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# Construction of a Physically Adequate Coordinate System with Help of an Observation on Earth's Ground

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## Abstract

The International Astronomical Union formulated the problem of finding a useful and adequate coordinate system (ACS) in astronomy. The ACS is crucial, in order to achieve adequate predictions about the time dilation of each clock onboard a spacecraft. For it, a general method for the construction of an ACS is provided. Thereby, the constructed ACS represents the properties of real space and of the theory of relativity. In particular, the ACS is applied to a precise measurement of a time dilation and of a potential at Earth. Hereby the ACS provides precise accordance of theory and observation. Moreover, at each point P in the universe, the ACS provides a zero point of the kinematic time difference. Similar to the zero point of temperature, zero Kelvin (1848), the achieved zero point of the kinematic time difference is based on a zero point of motion [1].

**Keywords:** Relativity, Adequate coordinate system, Frame, Observation, Earth, Clock, Motion

## Introduction

In nature, in space-navigation and in geodesy, the phenomenon of time dilation is essential. For instance, a gravitational potential difference between two points A and B causes a gravitational time dilation, see Pound and Rebka (1960) and Hobson et al. (2006) [2,3]. Similarly, a speed v can cause a kinematic time dilation, see Starker et al. (1985) [4]. General relativity theory (GRT) provides an equation for both forms of time dilation, see Einstein (1905, 1915), Hobson et al. (2006) [3,5,6]. Though GRT provides the key equations for time dilation, GRT does not provide sufficient information about the coordinate systems, in which these equations predict an observation correctly. This problem is outlined in the next section:

## Problem of Constructing an Adequate Coordinate System

The International Astronomical Union (IAU) realized, that the equation of time dilation of general relativity theory (GRT) is not sufficient for space navigation. In particular, in order to use the equation of time dilation in space navigation, a useful and adequate coordinate system is needed in addition, see Soffel et al. (2003), p. 2688 [7]. That adequate coordinate system (ACS) should be in accordance with the equation of time dilation of GRT, See Soffel et al. (2003), p. 2687. Accordingly, the IAU formulated the problem of finding a useful and adequate coordinate system, See Soffel et al. (2003), p. 2688) [7]. Additionally, for the physics community, the ACS is named adequate frame and the problem of finding an ACS is named frame problem, see Carmesin (2025a) [8]. For comparison, Newtonian mechanics is based on three equations or axioms, and on the coordinate system, in which these equations hold: an inertial frame, or inertial coordinate system, see Landau and Lifschitz (1960) [9]. In general, in order to provide a prediction in physics, at least two items are needed: a physical equation and a coordinate system, in which this equation holds.

## A Provisional Solution

As a provisional solution, the IAU recommended a Geocentric Celestial Reference System (GCRS) for the case of space navigation in the vicinity of Earth. This corresponds to the empirical finding, that an Earth centered coordinate system describes correctly the clocks onboard a spacecraft in the vicinity of Earth, see Starker et al. (1985) [4]. Additionally, the

IAU recommended a Barycentric Celestial Reference System (BCRS), for the case of space navigation in our planetary system. The BCRS is nearly heliocentric. This BCRS is in accordance with the empirical finding that space missions in the planetary system are adequately described by a heliocentric or barycentric coordinate system, see e. g. NASA (1963), Lavery (1972), See Soffel et al. (2003) [7,10,11].

Clearly, this proposal is not uniquely determined, as Earth is a part of our planetary system. Accordingly, essential questions remain: In what region are the GCRS and the BCRS useful and adequate? How can a useful and adequate coordinate system be measured at Earth or in a spacecraft?

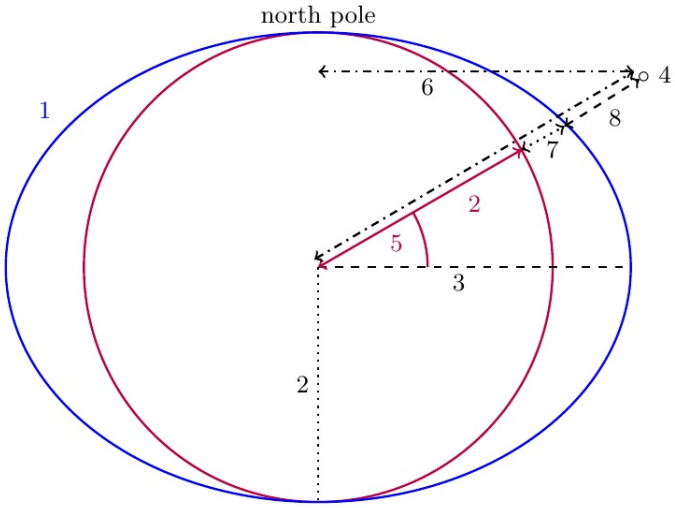


**Figure 1: Sketch of a Cross Section of Earth Through its Center: Geoid (Surface of Oceans at Sea Level) (1), Radius at the Pole (2), Radius at the Equator (3), Point *P* at Earth's Surface (4), Latitude beta_P (5), Radius $R_{p,\omega}$ of the Rotation of *P* Around The Axis of Earth (6), Additional Radius Caused by the Geoid (7), Height $H_p$ of *P* Above Sea Level (8).**

## General Solution and Particular Example

In this paper, a general solution of the frame problem is provided, see section A Solution of the Frame Problem. Additionally, an ACS is determined for the case of the surface of Earth (Fig. 1). For this particular determination, an experiment is evaluated. This experiment used measurement points at the Max Planck Institute of Quantum Optics (MPQ) near Munich and at the Physikalisch - Technische Bundesanstalt (PTB) in Braunschweig. In that experiment, the potential difference ΔΦ between the MPQ and PTB has been measured with help of an optical lattice clock Grotti et al. (2024) [12]. Additionally, that potential ΔΦ has been measured with help of geodetic methods, for such methods see Denker et al. (2018) [13].

## A Solution of the Frame Problem

In this section, a general and founded solution of the frame problem is presented.

## Concept of the Solution

In this section, the concept of the solution of the frame problem is presented. The concept is based on the fact, that a physical theory needs equations and coordinate systems, in which these equations hold. An example is Newton's mechanics: The equations can be summarized by Newton's axioms. These axioms hold in inertial coordinate systems only [9]. Consequently, each inertial coordinate system is an adequate coordinate system (ACS) of Newton's mechanics. Therefore, the concept of the presented solution of the frame problem is summarized and defined as follows:

• In general, the velocity $\vec{v}_{C,CS}$ of a clock *C* in a coordinate system CS, and the velocity $\vec{v}_{ACS,CS}$ of an ACS in a coordinate system CS, are described relative to an arbitrarily chosen coordinate system CS.

