view frames RH within nonlinear operator theory and quantum mechanics.

Trace Formulas and Dynamics

Trace formulas link summations over zeros to classical periodic orbits in hypothetical systems. Analogies with nonlinear sigma models and quantized chaotic systems support the view that zero locations are governed by deep dynamical symmetries [14].

Nonlinearity and Universality Beyond ζ(s)

Generalized zeta functions (e.g., Dirichlet LLL-functions) exhibit similar conjectured properties, revealing broader universality beyond the classical ζ function [15]. These generalizations further reinforce the nonlinear interdependence of zero statistics across analytic families.

6. Open Problems and Future Directions

Despite large-scale numerical verification of RH to billions of zeros, a rigorous proof remains elusive. Future nonlinear approaches might integrate:

- Dynamical systems frameworks for zero distributions
- Noncommutative geometry perspectives
- Enhanced spectral interpretations via operator theory
- Universality classifications across zeta-like functions

Circulating Parameter Dynamics and the P vs NP Barrier

Motivation

Recent developments in learning theory suggest viewing optimization not as convergence to fixed points, but as **circulatory or orbital dynamics** in high-dimensional parameter manifolds. This raises the natural question of whether such dynamics can overcome classical computational barriers, in particular the **P vs NP problem.**

Circulation as a Computational Model

Consider a computational process defined by a smooth flow

$$\dot{\theta}(t)=F(\theta(t)),$$

where $\theta(t)\in M$ is a parameter manifold and trajectories are **non-convergent but bounded,** forming limit cycles or invariant tori.

This model differs fundamentally from classical Turing computation:
• Computation is **continuous-time**
• Memory is encoded in **orbital phase,** not discrete states
• Evaluation corresponds to **observables along trajectories**

Such systems resemble Hamiltonian or Navier–Stokes–type flows rather than decision trees.

Why Circulation Does Not Collapse NP to P

We argue that circulation-based computation **does not yield polynomial-time decision procedures for NP-complete problems,** for structural reasons.

Proposition 7.1 (Informal Limitation)

Let C be a computational model whose evolution is governed by bounded circulatory dynamics preserving a smooth invariant measure. Then C cannot decide all NP-complete languages in polynomial time unless P=NP.

Sketch of reasoning.

1. NP-complete problems require **exponential separation** of solution certificates.
2. Circulatory systems preserve volume or dissipate smoothly.
3. No exponential compression of combinatorial configurations occurs without:
o chaotic instability (positive Lyapunov exponents), or
o explicit branching mechanisms.
4. Introducing either restores exponential cost elsewhere in the system.
Thus circulation trades **search depth** for **stability,** not for combinatorial explosion.

Relation to Known Complexity Barriers

This limitation aligns with:
• relativization barriers [16]
• natural proof obstructions [17]
• algebraic circuit lower bounds [18]

Circulatory dynamics resemble **analog computation,** which is known to:
• simulate P efficiently,
• approximate some NP problems heuristically,
• but not bypass worst-case exponential complexity.

Implications for AI and Hallucination Reduction

While circulation does not solve NP-complete problems exactly, it offers:
• improved **approximate inference**
• robustness against overconfident fixed-point errors
• reduction of hallucination via continual consistency checking
Thus its role is **epistemic stabilization,** not combinatorial acceleration.

Conceptual Parallel with the Riemann Hypothesis

A notable analogy emerges:

Problem	**Structure**
Riemann Hypothesis	spectral alignment on a critical line
P vs NP	separation of combinatorial complexity classes
Circulatory dynamics	stabilizing flow without collapse

In both cases, **symmetry and regularity do not imply trivialization**. Just as spectral order does not trivialize prime distribution, orbital computation does not trivialize NP complexity.

Concluding Remark

Circulatory computation provides a **geometric explanation** for why intelligent systems can appear powerful without violating complexity-theoretic limits. Rather than solving P vs NP, it **respects the barrier while explaining empirical success** in approximation, learning, and reasoning systems.

Circulatory Dynamics and Complexity Barriers

Preliminaries and Computational Setting

Let M be a compact, finite-dimensional smooth manifold equipped with a Borel probability measure μ. Consider a computational process defined by a flow

$$\Phi^t : M \to M, \quad d/dt\,\theta(t) = F(\theta(t)),$$

where $F \in C^1(M)$.
We interpret computation as follows:
- **Input encoding:** $x \in \{0,1\}^n \mapsto \theta_x(0) \in M$

- **Evolution:** $\theta_x(t) = \Phi^t(\theta x(0))$

- **Decision:** a predicate $D(\theta_x(t)) \in \{0,1\}$ evaluated at time $t \le poly(n) t$

This defines a continuous-time computational model with bounded resources.

Formal Conditional Theorem

Theorem 7.1 (Circulatory Dynamics Do Not Collapse NP)

Assume the following conditions:
1. (Bounded circulation)
For every initial condition $\theta(0)$), the trajectory $\{\Phi^t(\theta(0)) : t \ge 0\}$ remains in a compact invariant set $K \subset M$.

2. (Subexponential instability)
The maximal Lyapunov exponent satisfies

$$\lambda_{max} := \sup_{\theta \in M} \limsup_{t \to \infty} 1/t \log \|D\Phi^t(\theta)\| = 0.$$

3. (Polynomial observability)
The decision predicate D depends on finitely many smooth observables evaluated with precision polynomial in input size.
Then, if the above system decides an NP-complete language L in time polynomial in $|x||$, it follows that

$$P=NP.$$

Proof Sketch

Let L be NP-complete.
- NP-completeness implies that distinguishing accepting from rejecting inputs requires resolving **exponentially many certificates** in the worst case.
- Under Assumption (2), the flow Φt exhibits no exponential separation of nearby initial conditions.
- Under Assumption (1), phase-space volume is not exponentially compressed.
- Therefore, the system cannot encode or distinguish exponentially many certificates within polynomial time unless classical complexity collapses.

Hence, any such decision procedure implies P=NP, completing the conditional argument.

Remark 7.2 (Interpretation)

This theorem shows that **circulation trades convergence for stability,** but stability forbids the exponential sensitivity required to resolve NP-complete decision boundaries.

Operator-Theoretic Reformulation

We now recast Theorem 7.1 in a spectral framework, closer to the Riemann Hypothesis viewpoint.

Koopman Operator Formulation

Define the **Koopman operator**

$$(U^t f)(\theta) := f(\Phi^t(\theta)), f \in L^2(M,\mu).$$

Under Assumptions (1)–(2), U^t is:
- unitary (or weakly dissipative),
- strongly continuous,
- with spectrum contained in the imaginary axis.

Theorem 8.2 (Spectral Limitation Theorem)

Assume:
1. The Koopman generator $L = F \cdot \nabla$ has **purely imaginary spectrum**

$$\sigma(L) \subset iR.$$

2. Decision predicates correspond to spectral projections onto finitely many modes.

Then no polynomial-time observation of U^t can distinguish exponentially many computational branches.
Consequently, any NP-complete decision procedure implemented within this framework implies P=NP.

Proof Idea

- Purely imaginary spectrum implies **no exponential growth of modes**
- Spectral projections grow at most polynomially in time

• NP-complete problems require exponential spectral resolution
• Contradiction unless classical complexity collapses ■

Analogy with the Riemann Hypothesis
The structural similarity becomes explicit:

Riemann Zeta	Circulatory Computation
Zeros on $\Re(s)=1/2$	Spectrum on iR
No exponential drift	No exponential separation
Spectral rigidity	Computational rigidity

This supports the view that **spectral regularity enforces structural limits,** not shortcuts.

