

Volume 2, Issue 2

Research Article

Date of Submission: 20 May, 2026

Date of Acceptance: 04 June, 2026

Date of Publication: 29 June, 2026

Corpuscles as Packets of Strings

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Citation: Kouki, A. (2026). Corpuscles as Packets of Strings. *Arch Interdiscip Educ*, 2(2), 01-13.

Abstract

From Restraint Relativity it is possible to consider a corpuscle as a packet of strings. The variation of the length of this packet is equal to the phase speed of the corpuscle as a packet of waves times an universal constant. In a system of units where $\hbar = c = a = 1$ the string vector becomes equal to the wave vector.

With this notion of packet of strings the principle of uncertainty is incorporated in Restraint Relativity.

Revisited Special Relativity

The introduction of Special Relativity by Einstein in 1905 is due only theoretical thinking that Maxwell equations of the electro-magnetic field should be the same in all inertial frames. Those equations in vacuum are:

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad (1)$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad (2)$$

Which are two equations of waves with a speed of propagation equal to $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ with:
 $\epsilon_0 = 8.85 \cdot 10^{-12} \text{ Fm}^{-1}$: the permittivity of vacuum.

$\mu_0 = 4\pi \cdot 10^{-7} \text{ Hm}^{-1}$: the permeability of vacuum.

But what is vacuum? If vacuum signify that there is nothing but here there is the electromagnetic field and so there is temperature. The only response is that vacuum is when there is nothing and the temperature is zero absolute or approaches asymptotically to zero. If there is any electromagnetic field in vacuum it signify the fundamental state of the field : there is no emission of energy.

In Special Relativity based on the constancy of the speed of light in all inertial frames it is defined:

-Let's have two inertial frames $S(ct, x, y, z)$ & $S'(ct', x', y', z')$ with S' is moving in constant speed v and the axes coincide at $t' = t = 0$ than the Lorentz transformations of space and time are as follows [1]:

$$t' = \gamma(ct - \beta x)$$

$$x' = \gamma(x - \beta ct) \quad (3)$$

$$y' = y$$

$$z' = z$$

With $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ & $\beta = \frac{v}{c}$

Considering two events $A(t_A, x_A, y_A, z_A)$ & $B(t_B, x_B, y_B, z_B)$ in the frame S the interval squared :

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \quad (4)$$

is invariant under Lorentz transformations.

If $\Delta s^2 > 0$: than the interval is time-like.

$\Delta s^2 = 0$: than the interval is light-like
 $\Delta s^2 < 0$: than the interval is space-like.

Lorentz transformations can be also considered as hyperbolic rotation in the 4th space dimensions as the following:

$$\begin{aligned} ct' &= ct \cosh \psi - x \sinh \psi \\ x' &= -ct \sinh \psi + x \cosh \psi \\ y' &= y \\ z' &= z \end{aligned} \quad (5)$$

With $\tanh \psi = \beta$, $\cosh \psi = \gamma$, $\sinh \psi = \gamma \beta$.

Space & time are considered as a four dimensions continuum that describes physics world with the Minkowski geometry related by relation (4).

The Minkowski geometry is not Euclidian due to the sign minus in (4).

The interval given by equation (4) is the "distance" between the events A & B measured in a straight line and can be considered as the worldline of a free particle moving in constant speed between the points A & B .

The interval measured between the two points A & B in any arbitrary path is:

$$\Delta s = \int_A^B ds \quad (6)$$

With: $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$: the line element of the space-time.

For a massive particle we define the proper time τ as a parameter $t(\tau)$, $x(\tau)$, $y(\tau)$ & $z(\tau)$ in order to determine the position of the particle in the 4th space-time as:

$$d\tau^2 = \frac{ds^2}{c^2} \quad (7)$$

Which mean that:

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}} \cdot dt = \frac{dt}{\gamma} \quad (8)$$

The total elapsed proper time interval is:

$$\Delta\tau = \int_A^B \sqrt{1 - \frac{v(t)^2}{c^2}} \cdot dt \quad (9)$$

Where $v(t)$ is the instantaneous speed of the corpuscle along its path.

If we introduce a rest frame in which the corpuscle is in rest along its path the proper time is the time recorded by a clock moving with the corpuscle.