• In particular, the velocity $\vec{v}_{C,ACS}$ of a clock C relative to an adequate coordinate system ACS is the following difference

$$\vec{v}_{C,ACS} := \vec{v}_{C,CS} - \vec{v}_{ACS,CS}. \tag{1}$$

• In relativity theory, the equation of the kinematic time difference $\delta t_{kin}$ of a clock *C* represents a function of the velocity of *C* in a coordinate system:

$$\delta t_{kin} = \delta t_{kin}(\vec{v}_{C,CS}), \quad \text{in relativity theory.} \tag{2}$$

A coordinate system is an ACS, if and only if the time of a clock *C* is described correctly by $\delta t_{kin}$ as a function of the velocity $\vec{v}_{C,ACS}$ of *C* relative to an ACS:

$$\delta t_{kin} = \delta t_{kin}(\vec{v}_{C,ACS}) = \delta t_{kin}(\vec{v}_{C,CS} - \vec{v}_{ACS,CS}). \tag{3}$$

Hereby, more generally, the equations of the relativistic time dilation or time difference can be used, see Eq. (9) or the approximation in Eq. (19).

In order to derive the solution in detail, the equations of time dilation are summarized in the next section:

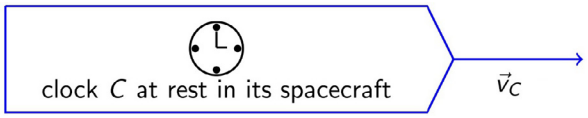


**Figure 2: A Clock C is at Rest Onboard a Spacecraft. In a Wider Sense, the Spacecraft Can be at Rest on a Celestial Body or the Clock Can be at a Ground Station on a Celestial Body**

## Equation of Time Dilation Used in the Solution

In this section, the *equation of time dilation* of general relativity theory (GRT) is derived. This includes the equation of kinematic time dilation of special relativity theory (SRT). Thereby, that equation is derived progressively for various useful cases and in corresponding forms. In particular, that equation is derived for the case of a relatively slow $(v/c \ll 1)$ clock $C$ in a relatively small absolute value of the potential $(2|\Phi|/c^2 \ll 1)$, see section (2.2.4). Basically, that equation of time dilation is based on the scalar product that Einstein (1916) proposed in his Eq. 3:

$$ds^2 := \sum_{i=0}^{i=3} \sum_{j=0}^{j=3} g_{ij} dx_i dx_j \tag{4}$$

This is an invariant with respect to transformations of the coordinate system [14]. Hereby, the increment $dx_0 = c \bullet dt$ describes the distance that light propagates during an increment $dt$, whereby $dt$ describes a coordinate time, describing a flat spacetime. Similarly, the increments $dx_1$, $dx_2$ and $dx_3$ describe the spatial coordinates in a flat spacetime. Thereby, the numbers $g_{ij}$ are the elements of the metric tensor. Moreover, in GRT, $ds^2$ is a short notation for $(ds)^2$, or $dx^2_j$ is a short notation for $(dx_j)^2$, see e. g. Hobson et al. (2006) [3]. Without loss of generality, we choose a locally orthogonal coordinate system. As a consequence, the scalar product in Eq. (4) has the following form

$$ds^2 = \sum_{j=0}^{j=3} g_{jj} dx_j^2 = g_{00} \cdot c^2 \cdot dt^2 - \sum_{j=1}^{j=3} |g_{jj}| dx_j^2. \tag{5}$$

In a typical analysis in GRT, a clock $C$ is considered. In the coordinate system or reference system of the clock C, the time increment measured by the clock is named $d\tau$, and the spatial increments are zero, $d\xi_1 = 0$, $d\xi_2 = 0$ and $d\xi_3 = 0$. Consequently, the scalar product $ds^2$ is equal to $c^2 d\tau^2$,

$$ds^2 = c^2 d\tau^2 \quad \text{in a clock's coordinate system} \tag{6}$$

Of course, the usual GRT and SRT notation with $\tau$ for a system with a clock is hardly specific by itself. The notation can be made specific in a context.

As a further consequence, the squared time increment $d\tau^2$ in the coordinate system of a clock $C$ in Eq. (6) is transformed to the squared time increment $dt^2$ in an *external coordinate system* (with respect to the clock C) in Eq. (5) by using the invariance of $ds^2$:

$$c^2 d\tau^2 = ds^2 = g_{00} \cdot c^2 \cdot dt^2 - \sum_{j=1}^{j=3} |g_{jj}| dx_j^2. \tag{7}$$

In order to solve for $d\tau^2$, the above Eq. is divided by $c^2$. Moreover, the above sum is multiplied by $1 = \frac{dt^2}{dt^2}$, and $\sum_{j=1}^{j=3} |g_{jj}| dx_j^2/dt^2$ is identied with the squared velocity of the clock $C$ in the external coordinate system. As a consequence, the squared time measured by the clock $C$ has the following form

$$d\tau^2 = dt^2 \cdot \left( g_{00} - \sum_{j=1}^{j=3} |g_{jj}| \frac{dx_j^2}{dt^2} \frac{1}{c^2} \right) = dt^2 \cdot \left( g_{00} - \frac{v^2}{c^2} \right). \tag{8}$$

In order to solve for $d\tau$, the square root is applied to the above Eq.

$$d\tau = dt \cdot \sqrt{g_{00} - \frac{v^2}{c^2}}. \tag{9}$$

In the case $v = 0$, there remains the gravitational time dilation:

$$d\tau_{grav} = dt \cdot \sqrt{g_{00}} \quad \text{or} \quad \frac{d\tau_{grav} - dt}{dt} = \sqrt{g_{00}} - 1. \tag{10}$$

## Equation of Time Dilation in SRT

In the particular case of a flat spacetime, the tensor element $g_{00}$ is one. As a consequence, the time $d\tau$ measured by a considered clock $C$ in Eq. (9) has the form

$$d\tau = dt \cdot \sqrt{1 - \frac{v^2}{c^2}}, \quad \text{in flat spacetime.} \tag{11}$$

This term is identified with the equation of time dilation in special relativity theory, SRT, see Einstein (1905), Hobson et al. (2006). Minkowski (1908) explained this fact by realizing that SRT describes a flat spacetime, which is now called Minkowski space [5, 15].

## Equation of Time Dilation in the Vicinity of a Mass

Schwarzschild (1916) derived the metric tensor elements $g_{ij}$ that describe spacetime in the vicinity of an uncharged and non-rotating mass $M$ [16]. Thereby, at a coordinate distance $r$ from $M$, the element $g_{00}$ is

$$g_{00} = 1 - \frac{R_S}{r} \quad \text{with} \quad R_S = \frac{2GM}{c^2} \tag{12}$$

Hereby, $R_s$ is named Schwarzschild radius, $G$ is Newton's gravitational constant $G$ = 6.674 30(15) $10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg \cdot s}^2}$, and c is the velocity of light in vacuum, $c$ = 299 792 458 $\frac{\mathrm{m}}{\mathrm{s}}$, see Workman et al. (2022) [17].