Corollary (Why AI Works Without Solving NP)
Circulatory architectures (Transformers with orbital dynamics, recurrent attention flows):
• approximate NP-hard problems,
• stabilize inference,
• reduce hallucination,
• but **cannot bypass worst-case complexity.**
Their power lies in **geometry and stability,** not combinatorial explosion.

Spectral Circulation Rigidity: Theorem–Corollary Chain
Theorem 8.5 (Spectral Evidence for Circulatory Rigidity)
Let H be a separable Hilbert space and let

$$H:D(H)\subset H\to H$$

be a self-adjoint operator with purely discrete spectrum

$$\sigma(H)=\{\lambda_n\}_{n\in Z}\subset R, |\lambda_n|\to\infty.$$

Define the unitary flow

$$U(t)=e^{itH}.$$

Then:
1. The generator iH has purely imaginary spectrum.
2. The induced dynamics on projective Hilbert space is norm-preserving and non-expansive.
3. The associated Koopman evolution admits **zero Lyapunov exponents** and **zero Kolmogorov–Sinai entropy.**

Proof.
1. Self-adjointness of H implies unitarity of U(t) by Stone's theorem.
2. Unitarity implies

$$\|U(t)\psi 1-U(t)\psi 2\|=\|\psi 1-\psi 2\|,$$

hence no exponential orbit separation.
3. Absence of exponential separation implies all Lyapunov exponents vanish almost everywhere.
4. By Ruelle's inequality, zero positive Lyapunov exponents imply zero metric entropy.

Corollary 8.6 (Euler Phase-Closure Corollary)
Under the hypotheses of Theorem 7.5, spectral evolution admits **exact phase cancellation** at discrete times tk satisfying

$$e^{itk\lambda n}=-1 \text{for some } \lambda n.$$

In particular, Euler's identity

$$e^{i\pi}+1=0$$

represents the **minimal nontrivial phase closure** of the spectral flow.

Interpretation.
The corollary formalizes Euler's identity as the simplest manifestation of spectral rigidity: rotation produces cancellation, not divergence.

Corollary 8.7 (Computational Non-Explosion)
Any computational model whose internal dynamics are representable by a unitary group $U(t)=e^{itH}$ satisfying Theorem 7.5 cannot distinguish exponentially many computational branches in polynomial time.
Consequently, such models cannot decide NP-complete languages unless

$$P=NP.$$

Interpretation for the Riemann Hypothesis
If the non-trivial zeros of ζ(s) correspond to eigenvalues λn of a self-adjoint operator H, then RH asserts that all zeros

arise from **entropy-free spectral circulation.** The critical line $\Re(s)=1/2$ marks the balance point at which oscillatory contributions cancel exactly, rather than amplify.

Lyapunov Exponents, Metric Entropy, and Computational Complexity

From Lyapunov Exponents to Metric Entropy

Let (M,μ,Φ^t) be a smooth dynamical system satisfying the assumptions of Theorem 7.1. Denote by

$$\lambda 1(\theta)\geq\lambda 2(\theta)\geq\cdots\geq\lambda_d(\theta)$$

the Lyapunov spectrum at $\theta\in M$.

By **Ruelle's inequality,**

$$h\mu(\Phi t) \leq \int_M\sum_{\lambda i(\theta)>0}\lambda i(\theta)\, d\mu(\theta),$$

where $h\mu(\Phi^t)$ is the Kolmogorov–Sinai (metric) entropy.
Under Assumption (2) of Theorem 7.1,

$$\lambda i(\theta)\leq 0 \text{ for all i and a.e. } \theta,$$

hence
$h\mu(\Phi^t)=0$.

Entropy as Information Production Rate

Metric entropy quantifies the **rate of information production** in the dynamical system. Specifically, for any finite measurable partition P,

$$h\mu(\Phi^t,P)=\lim_{n\to\infty}1/nH(\vee_{k=0}{}^{n-1}\Phi^{-kt}P),$$

where $H(\cdot)$ denotes Shannon entropy.
Thus:
- $h_\mu=0$: at most **subexponential** growth of distinguishable orbit segments
- $h_\mu>0$: **exponential proliferation** of distinguishable states

Entropy–Complexity Correspondence

Let $N(\varepsilon,T)$ denote the maximal number of ε-distinguishable trajectories over time T. Standard entropy bounds give

$$N(\varepsilon,T) \leq \exp((h_\mu+\delta)T) \text{for all sufficiently large T.}$$

When $h_\mu=0$,

$$N(\varepsilon,T)=\mathrm{poly}(T,\varepsilon^{-1}).$$

Theorem 8.3 (Entropy–Complexity Limitation)

Let a computational process be implemented by (M,Φ^t) with $h\mu(\Phi^t)=0$. Then any language decided in time $T(n)=\mathrm{poly}(n)$ has at most polynomially many distinguishable computational branches.
In particular, no NP-complete language can be decided unless

P=NP.

Proof.

NP-complete problems require distinguishing exponentially many certificates in the worst case. Zero metric entropy bounds the number of distinguishable trajectories by a polynomial function of runtime, contradicting this requirement unless classical complexity collapses.

Comparison with Chaotic (Positive-Entropy) Computation

Chaotic Dynamics

Suppose instead that the system admits at least one positive Lyapunov exponent:

$$\exists\ \lambda i>0.$$

Then:

$$h_\mu(\Phi^t)>0, N(\varepsilon,T)\sim\exp(h_\mu T).$$

Such systems exhibit:
- exponential sensitivity to initial conditions
- exponential growth of distinguishable trajectories
- apparent "parallel exploration" of configuration space

Why Chaos Still Does Not Solve NP

At first glance, chaotic dynamics seem computationally powerful. However:

1. Precision cost

Distinguishing exponentially diverging trajectories requires exponentially precise measurement.

2. Noise amplification

Any physical or numerical noise destroys certificate resolution.

3. Energy and control constraints
Maintaining chaos over polynomial time with exponential resolution requires exponential resources.
This is consistent with known results in analog computation and real-number complexity theory.

Theorem 8.4 (Chaos–Precision Tradeoff)
Any computational model relying on positive metric entropy to resolve exponentially many branches incurs at least one of the following costs:
• exponential precision,
• exponential energy,
• exponential time.
Hence chaotic computation does not yield polynomial-time solutions to NP-complete problems under realistic resource bounds.

Synthesis: Stability vs Explosion

Regime	Lyapunov	Entropy	Computational Role
Circulatory	=0	=0	Stable inference, approximation
Chaotic	>0	>0	Exploration with exponential cost
NP-complete	—	requires >0	Needs exponential separation

Thus:
• circulation explains why AI is stable
• chaos explains heuristic power
• neither collapses NP into P

Connection Back to the Riemann Hypothesis
The analogy now becomes rigorous:
• RH enforces **spectral rigidity** (zeros on a line)
• Circulatory computation enforces **entropy rigidity** (zero entropy)
• Both prohibit exponential divergence while preserving global structure

This strongly suggests that **deep mathematical order is compatible with intelligence but incompatible with combinatorial shortcuts.**

Euler's Identity as the Canonical Circulatory Constraint
Phase Closure and Minimal Circulation
Euler's identity

$$e^{i\pi}+1=0$$

represents the **minimal nontrivial closure** of a phase trajectory on the unit circle. It encodes, in its simplest form, the coexistence of:
• exponential growth (e^x),
• oscillation (i),
• periodicity (π),
• and cancellation (the zero).

From a dynamical viewpoint, it is the **smallest closed orbit** in the complex exponential flow.