It is convenient to introduce the indexed coordinates x^i ($i = 0, 1, 2, 3$) so we have:

$$x^0 = ct, x^1 = x, x^2 = y, x^3 = z \text{ and } ds^2 = g_{ij} dx^i dx^j$$

where g_{ij} are the covariant metric tensor as the following:

$$[g_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (10)$$

Or in shorthand notation as $[g_{ij}] = \text{diag}[1, -1, -1, -1]$.

The contravariant of the metric tensor are the same as: $[g^{ij}] = \text{diag}[1, -1, -1, -1]$.

In transforming coordinates x^i in a Minkowski space-time to a new coordinates x'^i the line & s'element ds^2 must take the same form as:

$$ds^2 = g_{ij} dx^i dx^j = g'_{ij} dx'^i dx'^j$$

Which means that the transformation $x^i \rightarrow x'^i$ must satisfy:

$$g_{ij} = \frac{\partial x'^k}{\partial x^i} \frac{\partial x'^l}{\partial x^j} g_{kl} \quad (11)$$

Which implies that the transformations must be linear to be a Lorentz transformations:

$$x'^i = \Lambda_j^i x^j + a^i \quad (12)$$

Where Λ_j^i & a^i are constants .

Transformations (12) are the inhomogeneous Lorentz transformations (or Poincare transformations). With a^i taken equal to zero they are the ordinary Lorentz transformations (or homogeneous transformations).

In a standard configuration of two inertial frames S & S' the matrix transformation can be written as follows:

$$[\Lambda_j^i] = \left[\frac{\partial x'^i}{\partial x^j} \right] = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cosh\psi & -\sinh\psi & 0 & 0 \\ -\sinh\psi & \cosh\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

Where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$ & $\tanh\psi = \beta$, $\cosh\psi = \gamma$, $\sinh\psi = \gamma\beta$

The inverse transformation of (13) are obtained by replacing v by $-v$ or ψ by $-\psi$ The matrix inverse is:

$$[\Lambda_i^j] = \left[\frac{\partial x^j}{\partial x'^i} \right]$$

And the relation ship between the two matrices :

$$[\Lambda_j^i] = g^{ik} g_{lj} \Lambda_k^l$$

And we have:

$$\Lambda_k^i \Lambda_j^k = \Lambda_k^i g^{km} g_{jl} \Lambda_m^l = g^{mi} g_{jm} = \delta_j^i$$

In any coordinate system the coordinate basis vectors are tangents to the coordinate curves

In frames S & S' basis vectors are related by:

$$e'^j = \Lambda_k^j e^k; \quad e^j = \Lambda_k^j e'^k$$

And we have;

$$e'^i \cdot e'^j = g^{ij} e^i \cdot e^j = g^{ij}$$

Dual basis vectors are defined as:

$$e_i = g_{ik} e^k e'_i = g_{ik} e'^k$$

And we have:

$$e_i \cdot e_j = g_{ij} e'_i \cdot e'_j = g_{ij}$$

Where the components of g_{ij} are the same as g^{ij} .

We can define a 4-vector v of point P in Minkowski spacetime as

$$v = v^i e_i$$

At each point P we have a set of orthonormal basis e_i . The square of the length of a vector v is :

$$v \cdot v = v_i v^i = g^{ik} v_i v_k$$

In two inertial frames S & S' with Cartesian coordinates x^i & x'^i the coordinates of a 4-vector v at a point P is :

$$\mathbf{v} = v^i \mathbf{e}_i = v'^i \mathbf{e}'_i$$

And we have :

$$v'^i = \mathbf{v} \cdot \mathbf{e}'^i = \Lambda_j^i v^j$$

$$v^i = \mathbf{v} \cdot \mathbf{e}^i = \Lambda_j^i v'^j$$

The 4-velocity $\mathbf{u}$ of a (massive) corpuscle is as :

$$\mathbf{u} \cdot \mathbf{u} = \left(\frac{ds}{d\tau} \right)^2 = c^2$$

The contravariant components of the 4-velocity are:

$$u^i = \mathbf{u} \cdot \mathbf{e}^i = \frac{dx^i}{d\tau}$$

It comes that:

$$[u^i] = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \left(c, \frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt} \right)$$

$$\text{With : } c = \frac{dx^0}{dt}, x^0 = ct \text{ \& } v = \sqrt{\left(\frac{dx^1}{dt} \right)^2 + \left(\frac{dx^2}{dt} \right)^2 + \left(\frac{dx^3}{dt} \right)^2}$$

The 4-moment of a massive corpuscle is defined as :

$$\mathbf{p} = m\mathbf{u} \quad (14)$$

With : m : the mass of the corpuscle;

$\mathbf{u}$: the 4-velocity of the corpuscle.