## Equation of Time Dilation at a Potential

In Newton's theory of gravitation, the gravitational potential at a coordinate r is as follows [18]. This term has also been derived in an exact theory of gravity, in which distances are determined according to the local curvature of spacetime, see Carmesin (2025a) [8].

$$\Phi(r) = -\frac{GM}{r}. \tag{13}$$

As a consequence, the tensor element $g_{00}$ in Eq. (12) as a function of the potential in Eq. (13) is

$$g_{00} = 1 - \frac{2|\Phi|}{c^2}. \tag{14}$$

As a consequence, the time $d\tau$ measured by a considered clock C in Eq. (9) has the form

$$d\tau = dt \cdot \sqrt{1 - \frac{2|\Phi|}{c^2} - \frac{v^2}{c^2}}, \quad \text{near a mass.} \tag{15}$$

## Equation of Time Dilation for A Slow Clock in a Small Potential

In this section, equation (15) of time dilation is analyzed for a clock $C$ that has a relatively small velocity v with respect to the external coordinate system, and that is in a relatively small absolute value of the potential |Φ|,

$$\frac{v}{c} \ll 1, \quad \text{and} \quad \frac{2|\Phi|}{c^2} \ll 1. \tag{16}$$

For instance, these conditions are fulfilled for vehicles including space crafts that travel in our planetary system, in the vicinity of Earth or on Earth. As a consequence of the condition in Eq. (16), the root in equation (15) of time dilation can be expanded at linear order, marked by $\doteq$ ,

$$d\tau \doteq dt - dt \cdot \frac{|\Phi|}{c^2} - dt \cdot \frac{v^2}{2c^2}. \tag{17}$$

In equation (17) of time dilation, the subtrahends are identified with the gravitational time difference $d\tau_{grav}$ and the kinematic time difference $d\tau_{kin}$,

$$d\tau \doteq dt + d\tau_{grav} + d\tau_{kin}, \quad \text{with} \quad d\tau_{grav} = -dt \cdot \frac{|\Phi|}{c^2}, \quad \text{and} \quad d\tau_{kin} = -dt \cdot \frac{v^2}{2c^2}. \tag{18}$$

The above time increments are divided by the increment of the coordinate time $dt$,

$$\frac{d\tau}{dt} \doteq 1 + \delta\tau_{grav,frac} + \delta\tau_{kin,frac}, \quad \text{with} \quad \delta\tau_{grav,frac} = -\frac{|\Phi|}{c^2} = \frac{\Phi}{c^2}, \quad \text{and} \quad \delta\tau_{kin,frac} = -\frac{v^2}{2c^2}. \tag{19}$$

Hereby, $\delta\tau_{grav,frac} := \frac{d\tau_{grav}}{dt}$ and $\delta\tau_{kin,frac} := \frac{d\tau_{kin}}{dt}$ are called gravitational fractional time difference and kinematic fractional time difference. The time difference and the fractional time difference are

$$\delta t = d\tau - dt \quad \text{and} \quad \delta\tau_{frac} := \frac{\delta t}{dt} = \delta\tau_{grav,frac} + \delta\tau_{kin,frac}. \tag{20}$$

Next, these equations of time difference are applied in order to derive the ACS.

## Direct Measurement, Existence and Uniqueness of the Solution

In this section, for each point *P*, an ACS is identified. While the ACS is a physically based coordinate system, an arbitrary conventional reference system is used as a tool. Similarly, in order to express the physically based absolute temperature with its Kelvin (1848) scale, the conventional Celsius scale is used: zero Kelvin equals minus 273.15 degree Celsius [1].

## Conventional Reference System

In general, each point *P* can be located relative to one or more celestial bodies. These celestial bodies are used as an arbitrary conventional reference system. Additionally, a clock at the conventional reference system can be used as a reference. Hereby, in general, P can move relative to the conventional reference system.

## Measurement of the Velocity of the ACS

In this section, a method for the measurement of the velocity $\vec{v}_{ACS}$ of the ACS is presented. Thereby, that velocity $\vec{v}_{ACS}$ is described relative to the conventional reference system in section Conventional Reference System.

At each point *P*, the velocity $\vec{v}_{ACS}$ of the ACS can be measured as follows:

A spacecraft with an onboard clock *C* and a velocity $\vec{v}_C$ is used (see Fig. 2). When the spacecraft is at rest in the ACS, then the velocity $\vec{v}_C$ of the clock is equal to the velocity $\vec{v}_{ACS}$ of the ACS:

$$\vec{v}_{ACS} - \vec{v}_C = 0, \quad \text{iff C is at rest in the ACS.} \tag{21}$$

Hereby and as usual, iff abbreviates 'if and only if'. The above equation is transformed equivalently by application of the square and division by $-2c^2$:

$$-\frac{(\vec{v}_{ACS} - \vec{v}_C)^2}{2c^2} = 0, \quad \text{iff C is at rest in the ACS.} \tag{22}$$

Moreover, when the spacecraft is at rest in the ACS, the law of kinematic time dilation holds, see Eq. (19) and see section Concept of the solution:

$$\delta\tau_{kin,frac} = \frac{-(\vec{v}_{ACS} - \vec{v}_C)^2}{2c^2} = 0, \quad \text{iff C is at rest in the ACS.} \tag{23}$$

In the above equation, the fraction is zero iff it takes its maximal value $\delta\tau_{kin,frac,max}$:

$$\delta\tau_{kin,frac} = \delta\tau_{kin,frac,max} = 0, \quad \text{iff C is at rest in the ACS.} \tag{24}$$

The above equation and statement are used for the measurement of $\vec{v}_{ACS}$ as follows: For each point P, a spacecraft, including another vehicle in a wider sense, with an onboard clock *C* is used, see Fig. (2). Thereby, the clock is at rest in the spacecraft. The spacecraft is moved so that the observed kinematic time difference of the clock C is maximal. This is tested by a permanent comparison with a clock at the conventional reference system. For it, a permanent radio transmission is used, whereby a possible Doppler effect is corrected. Moreover, the velocity $\vec{v}_C$ is measured (relative to the conventional reference system).

As a result, the spacecraft is at rest in the ACS. Consequently, the measured velocity $\vec{v}_C$ provides the velocity of the ACS (see Eq. 21):

$$\vec{v}_{ACS} = \vec{v}_C, \quad \text{as the clock C is at rest in the ACS.} \tag{25}$$

Altogether, this method can provide the ACS and its velocity in general, for each point *P* in the universe. More practical devices for that measurement are presented in Carmesin (2025a,b,c,d) [8,19,20,21].