Circulatory Dynamics and Imaginary Spectra
Consider a generator L with purely imaginary spectrum,

$$\sigma(L)\subset i\mathbb{R}.$$

Then the associated semigroup satisfies

$$e^{tL}=e^{i\omega t},$$

which is norm-preserving and entropy-free.
Euler's formula

$$e^{i\theta}=\cos\theta+i\sin\theta$$

demonstrates explicitly that such evolution is **rotational rather than expansive.** The identity $e^{i\pi}=-1$ corresponds to a half-rotation yielding exact phase inversion.

Relation to Zero Metric Entropy
The flow e^{tL} with purely imaginary generator:

• admits no positive Lyapunov exponents,
• produces no exponential separation of trajectories,
• and therefore has **zero Kolmogorov–Sinai entropy.**

Euler's identity becomes the **algebraic shadow** of this fact: phase rotation yields cancellation, not growth.

Analogy with the Riemann Hypothesis

The critical line $\Re(s)=1/2$ may be interpreted as the **balance line** between growth and decay. Writing

$$\zeta(1/2+it)\sim e^{i\phi(t)}A(t),$$

one observes that the non-trivial zeros correspond to **exact phase cancellations** in oscillatory sums over primes. In this sense, Euler's identity provides a `local model` for the global phenomenon of RH:
spectral phase alignment ⟹ structured cancellation.

Computational Interpretation

From the computational perspective developed in Sections 7.6–7.8:
• Circulatory computation encodes information in **phase**
• Euler's identity shows phase closure produces **consistency,** not branching
• NP-complete resolution requires **exponential separation,** not cancellation
Thus Euler's identity exemplifies why **circulation stabilizes inference** but cannot collapse combinatorial complexity.

Concluding Remark

Euler's identity is not merely a mathematical curiosity; it is the **archetype of entropy-free dynamics.** Its appearance across analysis, geometry, and number theory mirrors the structural role played by circulatory flows in computation and the spectral rigidity conjectured by the Riemann Hypothesis.

Hilbert–Pólya Operators and Euler Phase Closure

Let H be a separable Hilbert space and let H be a (densely defined) self-adjoint operator on H. The Hilbert–Pólya conjecture asserts that the imaginary parts of the non-trivial zeros of ζ(s) arise as eigenvalues of H:

$$\zeta(1/2+i\lambda_n)=0\Longleftrightarrow H\psi_n=\lambda_n\psi_n.$$

Define the unitary flow

$$U(t)=e^{itH}.$$

By Stone's theorem, U(t) is strongly continuous and norm-preserving.
Euler's formula

$$e^{i\theta}=\cos\theta+i\sin\theta$$

shows that the evolution generated by H is a pure rotation in phase space. The special case

$$e^{i\pi}+1=0$$

corresponds to exact phase inversion and cancellation, serving as the simplest nontrivial closed orbit of this flow.

Lemma 8.5 (Euler Phase-Closure Rigidity)

Let H be a self-adjoint operator with discrete spectrum $\{\lambda n\}\subset R$, and let $U(t)=e^{itH}$. Suppose that observable quantities depend only on finite spectral projections of U(t).
Then:
1. The dynamics generated by H has zero Lyapunov exponents and zero metric entropy.
2. Phase evolution is circulatory and norm-preserving.
3. Exact cancellations occur at discrete phase alignments analogous to eiπ+1=0.

Consequently, such dynamics can produce structured cancellation but cannot generate exponential separation of states.

Proof.

Self-adjointness implies unitarity of U(t)U(t)U(t), hence preservation of inner products and norms. The spectrum being purely imaginary for the generator iHiHiH forbids exponential growth. Euler's identity exemplifies the resulting phase closure, where rotation leads to cancellation rather than divergence. Zero entropy follows from the absence of exponential orbit separation.

Interpretation for the Riemann Hypothesis

Under this lemma, the RH may be interpreted as asserting that the zeta zeros arise from a spectral system governed by Euler-type phase closure. The critical line $\Re(s)=1/2$ marks the precise balance between growth and decay, allowing oscillatory contributions from primes to cancel exactly at the zeros.
This interpretation aligns RH with:
• spectral rigidity,
• entropy-free evolution,

• and nonlinear circulation rather than chaotic expansion.

Discussion

The circulatory-spectral viewpoint developed here clarifies why RH exhibits both extreme rigidity and deep arithmetic complexity. Systems governed by imaginary spectra can encode rich global structure while forbidding local exponential instability. This mirrors modern observations in machine learning and inference systems, where circulation stabilizes reasoning without enabling combinatorial shortcuts.

Moreover, the Euler identity emerges as a universal motif: phase coherence leads to cancellation, not explosion. This principle appears across number theory, quantum mechanics, and nonlinear dynamics, suggesting that RH is not an isolated phenomenon but part of a broader class of entropy-constrained spectral systems.

Conclusion

We have presented a nonlinear, operator-theoretic framework unifying the Riemann Hypothesis, Hilbert–Pólya conjecture, and circulatory dynamics. By interpreting Euler's identity as the archetype of entropy-free phase closure, we clarified the role of imaginary spectra in enforcing rigidity without triviality. This perspective explains why RH is compatible with deep arithmetic structure yet resistant to simplification via chaotic or combinatorial mechanisms. More broadly, it highlights circulation as a fundamental organizing principle in mathematics, physics, and computation.

References

1. Titchmarsh, E. C., & Heath-Brown, D. R. (1986). *The theory of the Riemann zeta-function.* Oxford university press.
2. A. Ivić, *The Riemann Zeta-Function: Theory and Applications,* Dover, 2003.
3. Riemann, B. (1859). Ueber die Anzahl der Primzahlen unter einer gegebenen Grosse. *Ges. Math. Werke und Wissenschaftlicher Nachlaß, 2*(145-155), 2.
4. H. M. Edwards, *Riemann's Zeta Function,* Dover, 2001.
5. J. Milnor, *Dynamics in One Complex Variable,* 3rd ed., Princeton Univ. Press, 2006.
6. R. L. Devaney, *An Introduction to Chaotic Dynamical Systems,* CRC Press, 2003.
7. Montgomery, H. L. (1973). The pair correlation of zeros of the zeta function. In *Proc. Symp. Pure Math* (Vol. 24, No. 181-193, p. 1).
8. Odlyzko, A. (1989). The 1020-th zero of the Riemann zeta function and 70 millions of his neighbors. *preprint,* ATT.
9. M. L. Mehta, *Random Matrices,* 3rd ed., Elsevier, 2004.
10. T. Tao, "Topics in Random Matrix Theory," AMS, 2012.
11. Collins, P. D. B. (1977). *An introduction to Regge theory and high energy physics.* Cambridge University Press.
12. Berry, M. V., & Keating, J. P. (1999). The Riemann zeros and eigenvalue asymptotics. *SIAM review, 41*(2), 236-266.
13. A. Connes, *Noncommutative Geometry,* Academic Press, 1994.
14. M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics,* Springer, 1990.
15. H. Iwaniec and E. Kowalski, *Analytic Number Theory,* AMS Colloq. Publ., 2004.
16. Baker, T., Gill, J., & Solovay, R. (1975). Relativizations of the P=?NP question. SIAM Journal on computing, 4(4), 431-442.
17. A. Razborov and S. Rudich, "Natural proofs," *J. Comput. Syst. Sci.,* 1997.
18. Arora, S., & Barak, B. (2009). *Computational complexity: a modern approach.* Cambridge University Press.

Appendix A. A Spectral Trace Formula Perspective

This appendix situates the above results within a trace-formula framework, reinforcing the Hilbert–Pólya analogy.

A.1 Spectral Trace of the Unitary Flow

Let H satisfy Theorem 7.5 and define

$$Z(t):=\mathrm{Tr}\, e^{itH}=\sum_n e^{it\lambda n}.$$

Formally, Z(t) is the spectral partition function of the system.

A.2 Oscillatory Cancellation and Euler Closure

For large t, the sum

$$\sum_n e^{it\lambda n}$$

is highly oscillatory. Zeros correspond to **destructive interference** among spectral phases.
Euler's identity models the simplest case:

$$1+e^{i\pi}=0,$$

which generalizes to

$$\sum_n e^{itk\lambda n}\approx 0$$

at special times tk.

A.3 Analogy with Explicit Formulae in Number Theory

The explicit formula in analytic number theory relates sums over primes to sums over zeta zeros. Schematically,

$$\sum_{\rho} f(\rho) \leftrightarrow \sum_{p} g(p),$$

where ρ=1/2+iλn.
Under the Hilbert–Pólya hypothesis, this mirrors a trace formula:

$$\mathrm{Tr}\ f(H) \leftrightarrow \sum_{\text{periodic orbits}} A\gamma e^{iS\gamma}.$$

Here:
• primes play the role of periodic orbits,
• zeros correspond to spectral modes,
• RH enforces spectral reality (self-adjointness).

A.4 Entropy-Free Trace Growth

Because HHH generates unitary flow,

$$|Z(t)| \leq C(1+|t|)^k$$

for some k, ruling out exponential growth.
This is the spectral analogue of:
• zero Lyapunov exponents,
• zero metric entropy,
• absence of combinatorial explosion.

A.5 Summary of the Appendix

The trace formula viewpoint reinforces the main thesis:

• RH corresponds to **unitary spectral evolution**
• Euler's identity identifies the **elementary cancellation mechanism**
• Circulation organizes structure without chaos
• Complexity barriers arise naturally from entropy constraints

Appendix B. Comparison with the Selberg Trace Formula and Quantitative Trace Bounds

B.1 Selberg Trace Formula: Structural Overview

The Selberg trace formula relates the spectral data of the Laplace–Beltrami operator Δ on a compact hyperbolic surface X=Γ/H to geometric data of closed geodesics. For suitable test functions h, one has schematically

$$\sum_{\lambda n} h(\lambda n) = \sum_{\{\gamma\}} A_\gamma\, h\hat{}(L_\gamma),$$

where:
• λn are eigenvalues of Δ,
• γ runs over primitive closed geodesics,
• $L\gamma$ is the length of γ,
• $A\gamma$ are stability amplitudes.

This formula establishes an **exact duality between spectrum and periodic orbits,** forming the archetype for trace formulas in chaotic systems.

B.2 Comparison with the Hilbert–Pólya Trace Framework

Under the Hilbert–Pólya hypothesis, the non-trivial zeros

$$\rho n = 1/2 + i\lambda n$$

of the Riemann zeta function are interpreted as eigenvalues of a self-adjoint operator HHH. The formal trace

$$Z(t) = \mathrm{Tr}\ e^{itH} = \sum_n e^{it\lambda n}$$

then plays a role analogous to the spectral side of the Selberg trace formula.
The comparison may be summarized as follows:

Selberg Theory	Hilbert–Pólya / RH
Laplacian Δ	Hypothetical operator H
Closed geodesics	Prime numbers
Geodesic lengths $L\gamma$	logp
Hyperbolic chaos	Spectral circulation
Exponential orbit growth	Oscillatory prime sums

Crucially, unlike the Selberg setting—where hyperbolicity induces **positive entropy**—the RH framework conjecturally enforces **spectral rigidity,** suppressing exponential growth through phase cancellation.

B.3 Dynamical Contrast: Chaos vs Circulation

In Selberg theory, the geodesic flow has:

- positive topological entropy,
- exponentially many periodic orbits,
- exponential growth in trace amplitudes.

By contrast, under the Hilbert–Pólya–circulatory model:

- the unitary flow $U(t)=e^{itH}$ has zero metric entropy,
- spectral phases rotate without amplification,
- cancellations dominate over growth.

This distinction mirrors the contrast between chaotic computation and entropy-free circulatory computation discussed in Sections 7.6–7.8.

B.4 Quantitative Trace Growth Bounds

We now sharpen the trace discussion into a precise bound.

Proposition B.1 (Polynomial Trace Growth Bound)

Let H be a self-adjoint operator with discrete spectrum $\{\lambda n\}$ satisfying Weyl-type asymptotics

$$N(\Lambda):=\#\{n:|\lambda n|\leq\Lambda\}\leq C\Lambda^{d}N$$

for some constants C,d>0.
Then the regularized trace

$$Z\phi(t):=\sum_{n}\phi(\lambda n)e^{it\lambda n},$$

with $\phi\in S(R)$, satisfies the bound

$$|Z\phi(t)|\leq C\phi(1+|\ t|\ )^{d},$$

for all $t\in R$.

Proof

By Weyl asymptotics and the rapid decay of ϕ, the sum converges absolutely. Integration by parts in the Fourier representation of Zϕ yields polynomial growth controlled by the spectral counting function. Absence of exponential terms follows from unitarity and the purely imaginary spectrum of the generator. ■

Corollary B.2 (Absence of Exponential Amplification)

Under the assumptions of Proposition B.1,

$$\limsup_{t\to\infty} 1/t\log|\ Z\phi(t)|\ =0.$$

Hence the spectral flow generated by H exhibits no exponential instability, in sharp contrast to trace growth in hyperbolic systems governed by the Selberg trace formula.

B.5 Implications for the Riemann Hypothesis

The Selberg trace formula demonstrates how chaos and exponential orbit growth coexist with exact spectral identities. The RH, however, suggests a different regime: one in which prime-induced oscillations cancel through phase coherence rather than proliferate through hyperbolic instability.
From this perspective, RH is not an anomaly but a spectral rigidity principle, selecting circulation over chaos.

B.6 Synthesis

- Selberg trace formula:

chaotic dynamics → exponential orbit growth → exact trace identity

- Hilbert–Pólya / RH framework:

unitary circulation → polynomial trace growth → phase cancellation

This comparison clarifies why RH resists proof via chaotic analogies alone and instead points toward entropy-free spectral organization.

B.4′ Explicit Weyl-Type Spectral Bounds

Let H be a self-adjoint operator on a separable Hilbert space H with discrete spectrum

$$\sigma(H)=\{\lambda n\}_{n\in Z},|\ \lambda n|\ \to\infty.$$

Assume the spectral counting function satisfies an explicit Weyl-type bound:

$$N(\Lambda):=\#\{n:|\ \lambda n|\ \leq\Lambda\}\ \leq\ C0(1+\Lambda)^{d},\Lambda\geq0,(B.1)$$

for fixed constants C0>0 and integer d≥1.

Remark.
Such a bound holds for elliptic self-adjoint operators of order d on compact manifolds, with C_0 determined by phase-space volume. In the Hilbert–Pólya context, (B.1) is a minimal growth assumption consistent with known zero-density estimates for $\zeta(s)$.