## Existence of the ACS and Uniqueness of its Velocity

For each point *P*, the quadratic equation in Eq. (23) has exactly one solution. Therefore, for each P, there exists an ACS. Moreover, the measured velocity $\vec{v}_{ACS}$ in Eq. (25) is uniquely determined, as any velocity of the spacecraft different from $\vec{v}_{ACS}$ would cause a kinematic time difference that is not maximal. Therefore, the ACS exists and its velocity is determined in a unique manner.

Thus, the essential remaining question is: How can the velocity of the ACS be predicted?

## Prediction of the Velocity of the ACS

In this section, a prediction of the velocity of the ACS is derived. In general, at different points $P_1$ and $P_2$, the ACS has different velocities $\vec{v}_{ACS}(P_1) \neq \vec{v}_{ACS}(P_2)$. Therefore, even a fully screened clock onboard a spacecraft (see Fig. 2) must experience an interaction caused by its surroundings. Only with help of such an interaction, the clock can show the correct kinematic time differenc, as this difference depends on the velocity of the ACS, see Eq. (23). As a consequence of the screening of the clock, the electromagnetic interaction, the weak interaction, and the strong interaction can be excluded, see Workman et al. (2022). Consequently, the clock must experience the gravitational interaction, which can be expressed in terms of the local gravitational field $\vec{G}^*(\vec{r})$. This vector field represents a pattern. Accordingly, the velocity that enters the equation of the kinematic time difference in Eq. (23) is the velocity relative to this pattern of the vector field $\vec{G}^*(\vec{r})$. (Alternatively, gravity can be expressed with help of curvature of spacetime or with help of additional volume, see Carmesin (2025a) [8]).

In general, the field is moving. And a screened clock is at rest with all of its interaction with its surroundings, iff the clock is at rest at the field. Correspondingly, at each point *P*, the ACS is at rest at that pattern of the vector field $\vec{G}^*(\vec{r})$ at the point *P*, at which the ACS is determined.

For instance, in the vicinity of Earth, the vector field $\vec{G}^*(\vec{r})$ is at rest at the Earth Centered Inertial frame (ECI). Consequently, near Earth, the ACS is at rest at the ECI. This prediction is tested with help of an observation provided by Grotti et al. (2024).

## Study of the ACS With Help of an Observation on the Ground

In this section, the observation provided by Grotti et al. (2024) is used in order to test the prediction of the ACS in section Prediction of the velocity of the ACS. The considered experiment by Grotti et al. (2024) had the purpose to test optical atomic clocks at two locations. This experiment used measurement points at the Max Planck Institute of Quantum Optics (MPQ) in Garching and at the Physikalisch - Technische Bundesanstalt (PTB) in Braunschweig [12]. In that experiment, the potential difference $\Delta\Phi_{chronometric}$ between the MPQ and PTB has been measured in a chronometric manner with help of optical lattice clocks. A corresponding function is derived and marked by $\Delta\Phi_c$. Additionally, that potential difference $\Delta\Phi_{geodetic}$ has been measured with help of geodetic methods. A respective function is denoted by $\Delta\Phi_g$.

Here, the maximal possible speed of the ACS relative to the ECI is evaluated with help of these measurements. For it, the geodetically measured potential at the PTB and at the MPQ are expressed in terms of the angular frequency $\omega_{ECI}$ of Earth relative to the Earth Centered Inertial coordinate system, ECI. Similarly, the chronometrically measured potential at the PTB and at the MPQ are expressed in terms of the angular frequency $\omega_{ACS}$ of Earth relative to the ACS. Hereby, the celestial body Earth is used as a conventional reference system see section 2.3.1. Accordingly, and in general, the ACS exhibits a nonzero angular frequency relative to the conventional reference system.

Altogether, the measured potentials are functions of the respective angular frequencies:

$$\Delta\Phi_g = \Delta\Phi_g(\omega_{\circlearrowleft,ECI}) \quad \text{and} \quad \Delta\Phi_c = \Delta\Phi_c(\omega_{\circlearrowleft,ACS}) \tag{26}$$

Based on these relations and measurements, the difference $\omega_{\circlearrowleft,ACS} - \omega_{\circlearrowleft,ECI}$ can be derived and investigated. Next, the evaluation is presented in detail:

| quantity | variable | result | unit | Ref. |
|---|---|---|---|---|
| radius | $R_e$ | 6 378 137 | m | WGS-84 |
| radius at the pole | $R_p$ | 6 356 752.3142 | m | WGS-84 |
| latitude | $\beta_{PTB}$ | 52.296 | $^o$ | PTBB00DEU |
| latitude | $\beta_{MPQ}$ | 48.2597 | $^o$ | Denker et al. (2019) |
| height above sea level | $H_{PTB}$ | 83 | m | Ketzler (2025) |
| potential difference | $\Delta\Phi_{geodetic}$ | 3915.88(0.30) | $\frac{m^2}{s^2}$ | Grotti et al. (2024) |
| potential difference | $\Delta\Phi_{chronometric}$ | 3918.1(2.6) | $\frac{m^2}{s^2}$ | Grotti et al. (2024) |
| height above sea level | $H_{MPQ}$ | 481.8 | m | Eq. (43) |
| radial length of ellipsoid | $R_{PTB}$ | 6 364 759 | m | Eq. (27) |
| radial length of ellipsoid | $R_{MPQ}$ | 6 366 239.51 | m | Eq. (27) |
| speed of rotation | $v_{PTB,ECI}$ | 283.85 | $\frac{m}{s}$ | Eq. (29) |
| speed of rotation | $v_{MPQ,ECI}$ | 309.07 | $\frac{m}{s}$ | Eq. (29) |

| | | | | |
|---|---|---|---|---|
| potential | $\Phi_{PTB,g}$ | 815.63 | $\frac{m^2}{s^2}$ | Eq. (32) |
| potential | $\Phi_{MPQ,g}$ | 4730.93 | $\frac{m^2}{s^2}$ | Eq. (39) |
| potential difference | $\Delta\Phi_g$ | 3915.3 | $\frac{m^2}{s^2}$ | Eq. (42) |
| ACS to ECI difference | $\frac{\omega_{♁,ACS}^{♁}-\omega_{♁,ECI}^{♁}}{\lvert\omega_{♁,ECI}^{♁}\rvert}$ | $\leq 0.5862$ | | Eq. (48) |
| speed difference | $\frac{\lvert\vec{v}_{♁,ACS}^{♁}-\vec{v}_{♁,ECI}^{♁}\rvert}{\lvert\vec{v}_{♁,ECI}^{♁}\rvert}$ | $\leq 0.5862$ | | |
| speed difference | $v_{PTB} \cdot 0.5862$ | 166 | $\frac{m}{s}$ | Eq. (29) |

**Table 1: Overview of the Evaluation. Wgs-84 Abbreviates the World Geodetic System 1984. Ptbb00deu is a Station for Which Data are Provided by Global Navigation Systems Such as the GPS**

## Calculation

• Measured values about this experiment are listed in lines 1 - 7 in table (1): Earth's radius $R_e$ at the equator. Earth's radius $R_p$ at the pole. The latitude $\beta_{PTB}$ at the PTB. The latitude $\beta_{MPQ}$ at the MPQ. The height $H_{PTB}$ above sea level at the PTB. The geodetically measured potential difference $\Delta\Phi_{geodetic}$. The chronometrically measured potential difference $\Delta\Phi_{chronometric}$.