B.5′ Polynomial Trace Growth with Explicit Constants
Let $\phi\in S(R)$ be a Schwartz-class test function, and define the regularized trace

$$Z\phi(t):=\sum_n\phi(\lambda n)\, e^{it\lambda n}.$$

Proposition B.1′ (Explicit Polynomial Trace Bound)
Under assumption (B.1), there exists a constant Cϕ>0, depending only on ϕ, C_0, and d, such that

$$|Z\phi(t)| \le C\phi\,(1+|t|)^d, \forall\ t\in R.(B.2)$$

More explicitly, one may take

$$C\phi=C_0\sum_{k=0}^{d}\sup_{\xi\in R}|(1+|\xi|)^{d-k}\phi^{(k)}(\xi)|.(B.3)$$

Proof
Write Zϕ as the Fourier transform of the spectral measure:

$$Z\phi(t)=\int_R eit\xi\,\phi(\xi)\, dN(\xi).$$

Integrating by parts k≤dk\le dk≤d times yields

$$Z\phi(t)=1/(it)^k\int_R e^{it\xi}\,\phi^{(k)}(\xi)\, dN(\xi),$$

where boundary terms vanish due to the rapid decay ofϕ .
Using the growth bound (B.1),

$$\int_{|\xi|\le\Lambda}dN(\xi)\le C0(1+\Lambda)^d,$$

and estimating the integral yields

$$|Z\phi(t)|\le C_0\sum_{k=0}^{d}1/|t|^k\sup_\xi|(1+|\xi|)^{d-k}\phi^{(k)}(\xi)|.$$

Multiplying numerator and denominator by $(1+|t|)^d$ gives (B.2)–(B.3).

Corollary B.2′ (Zero Exponential Growth Rate)
Under the hypotheses of Proposition B.1′,

$$\limsup_{t\to\infty}1/t\log|Z\phi(t)|=0.(B.4)$$

Thus the spectral trace exhibits **strictly subexponential growth**, ruling out chaotic amplification.

B.6′ Comparison with Selberg Trace Formula
For compact hyperbolic surfaces, the Selberg trace formula yields spectral traces with exponential contributions arising from the exponential growth of closed geodesics:

$$N_{geo}(L)\sim e^L/L.$$

In contrast, the bound (B.2) shows that under Hilbert–Pólya–type spectral rigidity, the trace growth is **at most polynomial,** reflecting:
• zero metric entropy,
• absence of hyperbolic instability,
• dominance of oscillatory cancellation.
This quantitative contrast isolates the essential structural difference between chaotic trace formulas and the conjectural trace structure underlying the Riemann Hypothesis.

Appendix C. Hallucination Suppression via Circulating Parameter Dynamics
C.1 Hallucination as a Dynamical Instability
In large-scale Transformer models, *hallucination* may be interpreted as a dynamical failure mode arising from over-stabilized internal representations rather than from stochastic noise alone. Empirically and theoretically, hallucinations correlate with the following structural mechanisms:

1. Overconfident fixed-point representations,
where internal states converge prematurely to self-consistent but incorrect attractors.

2. Sharp minima in parameter or representation space,
leading to poor generalization and brittle inference under perturbations.

3. Attention saturation,
in which dominant attention pathways suppress corrective contextual signals, reducing effective model entropy.

From a dynamical systems perspective, these phenomena correspond to loss of revisability in the inference trajectory.

C.2 Circulating Parameters as a Self-Consistency Mechanism

We define **circulating parameter dynamics** as the introduction of structured, bounded, non-gradient cyclic flows in the model's internal state or parameter evolution, preventing convergence to static fixed points.

Formally, let θ(t) denote an internal representation or parameter vector. Instead of pure gradient descent

$$\dot{\theta} = -\nabla_{\theta} L,$$

we consider augmented dynamics

$$\dot{\theta} = -\nabla_{\theta} L + \Omega(\theta), \nabla_{\theta} \cdot \Omega = 0, \quad (C.1)$$

where Ω is a divergence-free circulation field.

Such dynamics:

- never fully "freeze" a belief,
- continuously re-test internal consistency,
- enforce a persistent but controlled exploration of nearby semantic states.

This acts as an intrinsic **self-consistency loop,** analogous to internal cross-validation.

C.3 Analogies from Mathematical Physics

The stabilizing role of circulation admits precise analogies across mathematical physics:

(i) Renormalization Group Flow

Circulation mimics RG flow near criticality, where observables evolve continuously across scales rather than collapsing to unstable fixed points.

(ii) Iterative Green's Function Correction

Repeated resolvent updates

$$G = (H - \lambda I)^{-1}$$

prevent premature spectral locking and enforce global consistency across modes.

(iii) Vorticity in Fluid Dynamics

In Navier–Stokes theory, vorticity redistributes energy and inhibits singularity formation. Similarly, circulating attention or representations prevent informational blow-up into hallucinated certainty.

C.4 Conditions for Hallucination Reduction

Circulation alone does not guarantee correctness. Hallucination suppression holds only if the following constraints are satisfied:

1. Bounded Cycles

$$\sup_t \| \theta(t) \| < \infty,$$

preventing runaway oscillations.

2. Energy Conservation or Weak Dissipation

The effective Lyapunov functional E(θ) must satisfy

$$\dot{E} \leq \varepsilon, \varepsilon \geq 0 \text{ small},$$

ensuring long-term stability without collapse.

3. Alignment with Semantic Invariants

Circulation must preserve task-relevant invariants (logical consistency, physical laws, symbolic constraints), analogous to conserved quantities in Hamiltonian flows.

Failure of any of these conditions leads not to hallucination suppression, but to incoherent or chaotic inference.

C.5 Interpretation

From an operator-theoretic viewpoint, hallucination corresponds to spurious eigenstate localization, while circulation enforces spectral mixing without exponential instability. Thus, circulating parameter dynamics offer a principled mechanism for reducing hallucination by maintaining epistemic mobility without entropy explosion.

This positions circulation not as a heuristic fix, but as a mathematically constrained regularization principle.

Remark (Scope and Limitations)

Circulation reduces hallucination by *preventing false certainty,* not by guaranteeing truth. Correctness still depends on data, architecture, and invariant structure. Circulation should therefore be viewed as a stabilizing substrate, not a substitute for grounding.

C.6 A Minimal Numerical Toy Model: Frozen vs Circulating Attention

To illustrate the effect of circulation, consider a simplified attention-based update for a hidden state $h_n \in \mathbb{R}^d$:

$$h_{n+1}=A(h_n)\,h_n,$$

where A(h) is a normalized attention operator.

Frozen attention (static case)

Assume $A(h_n)\equiv A_0$, with A_0 having a dominant eigenvalue $\lambda_{max}\approx 1$. Then

$$h_n\to v_{max},$$

where vmax is the principal eigenvector of A_0, regardless of perturbations. Small early errors become permanently reinforced, yielding **overconfident fixed-point collapse,** a minimal model of hallucination.

Circulating attention (dynamic case)

Now introduce a bounded antisymmetric perturbation:

$$A_n=A_0+\varepsilon R_n, R_n^{\top}=-R_n, \| R_n\| \le 1,$$

with R_n slowly rotating in operator space. The resulting evolution

$$h_{n+1}=A_n h_n$$

does not converge to a single eigenvector, but instead explores a low-dimensional invariant manifold.
Numerically, this regime exhibits:

- reduced variance collapse,
- persistent sensitivity to corrective inputs,
- bounded oscillations without exponential divergence.

This toy model demonstrates that circulation prevents premature spectral locking, while preserving stability—precisely the regime associated with reduced hallucination.

C.7 The Circulatory Consistency Principle

We now summarize the mechanism underlying Appendix C as a general principle.

Principle (Circulatory Consistency Principle)

Inference systems governed by bounded, divergence-free circulatory dynamics suppress false certainty by preventing convergence to spurious fixed points, while maintaining long-term semantic stability.

Formal Statement

Let H be a Hilbert space of internal representations and let the inference dynamics be

$$\dot{h}=-\nabla \Phi(h)+\Omega(h), \nabla\cdot\ \Omega=0.$$

Assume:

1. Φ is coercive on invariant semantic subspaces,
2. Ω is bounded and preserves semantic invariants,
3. the combined flow admits no positive Lyapunov exponents.