• The radial coordinate of Earth's ellipsoid is determined with help of the WGS-84 model as follows, see lines 9-10 in table (1):

$$R_{PTB} = \sqrt{[R_e \cdot \cos(\beta_{PTB})]^2 + [R_p \cdot \sin(\beta_{PTB})]^2} \tag{27}$$

The corresponding Eq. holds for the radial coordinate $R_{MPQ}$ at the MPQ.

• The gravitational acceleration at the PTB is

$$g_{PTB} = \frac{G \cdot M_{♁}^{♁}}{R_{PTB}^2} = 9.8395 \ \frac{m}{s^2}. \tag{28}$$

Hereby, $M_{♁}^{♁}$ is Earth's mass. The corresponding Eq. holds for the gravitational acceleration at the MPQ $g_{MPQ} = \frac{G \cdot M_{♁}^{♁}}{R_{MPQ}^2}$.

• The speed of the rotation relative to the ECI is the product of the radial coordinate, the cosine of the latitude and the angular frequency, see lines 11-12 in table (1):

$$v_{PTB,ECI} = \omega_{♁,ECI}^{♁} \cdot R_{PTB} \cdot \cos(\beta_{PTB}) = \omega_{♁,ECI}^{♁} \cdot R_{PTB,\omega}, \quad \text{with } R_{PTB,\omega} := R_{PTB} \cdot \cos(\beta_{PTB}) \tag{29}$$

Thereby, the angular frequency $\omega_{♁,ECI}^{♁}$ is $2\pi$ divided by the duration of the siderial day, $d_{sid}$ = 86164.09 s. The corresponding Eq. holds for the speed $v_{MPQ}$ at the MPQ.

• At the sea level, all points have the same effective potential. Moreover, the relativistic time differences caused by Earth's rotation cancel each other at the sea level, see e. g. Ashby (2003). Accordingly, the GPS uses clocks at Earth at the sea level, see Zogg (2009). Therefore, the height above sea level causes a potential and time difference. Hereby, without loss of generality, we set the potential at the sea level to zero, in this context and investigation.

For instance, at the PTB, that potential depends on the height $H_{PTB}$, the radius $R_{PTB}$, and the gravitational acceleration $gPTB$. The potential can be analyzed with a probe mass $m$ at the PTB: The probe mass experiences the gravitational force $F_{gr}$ = m $gPTB$, directed downwards. Additionally, at the rotating Earth, the probe mass experiences the centrifugal force $F_{cf} = m \cdot v_{PTB,ECI}^2 / R_{PTB,\omega}$. It is directed outwards from Earth's axis. The energy of $m$ is the force multiplied by the height above the reference point at sea level. In the case of the centrifugal force, the factor $\cos(\beta_{PTB})$ accounts for the angle between the force and the upward direction:

$$E_g = m \cdot H_{PTB} \cdot (g_{PTB} - v_{PTB,ECI}^2 / R_{PTB,\omega} \cdot \cos(\beta_{PTB})) \tag{30}$$

The potential is the energy divided by $m$:

$$\Phi_{PTB,g} = H_{PTB} \cdot (g_{PTB} - v_{PTB,ECI}^2 / R_{PTB,\omega} \cdot \cos(\beta_{PTB})) \quad \text{or} \tag{31}$$

$$\Phi_{PTB,g}(\omega_{♁,ECI}^{♁}) = H_{PTB} \cdot [g_{PTB} - \omega_{♁,ECI}^{♁2} \cdot R_{PTB} \cdot \cos^2(\beta_{PTB})] \tag{32}$$

This potential is determined according to Newton's mechanics, as it is based on the inertial system. Therefore, it is the geodetically measured potential at the PTB.

In the chronometrically measured potential, the gravitational part is the same, $H_{PTB} \cdot g_{PTB}$, as the gravitational potential is the same as above,. However, in the chronometrically measured potential, the speed of the angular frequency $\omega_{♁,ACS}$ is

$$v_{PTB,ACS} = \omega_{♁,ACS} \cdot R_{PTB,\omega}. \tag{33}$$

Consequently, the kinematic fractional time difference is as follows:

$$\delta t_{kin,frac} = \frac{-v_{PTB,ACS}^2}{2c^2} \tag{34}$$

Starting from the reference point at sea level, there occurs only the speed difference

$$\Delta v_{PTB,ACS} = \omega_{♁,ACS} \cdot H_{PTB} \cdot \cos(\beta_{PTB}), \tag{35}$$

and the time difference

$$\Delta\delta t_{kin,frac} = \frac{\partial \delta t_{kin,frac}}{\partial v_{PTB,ACS}} \cdot \Delta v_{PTB,ACS} = \frac{-v_{PTB,ACS} \cdot \Delta v_{PTB,ACS}}{c^2} \tag{36}$$

This time difference is expressed in terms of the angular frequency, see Eqs. (33 and 35):

$$\Delta\delta t_{kin,frac} = \frac{-\omega_{♁,ACS}^2 \cdot H_{PTB} \cdot \cos(\beta_{PTB}) \cdot R_{PTB,\omega}}{c^2} \tag{37}$$

The corresponding potential is obtained by multiplication with $c^2$:

$$\Phi = -\omega_{♁,ACS}^2 \cdot H_{PTB} \cdot \cos(\beta_{PTB}) \cdot R_{PTB,\omega} \tag{38}$$

The complete potential is this kinematic potential plus the gravitational potential $H_{PTB} \cdot g_{PTB}$:

$$\Phi_{PTB,c}(\omega_{♁,ACS}) = H_{PTB} \cdot [g_{PTB} - \omega_{♁,ACS}^2 \cdot R_{PTB} \cdot \cos^2(\beta_{PTB})] \tag{39}$$