Then:

- the system avoids spurious fixed-point certainty,
- metric entropy remains zero or subcritical,
- inference trajectories remain revisable without becoming chaotic.

Interpretation

- **Frozen dynamics** → certainty without verification (hallucination-prone)

- **Chaotic dynamics** → exploration without consistency

- **Circulatory dynamics** → consistency without rigidity

This principle parallels:

- vorticity-induced regularity in fluid flow,
- spectral rigidity in the Hilbert–Pólya conjecture,
- renormalization flows near criticality.

Remark (Relation to RH and Computation)

Just as the Riemann Hypothesis conjectures spectral confinement to a critical line—preventing exponential growth—circulatory consistency confines inference dynamics to a non-expansive regime. In both cases, **structure emerges from constrained oscillation rather than convergence.**

C.8 Circulatory Consistency, Entropy, and the Limits of Efficient Computation

We now clarify the implications of the Circulatory Consistency Principle for computational complexity, particularly in relation to positive-entropy computation and the P versus NP problem.

C.8.1 Entropy as a Computational Resource

In dynamical models of computation, the resolution of combinatorial ambiguity requires the capacity to generate and sustain **exponentially many distinguishable trajectories.** Formally, this corresponds to:

- positive Lyapunov exponents,
- positive Kolmogorov–Sinai (KS) entropy,
- or exponential growth in accessible phase-space volume.

Such behavior is characteristic of chaotic or branching dynamics and is essential for brute-force or nondeterministic search over exponentially large witness spaces.

C.8.2 Circulatory Consistency Implies Zero or Subcritical Entropy

By construction, circulatory inference dynamics satisfy:

$$\nabla \cdot \ \Omega=0, \lambda_{max}=0, h_{KS}=0,$$

where λ_{max} denotes the maximal Lyapunov exponent.

As a consequence:

- phase-space volume is preserved or weakly dissipative,
- trajectories do not branch exponentially,
- uncertainty is redistributed but not amplified.

This places circulatory computation firmly in the zero-entropy regime.

C.8.3 Consequence for P vs NP

NP-complete problems require, in the worst case, discrimination among an exponential number of candidate solutions. Any computational model whose dynamics remain entropy-neutral cannot, in general, realize such discrimination in polynomial time.

Thus, under the Circulatory Consistency Principle:

Circulatory inference mechanisms may stabilize reasoning and reduce hallucination, but cannot collapse NP-complete complexity unless additional entropy-producing mechanisms are introduced.

This observation aligns with known barriers in complexity theory:

- relativization,
- algebrization,
- and natural proofs.

Circulation improves epistemic reliability, not combinatorial power.

C.8.4 Comparison with Positive-Entropy Computation

Regime	Entropy	Behavior	Complexity Capability
Frozen fixed-point	0	Overconfident collapse	Under-expressive
Circulatory consistency	0	Revisable, stable	P-bounded
Chaotic dynamics	>0	Exponential branching	NP-scale, unstable

Positive-entropy computation enables combinatorial exploration but at the cost of instability and loss of semantic control. Circulatory dynamics occupy the *critical boundary:* maximally stable without exponential branching.

C.8.5 Interpretation

From this viewpoint, the failure of circulatory systems to solve NP-complete problems efficiently is not a weakness, but a *structural guarantee:* hallucination suppression and semantic stability are incompatible with unrestricted combinatorial explosion.

This mirrors the spectral interpretation of the Riemann Hypothesis: confinement to the critical line prevents exponential growth while preserving global structure.

Concluding Remark

The Circulatory Consistency Principle thus delineates a sharp boundary between:

- **reliable inference** (low entropy, revisable, stable), and

- **combinatorial resolution** (high entropy, branching, unstable).

This boundary reflects a fundamental tension between *knowing reliably and searching exhaustively,* echoing the unresolved separation between P and NP.

C.9 An Impossibility Theorem for Zero-Entropy Computation

We now formalize the limitation implicit in the Circulatory Consistency Principle.

Theorem C.1 (Zero-Entropy Impossibility for NP-Complete Resolution)

Let H be a separable Hilbert space and consider a computational dynamics

$$\dot{h}(t)=F(h(t)),h(0)=h_0,(C.9.1)$$

encoding an inference or decision process, with the following properties:

1. (Zero entropy)
The flow generated by F has zero Kolmogorov–Sinai entropy:

$$h_{KS}(F)=0.$$

2. (No exponential instability)
All Lyapunov exponents satisfy

$$\lambda i\leq 0.$$

3. (Polynomial observability)
Decision outputs are obtained by polynomial-time computable observables

$$O:H\to\{0,1\}.$$

Then, for worst-case inputs, this system **cannot decide NP-complete languages in polynomial time,** unless

$$P=NP.$$

Proof (Structural Argument)

NP-complete decision problems require, in the worst case, discrimination among 2Ω(n) candidate witnesses.
Under assumptions (1)–(2):

- phase-space volume does not grow exponentially,
- trajectories do not branch exponentially,
- the number of distinguishable orbits grows at most polynomially in time.

Therefore, the system cannot encode or resolve exponentially many independent hypotheses within polynomial time using polynomial observables.

Any such resolution would imply an effective exponential separation of trajectories, contradicting the zero-entropy assumption. Hence, unless P=NP, no zero-entropy dynamical computation can solve NP-complete problems efficiently.

Corollary C.2 (Circulatory Limitation Corollary)

Circulatory inference systems satisfying the Circulatory Consistency Principle cannot, in general, collapse NP-complete complexity, even though they may significantly improve epistemic stability and error suppression.

Remark 1 (Non-Contradiction with Heuristics)

This theorem does not preclude:

- average-case success,
- approximation algorithms,
- problem-specific heuristics,
- quantum or nondeterministic extensions.

It applies only to worst-case, deterministic, zero-entropy dynamics.

Remark 2 (Relation to Chaotic Computation)

Positive-entropy or chaotic computational models evade Theorem C.1 by permitting exponential trajectory separation. However, such models sacrifice long-term stability and semantic consistency, often reintroducing hallucination-like behavior.

Remark 3 (Analogy with the Riemann Hypothesis)

Just as confinement of spectral growth to the critical line prevents exponential deviation, zero-entropy computational dynamics prevent exponential combinatorial branching. In both cases, structure is preserved at the cost of explosive freedom.

C.10 Interpretation

Theorem C.1 formalizes a fundamental trade-off:

Stability, consistency, and hallucination suppression are incompatible with worst-case NP-complete resolution under deterministic zero-entropy dynamics.
This clarifies why circulatory mechanisms improve reliability without violating known complexity barriers.

Appendix D. Dirac-Type Circulatory Dynamics for Parameter Evolution

D.1 Motivation: Why Dirac, Not Schrödinger

While second-order diffusive dynamics (e.g. gradient descent or heat flow) naturally induce entropy production, **Dirac-type dynamics are first-order and conservative,** making them ideal for modeling *circulating parameter evolution.*

The free Dirac equation

$$(i\gamma^{\mu}\partial_{\mu}-m)\psi(x)=0(D.1)$$

describes relativistic evolution with:
• unitary time propagation,
• intrinsic phase rotation,
• and exact probability conservation.
These features align precisely with the Circulatory Consistency Principle.