The geodetic potential function $\Phi_{PTB,g}(\omega_{♁,ECI})$ in Eq. (32) and the chronometric potential function $\Phi_{PTB,c}(\omega_{♁,ACS})$ in Eq. (39) differ only by a the reference frames ECI and ACS. The corresponding potential functions hold for the MPQ:

$$\Phi_{MPQ,g}(\omega_{♁,ECI}) = H_{MPQ} \cdot [g_{MPQ} - \omega_{♁,ECI}^2 \cdot R_{MPQ} \cdot \cos^2(\beta_{MPQ})] \tag{40}$$

$$\Phi_{MPQ,c}(\omega_{♁,ACS}) = H_{MPQ} \cdot [g_{MPQ} - \omega_{♁,ACS}^2 \cdot R_{MPQ} \cdot \cos^2(\beta_{MPQ})] \tag{41}$$

Furthermore, the height at the MPQ can be determined with help of the geodetically measured potential as follows: For the case of the PTB and the MPQ, the radii are nearly equal. Therefore, the potential difference is as follows, and it is identified with $\Delta_{\Phi g}$:

$$\Delta\Phi_g := \Phi_{MPQ,g} - \Phi_{PTB,g} \approx (H_{MPQ} - H_{PTB}) \cdot [g_{MPQ} - \omega_{♁,ECI}^2 \cdot R_{MPQ} \cdot \cos^2(\beta_{MPQ})] \approx \Delta\Phi_{geodetic} \tag{42}$$

This equation is solved for $H_{MPQ}$, see line 8 in table (1):

$$H_{MPQ} = H_{PTB} + \frac{\Delta\Phi_{geodetic}}{g_{MPQ} - \omega_{♁,ECI}^2 \cdot R_{MPQ} \cdot \cos^2(\beta_{MPQ})} \tag{43}$$

Moreover, for the case of the PTB and the MPQ, the potentials are in lines 13-14 in table (1). The difference $\Delta\Phi g$ of these potentials is shown in line 15 in table (1). It is nearly equal to the measured value $\Delta\Phi_{geodetic}$. This confirms the present analysis. On that basis, the velocity of the ACS relative to the ECI is estimated:

The difference of the chronometric potential functions is formed:

$$\Delta\Phi_c := \Phi_{MPQ,c} - \Phi_{PTB,c} \tag{44}$$

The calculated difference $\Delta\Phi_c - \Delta\Phi_g$ corresponds to the observed difference $\Delta\Phi_{chronometrical} - \Delta\Phi_{geodetic}$, see Fig. 3:

$$\Delta\Phi_{c,g} := \Delta\Phi_c - \Delta\Phi_g \hat{=} \Delta\Phi_{chronometrical} - \Delta\Phi_{geodetic} \in \left[-0.68\ \frac{\mathrm{m}^2}{\mathrm{s}^2}, 5.12\ \frac{\mathrm{m}^2}{\mathrm{s}^2}\right] \tag{45}$$

Therefore, those values of $\omega_{♁,ACS}$ are compatible with observation, for which

$$\Delta\Phi_{c,g} = \Delta\Phi_c(\omega_{♁,ACS}) - \Delta\Phi_g(\omega_{♁,ECI}) \in \left[-0.68\ \frac{\mathrm{m}^2}{\mathrm{s}^2}, 5.12\ \frac{\mathrm{m}^2}{\mathrm{s}^2}\right] \tag{46}$$

In particular, for the case $\omega_{♁,ACS} = \omega_{♁,ECI}$, the above difference is zero, $\Delta\Phi_{c,g} = 0$. This is compatibel with observation, Fig. (3). This confirms the recommendation of the IAU, and it confirms the Prediction of the velocity of the ACS.

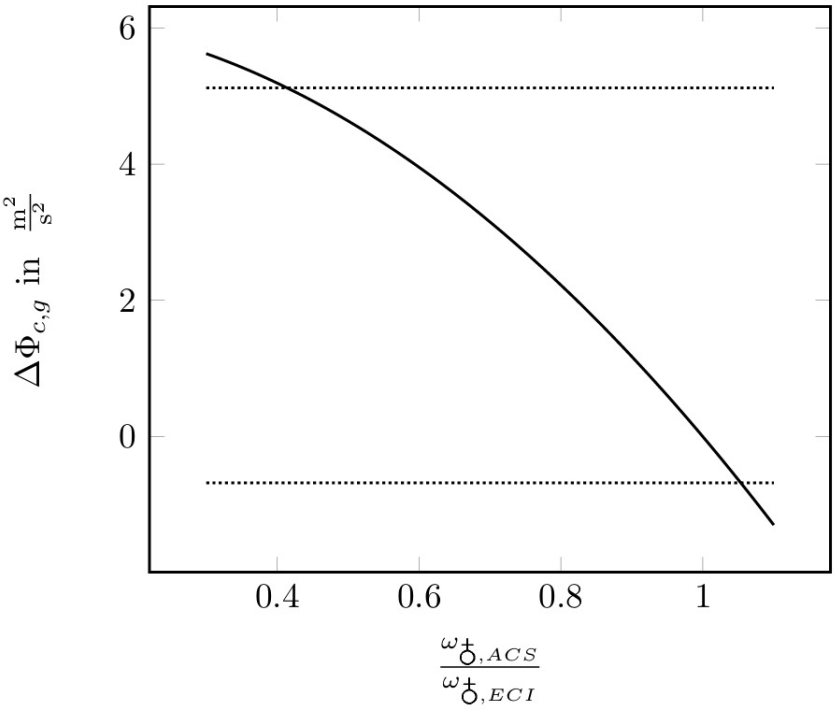


**Figure 3: The Solid Line Shows the Difference $\Delta\Phi_{c,g}$ in eq. (45) as a Function of Earth's Angular Frequency $\omega_{♁,ACS}$ Relative to the ACS, in Units of Earth's Angular Frequency $\omega_{♁,ECI}$ Relative to the ECI**

The maximal and minimal values of that difference, that are compatible with the observation in the studied experiment, are marked by dotted lines. The points of intersection are $\left(0.4138, 5.12\ \frac{\mathrm{m}^2}{\mathrm{s}^2}\right)$ and $\left(1.0535, -0,68\ \frac{\mathrm{m}^2}{\mathrm{s}^2}\right)$. The large difference between the dotted bounds 5.12 $\frac{\mathrm{m}^2}{\mathrm{s}^2}$ and $-0.68\ \frac{\mathrm{m}^2}{\mathrm{s}^2}$ is partially caused by the heterogeneity of Earth's density, which is not fully included in detail in the measurements by Grotti et al. (2024).

More generally, there is an interval of $\omega_{♁,ACS}$ values, that are compatible with observation. This interval is limited by the two points of intersection in Fig. (3). Therefore, this interval is as follows:

$$\omega_{♁,ACS} \in [0.4138 \cdot \omega_{♁,ECI}, 1.0535 \cdot \omega_{♁,ECI}] \tag{47}$$

This implies the following weaker relation:

$$\frac{|\omega_{♁,ACS} - \omega_{♁,ECI}|}{|\omega_{♁,ECI}|} \leq 0.5862 \tag{48}$$

The analogous result holds for the velocities at Earth's surface, in the considered experiment:

$$\frac{|\vec{v}_{♁,ACS} - \vec{v}_{♁,ECI}|}{|\vec{v}_{♁,ECI}|} \leq 0.5862 \tag{49}$$

With it, for the example of the PTB, the possible difference of the velocity relative to the ECI and to the ACS is obtained:

$$|\Delta v_{PTB}| \leq v_{PTB,ECI} \cdot 0.5862 = 166 \ \frac{\mathrm{m}}{\mathrm{s}}. \tag{50}$$

### Comparison with an observation in space

For comparison, the ACS is applied to a GPS satellite:

### Calculation of the solution

In this section, the essential quantities of a GPS satellite $E_1$ are calculated.
The *orbital period T* of a GPS satellite is one half of a sidereal day $d_{sid}$, see Zogg (2009) [21].

$$T = \frac{1}{2} \cdot d_{sid} = 43\ 082.045 \ \mathrm{s}. \tag{51}$$

The orbital radius of a GPS satellite is based on Newton's mechanics, see Landau and Lifschitz (1960) [9]. As a very good approximation, the orbit of a GPS satellite is described by a circle, see Zogg (2009):

$$R_{orbit} = \sqrt[3]{\frac{G \cdot M_{\oplus} \cdot T^2}{4\pi^2}} = 26\ 561.762 \ \mathrm{km}. \tag{52}$$

The speed of $E_1$ in the *ECI* $v_{E_{1,ECI}}$ is its circumference is 2π • $R_{orbit}$ divided by the orbital period:

$$v_{E_1,ECI} = \frac{2\pi \cdot R_{orbit}}{T} = 3\ 873.829 \ \frac{\mathrm{m}}{\mathrm{s}}. \tag{53}$$

The speed $v_{E_{1,ACS}}$ of $E_1$ in the ACS is studied with help of the observed fractional time difference, Ashby (2003), p. 16:

$$\delta t_{frac} = 4.425(0.04) \cdot 10^{-10}. \tag{54}$$

In order to obtain the gravitational fractional time difference, the potential is derived first. It is based on Newton's mechanics, see Landau and Lifschitz (1960):

$$\begin{aligned} \Phi(R_{orbit}) &= \frac{-G \cdot M_{\oplus}}{R_{orbit}} = -1.500\ 655 \cdot 10^7 \ \frac{\mathrm{J}}{\mathrm{kg}} \\ \Phi(R_e) &= \frac{-G \cdot M_{\oplus}}{R_e} = -6.249\ 481 \cdot 10^7 \ \frac{\mathrm{J}}{\mathrm{kg}} \\ \Delta\Phi := \Phi(R_{orbit}) - \Phi(R_e) &= 4.748\ 826 \cdot 10^7 \ \frac{\mathrm{J}}{\mathrm{kg}} \end{aligned} \tag{55}$$

Hereby, Sun, Moon and other celestial bodies except Earth are neglected in a relatively good approximation. With it, the fractional gravitational time difference of $E_1$ relative to a clock on the ground is calculated:

$$\delta t_{grav,frac} = \frac{\Delta\Phi}{c^2} = 5.512\ 493 \cdot 10^{-10}. \tag{56}$$

The fractional kinematic time difference of $E_1$ relative to a clock on the ground is as follows:

$$\delta t_{kin,frac} = \delta t_{frac} - \delta t_{grav,frac} = -8.588(0.4) \cdot 10^{-11}. \tag{57}$$

According to the solution of the frame problem, the coordinate system, in which the equation (19) of time difference holds, is the ACS:

$$\delta\tau_{kin,frac} = -\frac{v^2_{E_1,ACS}}{2c^2}. \tag{58}$$

This Eq. is solved for the speed $E_{1,ACS}$, and the kinematic time difference in Eq. (56) is used:

$$v_{E_1,ACS} = \sqrt{-2 \cdot \delta\tau_{kin,frac} \cdot c^2} = 3\ 929(93) \ \frac{\mathrm{m}}{\mathrm{s}}. \tag{59}$$

Therefore, the difference of the speed $v_{E_1,ACS}$ of $E_1$ relative to the ACS and of the speed $v_{E_1,ECI}$ of $E_1$ relative to the ECI is

$$\Delta v_{E_1} = v_{E_1,ACS} - v_{E_1,ECI} = 55(93)\ \frac{\mathrm{m}}{\mathrm{s}}, \text{or} \frac{\Delta v_{E_1}}{v_{E_1,ECI}} = 1.4(2.4)\%. \tag{60}$$

For comparison, Ashby (2003), see pages 16 and 17, estimated the error of measurement:

$$\frac{4.465 \cdot 10^{-10} - 4.425 \cdot 10^{-10}}{4.425 \cdot 10^{-10}} = 0.904\%. \tag{61}$$

Therefore, the speed $v_{E_1,ACS}$ of $E_1$ relative to the ACS and the speed $v_{E_1,ECI}$ of $E_1$ relative to the ECI can be regarded as equal within the error of measurement estimated by Ashby (2003).

## Comparison

The observation at the ground and the observation at a GPS satellite are both compatible with the equality of the velocities relative to the ECI and to the ACS. This is in accordance with the IAU recommendation. In the observation at the ground and in the observation at a GPS satellite, the maxima of the difference $\Delta v_C = v_{C,ACS} - v_{C,ECI}$ of the velocity of a clock relative to the ACS and of the velocity of a clock relative to the ECI differ by one order of magnitude: These differences are $|\Delta v_C| \leq 166$ m/s and $|\Delta v_C| \leq 55$ m/s, respectively. In the experiment at the ground, $|\Delta vC| \leq 166$ m/s is relatively large. This low precision is partially caused by a small difference of the kinematic time difference of the two clocks, $\delta t_{in,frac,PTB} - \delta t_{in,frac,MPQ} = 8.3 \cdot 10^{-14}$. In the study of GPS satellites, the respective differencefrac, $\delta t_{in,frac,satellite} - \delta t_{in,frac,ground}$ is three orders of magnitude larger.