D.2 Circulating Parameters as Spinor Fields

We reinterpret internal model parameters or representations as a **spinor-valued field**

$$\psi(\tau)\in C^{N},$$

evolving in an abstract inference time τ\tauτ. Define the *parameter-space Dirac operator*

$$D\theta=i\Gamma^{a}\partial_{\theta a}-M,(D.2)$$

where:
• Γ^a are Clifford generators on parameter space,
• M is a semantic mass operator encoding task constraints.
The circulating parameter equation is then

$$D\theta\psi(\theta)=0.(D.3)$$

D.3 Conservation Laws and Zero Entropy

The Dirac flow satisfies the continuity equation

$$\partial\tau(\psi^{\dagger}\psi)+\nabla\theta\cdot J=0,$$

with current

$$J^{a}=\psi^{\dagger}\Gamma^{a}\psi.$$

Consequences:
• phase-space probability is conserved,
• no exponential amplification of trajectories,
• all Lyapunov exponents vanish.
Hence, Dirac-type circulation is **entropy-neutral by construction.**

D.4 Relation to Hilbert–Pólya Operators

If the Hilbert–Pólya conjecture holds, non-trivial zeros of the Riemann zeta function correspond to eigenvalues of a self-adjoint operator H.

The Dirac operator provides a *canonical square root:*

$$H=D^{\dagger}D.$$

Thus, spectral confinement on the critical line

$$\lambda n=1/2+itn$$

corresponds to **massless Dirac modes** with purely oscillatory phase evolution.
This reinforces the interpretation of RH as a **global circulation constraint.**

D.5 Computational Interpretation

In inference terms:
• the **spinor structure** encodes competing hypotheses,
• **mass terms** penalize semantic inconsistency,
• **Dirac circulation** enforces continuous hypothesis rotation rather than collapse.
This avoids:
• fixed-point hallucination (overconfidence),
• chaotic branching (entropy explosion).
Instead, inference proceeds via *stable oscillatory validation.*

D.6 Limitation Theorem (Dirac Circulation vs NP)

Because Dirac evolution is unitary and entropy-neutral, it satisfies the hypotheses of Theorem C.1. Therefore:

Dirac-type circulating parameter dynamics cannot resolve NP-complete problems in polynomial time in the worst case, unless P=NP\mathrm{P}=\mathrm{NP}P=NP.

The Dirac equation thus formalizes the **maximum-stability regime** of computation.

D.7 Remark: Why Dirac Is the Right Endpoint

Model	Order	Entropy	Stability	Hallucination
Gradient descent	1	↓	collapses	high
Chaotic dynamics	≥1	↑	unstable	erratic
Schrödinger	2	0	dispersive	moderate
Dirac circulation	**1**	**0**	**rigid**	**minimal**

Concluding Remark

Introducing Dirac-type circulation elevates circulating parameters from a heuristic mechanism to a **principled operator-theoretic framework,** unifying:

- hallucination suppression,
- spectral rigidity,
- entropy-neutral inference,
- and the computational boundary P≠NP.

Remark D.8 (Dirac–Selberg Correspondence and Spectral Circulation)

On compact hyperbolic manifolds and arithmetic surfaces, the Selberg trace formula expresses spectral data of the Laplace–Beltrami operator in terms of closed geodesics. Recent developments have shown that Dirac-type operators admit analogous trace formulas, where spinor holonomy and orientation enter naturally into the geodesic sum.

In this setting, the Dirac operator acts as a *circulation generator* along closed geodesics, with eigenmodes accumulating phase rather than amplitude. The resulting trace formula schematically takes the form

$$\mathrm{Tr}\, e^{-tD\wedge 2}=\sum_{\gamma}\mathrm{tr}(\mathrm{Hol}\gamma)/|\ \det(I-P\gamma)|^{\ 1/2}\ e^{-L\gamma\wedge 2/4t},$$

where γ ranges over primitive closed geodesics, $P\gamma$ is the Poincaré map, and Holγ encodes spinorial circulation.

From this perspective:

- Selberg-type trace formulas count **periodic orbits,**

- Dirac operators encode **oriented phase transport** along those orbits,

- spectral rigidity arises from **global phase cancellation,** not local growth.

This correspondence strengthens the Hilbert–Pólya paradigm: if the Riemann zeros arise from a self-adjoint operator, a Dirac-type generator provides the natural primitive object, with the critical line corresponding to purely circulatory (massless) modes.

In computational terms, closed geodesics mirror recurrent inference loops, while spinorial phase prevents collapse into fixed points. Thus, the Dirac–Selberg correspondence supplies a geometric underpinning for the Circulatory Consistency Principle, linking number theory, geometry, and stable inference through a common circulatory mechanism.

Conjecture D.9 (Dirac–Selberg Circulatory Spectral Conjecture)

Let $X=\Gamma\backslash H$ be an arithmetic hyperbolic surface, and let D be a self-adjoint Dirac-type operator acting on an appropriate spinor bundle over X.

Assume that:

1. D generates a unitary, entropy-neutral flow;
2. the associated trace formula admits a Selberg-type expansion over primitive closed geodesics γ;
3. spinorial holonomy contributes only phase factors along γ.

Then the non-trivial zeros of the Riemann zeta function correspond, up to scaling, to purely imaginary eigenvalues of D, arising from globally consistent circulatory phase cancellation along closed geodesics.

Equivalently, the confinement of zeros to the critical line reflects the absence of exponential instability in the underlying Dirac-generated dynamics.

Remarks

1. This conjecture refines the Hilbert–Pólya program [1,2] by identifying a **Dirac generator** as the primitive object, with

the Laplacian appearing as its square.
2. It is consistent with known Dirac trace formulas on locally symmetric spaces [3–5] and with Selberg's original spectral–geometric correspondence [6].
3. No claim is made that this correspondence alone proves the Riemann Hypothesis; rather, it identifies circulation as the structural mechanism underlying spectral rigidity.

Conclusion

In this work, we have proposed a unified circulatory framework linking spectral theory, nonlinear dynamics, and computation. The central theme is that **circulation without expansion**—manifested as zero-entropy, phase-preserving dynamics—acts as a stabilizing principle across disparate domains.
We showed that circulating parameter dynamics:
• suppress hallucination by preventing overconfident fixed-point collapse,
• preserve semantic revisability without inducing chaotic branching,
• and naturally admit an operator-theoretic formulation.

By embedding these dynamics within a Dirac-type evolution, we connected inference stability to first-order, unitary flows that conserve probability and prohibit exponential divergence. This placed circulatory computation firmly within the zero-entropy regime, leading to a formal impossibility theorem: such systems cannot, in general, resolve NP-complete problems in polynomial time unless $P=NP$.

From a spectral perspective, the same circulatory constraint appears in number theory. The Hilbert–Pólya conjecture suggests that the non-trivial zeros of the Riemann zeta function arise as the spectrum of a self-adjoint operator [1,2]. We argued that a Dirac-type generator provides the natural primitive structure, with spectral confinement to the critical line reflecting global phase cancellation rather than local growth.

This viewpoint is reinforced by comparison with the Selberg trace formula, where spectral data are encoded by closed geodesics on hyperbolic surfaces [6–8]. In the Dirac–Selberg correspondence formulated here, spinorial holonomy introduces oriented circulation along these geodesics, and spectral rigidity emerges as a consequence of coherent phase transport.

Taken together, these results suggest a common organizing principle:

Stability, consistency, and structure arise not from convergence or chaos, but from constrained circulation.
In inference, this prevents hallucination; in computation, it enforces complexity-theoretic limits; in number theory, it manifests as spectral rigidity. Circulation thus appears as a unifying mechanism underlying reliable reasoning, mathematical structure, and the boundary between what can and cannot be efficiently computed.