## The Absolute Zero of the Kinematic Time Dilation

In this section, an especially useful and insightful result is elaborated:
At each point P in the universe, there exists a measurable ACS with a measurable velocity $\vec{v}_{ACS}$.
Therefore, each clock C with its velocity $\vec{v}_C$ has the following kinematic fractional time difference, see Eq. (23):

$$\delta\tau_{kin,frac} = \frac{-(\vec{v}_C - \vec{v}_{ACS})^2}{2c^2} \tag{62}$$

Therefore, at each P, there is the absolute zero of the kinematic fractional time differences, $\delta\tau_{kin,frac} = 0$. It occurs for each clock C at P, that has the same velocity as the ACS, $\vec{v}_C = \vec{v}_{ACS}$. Thereby, $\vec{v}_{ACS}$ is a function of P: The ACS is at rest at the local gravitational field at P. Accordingly, $\vec{v}_{ACS}$(P) can be predicted, for more details see Carmesin (2025a) [8]. As a consequence, $\vec{v}_{ACS}$ can be used for space navigation and for the synchronization of clocks in a predictable manner. It is insightful to realize that the kinematic time difference is related to a local property of space and gravity, for more details see Carmesin (2025a) [8]. The absolute zero of the kinematic time differences represents a zero point of motion relative to the ACS. This is very valuable, as physical laws achieve a relatively clear structure and short form at the zero point of motion, for more details see Carmesin (2025a) [8,24,25].

## Discussion

Clocks can be very precise with a relative error below $10^{-18}$. Time measurements are very useful. In geophysics, these can be used to determine potentials, speeds, densities or heights in a very precise manner. Results can be used for the measurement of the continental drift or the prediction of earthquakes. In astrophysics and space flight, time can be used for the determination of fields, potentials, motions, clock synchronization and precise space navigation.

However, the IAU realized that the theory, even the very successful theory of relativity, does not provide sufficiently realistic coordinate systems for space navigation. Accordingly, the IAU proposed the problem of finding an adequate coordinate system, ACS, See Soffel et al. (2003). Moreover, the IAU recommended the GCRS and the BCRS for space navigation near Earth or in the planetary system, respectively.

This problem is solved here as follows: A concept for finding the ACS is formulated. For each point P, the existence of the ACS as well as the uniqueness of its velocity are derived, and a prediction about the ACS is provided. It is shown that the ACS is at rest at the pattern of the gravitational field (or of the curvature of space or of additional volume), more details about corresponding calculations are provided in Carmesin (2025a) [8]. The ACS is analyzed for the case of an experiment by Grotti et al. (2024) on the ground, and for an observation at space. In both cases, the measured data are used in order to investigate the velocities of a clock C in the Earth Centered Inertial system, ECI and in the ACS. The difference of these velocities $\vec{v}_{C,ECI} - \vec{v}_{C,ACS}$ is determined. It is smaller than the error of measurement. Therefore, in both cases, the recommendation of the IAU and the above prediction are confirmed.

In the future, significantly nonzero values of $\Delta\vec{v}_C = \vec{v}_{C,ACS} - \vec{v}_{C,ECI}$ should be discovered. This could happen at larger distances to Earth, when the gravitational fields of Sun, Moon or other celestial bodies become essential. Moreover, nonzero values of $\vec{v}_{C,ACS} - \vec{v}_{C,ECI}$ could be achieved with even more precise clocks or other measurement devices, for

instance. In the present two investigated experiments on the ground and in space, the difference has been determined up to errors of measurement (including very small real differences) of the amount $|\Delta vC| < 166$ m/s and $|\Delta v_C| = 55(93)$ ms , respectively.

Moreover, a very useful and enlightening result is the discovery of the absolute zero of the kinematic time dilation, for each point P in the universe. This could be used as a zero point of motion in technology, navigation and science. With that zero of motion, the precision, measurement, derivation and discovery of new results could be improved.

## References


1. Kelvin, W. (1848). On an Absolute Thermometric Scale - Founded on Carnot's Theory of the Motive Power of Heat - and Calculated from Regnault's Observations. Philosophical Magazine, October:100– 106.
2. Pound, R. V., & Rebka Jr, G. A. (1960). Apparent weight of photons. Physical review letters, 4(7), 337.
3. Hobson, M. P., Efstathiou, G. P., & Lasenby, A. N. (2006). General relativity: an introduction for physicists. Cambridge university press.
4. Starker, S., Nau, H., & Hammes, J. (1985, December). The Shuttle Experiment Navex Completed on Spacelab Mission D1. In Proceedings of the 17th Annual Precise Time and Time Interval Systems and Applications Meeting (pp. 405-412).
5. Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. Annalen der Physik, 17(10), 891-921.
6. Einstein, A. (1915). Die Feldgleichungen der Gravitation. Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 844-847.
7. Kopeikin, S. M. (2003). The IAU 2000 resolutions for astrometry, celestial mechanics and metrology in the relativistic framework: explanatory supplement.
8. Carmesin, H. O. (2025). On the Dynamics of Time, Space and Quanta-Essential Results for Space Flight and Navigation. Dr. Köster.
9. Landau, L. and Lifschitz, J. (1960). Course of Theoretical Physics - Mechanics. Pergamon Press, Oxford.
10. NASA (1963). Voyager design studies. Technical Report.
11. Lavery, J. E. (1972). Relativistic time corrections for Apollo 12 and Apollo 13 (No. E-1049).
12. Grotti, J., Nosske, I., Koller, S. B., Herbers, S., Denker, H., Timmen, L., ... & Lisdat, C. (2024). Long-distance chronometric leveling with a portable optical clock. Physical Review Applied, 21(6), L061001..
13. Denker, H., Timmen, L., Voigt, C., Weyers, S., Peik, E., Margolis, H. S., ... & Petit, G. (2018). Geodetic methods to determine the relativistic redshift at the level of $10^{-18}$ in the context of international timescales: a review and practical results. Journal of Geodesy, 92(5), 487-516.
14. Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik49: 769–822. The collected works of Albert Einstein, 6, 283-339.
15. Minkowski, H. (1908). Raum und Zeit. Jahresbericht der Deutschen Mathematiker-Vereinigung, 18:75–88.
16. Schwarzschild, K. (1916). Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. Sitzungsberichte der königlich preussischen Akademie der Wissenschaften, 189-196.
17. Workman, R. L. et al. (2022). Review of Particle Physics (by Particle Data Group). Progr. Theor. Exp. Phys., 083C01:1–2270.
18. Karttunen, H., Kröger, P., Oja, H., Poutanen, M., & Donner, K. J. (Eds.). (2007). Fundamental astronomy. Berlin, Heidelberg: Springer.
19. Carmesin, H.-O. (2025b). Patentanmeldung April AKZ 102025001259.4.
20. Carmesin, H.-O. (2025c). Patentanmeldung August AKZ 102025002783.4.
21. Carmesin, H.-O. (2025d). Patentanmeldung September AKZ 102025003071.1.
22. Ashby, N. (2003). Relativity in the global positioning system. Living Reviews in Relativity, 6(1), 1-42.
23. Zogg, J.-M. (2009). GPS Essentials of Satellite Navigation. u-blox-AG, Chemnitz.
24. Denker, H. et al. (2019). Report on levelling and Large letters in GNSS, MPQ, Garching and PTB results for stations on the mpq campus in garching. Technical Report
25. Ketzler, R. (2025). Earth's magnetic field at ptb. Technical Report.