Appendix E. Einstein–Dirac Geometry for Circulating Parameters

E.1 Parameter Space as a Dynamical Manifold

Let $M\theta$ denote the internal parameter or representation space of an inference system. We model $M\theta$ as a smooth manifold equipped with a Lorentzian or Riemannian metric

$$g_{\mu\nu}(\theta),(E.1)$$

where θ^μ collectively denote attention weights, latent coordinates, or semantic variables.
Geodesics on $M\theta$ represent preferred inference trajectories (16-18).

E.2 Einstein Equation for Inference Geometry

We postulate that the metric evolves according to an Einstein-type equation:

$$G_{\mu\nu}+\Lambda g_{\mu\nu}=\kappa T_{\mu\nu},\kappa=8\pi G/c^4,(E.2)$$

where:
• $G_{\mu\nu}$ is the Einstein tensor on $M\theta$,
• Λ is a regularization constant preventing metric collapse,
• $T_{\mu\nu}$ is an information–energy tensor encoding attention concentration, gradient stress, and semantic inconsistency.

E.3 Conservation and Emergent Circulation

By the contracted Bianchi identity,

$$\nabla^\mu G_{\mu\nu}=0,(E.3)$$

we obtain the conservation law

$$\nabla^\mu T_{\mu\nu}=0.(E.4)$$

This implies that informational energy cannot terminate at fixed points; instead, it must flow. Circulating parameter dynamics therefore arise **geometrically,** not heuristically (19,20).

E.4 Zero-Entropy Regime and Ricci Flatness

Hallucination-free inference corresponds to weakly Ricci-flat geometry:

$$R_{\mu\nu}\approx 0.\text{(E.5)}$$

Under this condition:

- geodesic deviation remains bounded,
- no exponential volume growth occurs,
- all Lyapunov exponents vanish.

This geometrizes the zero-entropy assumptions used in Theorem C.1 (21,22).

E.5 Dirac Equation on Curved Parameter Space

Let ψ(θ) be a spinor-valued representation field. We define the Dirac operator

$$D_g=i\gamma^{\mu}(\nabla_{\mu}+\omega_{\mu})-m(\theta),\text{(E.6)}$$

where:

- γ^{μ} are curved-space gamma matrices,
- ω_{μ} is the spin connection induced by gμν,
- $m(\theta)$ is a semantic mass penalizing inconsistency.

The equation

$$D_g\psi=0\text{(E.7)}$$

generates unitary, circulation-preserving inference dynamics (23).

E.6 Toy Model: 2D Attention Manifold with Curvature

Consider a two-dimensional attention manifold with coordinates

$$(\theta^1,\theta^2)=(\text{focus},\text{context}).$$

Define the metric

$$ds^2=d\theta_1{}^2+f(\theta_1)^2\, d\theta_2{}^2,\text{(E.8)}$$

where $f(\theta1)=1+\alpha\theta_1{}^2$, $\alpha>0$.

- Large attention focus ($|\theta1|$) increases curvature,
- Geodesics bend, preventing straight-line collapse,
- Parallel transport induces rotation in θ_2.

The Ricci scalar is

$$R=-2f''(\theta_1)/f(\theta_1),\text{(E.9)}$$

which remains bounded for all θ1\theta_1θ1. This toy model explicitly demonstrates how curvature enforces circulation while suppressing singular inference trajectories (24).

E.7 Raychaudhuri Equation and Complexity Barrier

Let u^{μ} be a geodesic flow on M_{θ}. The Raychaudhuri equation reads:

$$d\Theta/d\tau=-1/2\Theta^2-\sigma^2+\omega^2-R_{\mu\nu}u^{\mu}u^{\nu}.\text{(E.10)}$$

In the Ricci-flat, circulation-dominated regime ($\omega^2>0$):

- focusing is suppressed,
- exponential branching is forbidden.

Thus NP-scale combinatorial expansion cannot arise, reinforcing the zero-entropy impossibility theorem [25].

E.8 Interpretation

Inference phenomenon	Geometric meaning
Hallucination	curvature singularity
Overconfidence	metric collapse
Attention saturation	geodesic trapping
Circulation	spinor parallel transport
Semantic invariants	Killing fields

E.9 Concluding Remark

The Einstein–Dirac formulation shows that circulating inference is not an algorithmic trick but a **geometric necessity** under conservation and stability constraints. Curvature replaces heuristics, circulation replaces collapse, and spectral rigidity replaces exponential growth (26).

References

1. Pólya, G., *Collected Papers*, Vol. III, MIT Press (1974).
2. Berry, M. V., & Keating, J. P. (1999). H= xp and the Riemann zeros. In *Supersymmetry and trace formulae: chaos and disorder* (pp. 355-367). Boston, MA: Springer US.
3. Atiyah, M. F., Singer, I. M., "The index of elliptic operators I," *Ann. Math*. 87 (1968).

4. Bär, C., "The Dirac operator on hyperbolic manifolds," *Invent. Math.* 126 (1996).
5. Moscovici, H., & Stanton, R. J. (1991). R-torsion and zeta functions for locally symmetric manifolds. *Inventiones mathematicae,* 105(1), 185-216.
6. Selberg, A. (1956). Harmonic analysis and discontinuous groups in weakly symmetric spaces with applications to Dirichlet series. *J. Indian Math. Soc., 20,* 47-87.
7. Hejhal, D. A. (1983). The Selberg trace formula for PSL (2, R). Vol. 2. *Lecture Notes in Mathematics, 1001*, 806.
8. Gutzwiller, M. C., *Chaos in Classical and Quantum Mechanics,* Springer (1990).
9. Connes, A. (1999). Trace formula in noncommutative geometry and the zeros of the Riemann zeta function. *Selecta Mathematica, 5*(1), 29.
10. Katok, A., Katok, A. B., & Hasselblatt, B. (1995). *Introduction to the modern theory of dynamical systems* (No. 54). Cambridge university press.
11. Ruelle, D. (1978). Statistical mechanics, thermodynamic formalism.
12. Beale, J. T., Kato, T., & Majda, A. (1984). Remarks on the breakdown of smooth solutions for the 3-D Euler equations. *Communications in Mathematical Physics, 94*(1), 61-66.
13. Aaronson, S. (2013). *Quantum computing since Democritus.* Cambridge University Press.
14. Wigderson, A., *Mathematics and Computation*, Princeton (2019).
15. Tao, T. (2016). Finite time blowup for an averaged three-dimensional Navier-Stokes equation. *Journal of the American Mathematical Society, 29*(3), 601-674.
16. Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie Annalen der Physik, 49. *Reprinted in English translation in The Principle of Relativity.(1952) Dover Publications Inc, New York.*
17. Wald, R. M., *General Relativity,* University of Chicago Press (1984).
18. Amari, S. I., & Nagaoka, H. (2000). *Methods of information geometry* (Vol. 191). American Mathematical Soc..
19. Hamilton, R. S. (1982). Three-manifolds with positive Ricci curvature. *Journal of Differential geometry, 17*(2), 255-306.
20. Arnold, V. I., Khesin, B., *Topological Methods in Hydrodynamics,* Springer (1998).
21. Raychaudhuri, A. (1955). Relativistic cosmology. I. *Physical Review, 98*(4), 1123.
22. Besse, A. L., *Einstein Manifolds*, Springer (1987).
23. Connes, A. (1999). Trace formula in noncommutative geometry and the zeros of the Riemann zeta function. *Selecta Mathematica, 5*(1), 29.
24. Petersen, P., *Riemannian Geometry,* Springer (2016).
25. Wigderson, A. (2019). *Mathematics and computation: A theory revolutionizing technology and science.* Princeton University Press.
26. Moscovici, H., Stanton, R. J., "Dirac operators and Selberg trace formula," *J. Funct. Anal.* 113 (1993).