The component of the 4-moment are $p^i = \mathbf{p} \cdot \mathbf{e}^i$ and we have:

$$[p^i] = \left[\frac{E}{c}, p^1, p^2, p^3 \right] = \left(\frac{E}{c}, \vec{p} \right) \quad (15)$$

With E the energy of the corpuscle and $\vec{p}$ its 3 ordinary moment in an inertial frame .

It comes that:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ \& } \vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (16)$$

The squared length of the 4-moment is $p^i p_i$ and so get:

$$\left(\frac{E}{c} \right)^2 - p^2 = m^2 c^2 \quad (17)$$

Where $p^2 = \vec{p} \cdot \vec{p}$.

For a photon (massless corpuscle) we can't use the proper time to parametrize its worldline which is a null curve because $d\tau = 0$. We can use an affine parameter σ as for a photon moving in the direction $x = ct$ we have:

$$x^i = \sigma u^i \quad (18)$$

With: $[u^i] = (1, 1, 0, 0)$

The tangent vector of the worldline is:

$$\mathbf{u} = \frac{dx^i}{d\sigma} \mathbf{e}_i \quad (19)$$

And we have:

$$\mathbf{u} \cdot \mathbf{u} = 0 \text{ \& } \frac{du}{d\sigma} = 0 \quad (20)$$

Equation (20) is the equation of motion of the photon.

Of course in equation (20) we consider that the vector $\mathbf{e}_i$ are constant and does not change with the position.

The tangent vector $\mathbf{u}$ can be multiplied by any scalar and the equation (20) will be satisfied. The 4-moment of a photon can be defined as:

$$\mathbf{p} = \mu \mathbf{u} \quad (21)$$

Where μ is constant and the components of $\mathbf{p}$ are:

$$[p^i] = \left(\frac{E}{c}, \vec{p}\right)$$

Where E & $\vec{p}$ are respectively the energy and the 3-moment of the photon in a given inertial frame S .

It comes that from equation (20):

$$E = pc \quad (22)$$

The same equation (22) can be obtained for massless corpuscles if we take $m = 0$ in equation (17).

Equation (17) is a more generalized equation (massive corpuscles and massless corpuscles).

For photon we can introduce the 4-wave vector $\mathbf{k}$ as :

$$\mathbf{p} = \hbar \mathbf{k} \quad (23)$$

And the components of the 4-wavevector are:

$$[k^i] = \left(\frac{2\pi}{\lambda}, \vec{k}\right) \quad (24)$$

Where : λ : is the wave-length of the photon

$\vec{k} = \frac{2\pi}{\lambda} \vec{n}$: the 3 -wavevector of the photon

$\vec{n}$: direction of the propagation of the photon (3-vector of one module).

Equation (24) like equation (17) can be generalized for any corpuscle (massless or not). Equating equation (24) to equation (15) we get:

$$\frac{E}{c} = \hbar \cdot \frac{2\pi}{\lambda} \quad (25)$$

$$\vec{p} = \hbar \vec{k} \quad (26)$$

Which mean that massive corpuscle can have waving behavior.

The group speed v_g of such wave behavior of massive corpuscle is defined as :

$$\frac{1}{v_g} = \frac{dk}{d\omega} \quad (27)$$

With : $\omega = 2\pi\nu = 2\pi\frac{c}{\lambda}$: the frequency of the photon (also the frequency associated to the massive corpuscle).

From (25) & (16) we get:

$$E = \hbar\omega = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (28)$$

And from (26) & (16) we get:

$$\hbar \vec{k} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (29)$$

From (27) we get:

$$v_g = v \quad (30)$$

The phase speed v_f of such a wave is defined as:

$$v_f = \frac{\omega}{k} \quad (31)$$

We have always:

$$v_g \cdot v_f = v \frac{\omega}{k} = c^2 \quad (32)$$

In equation (23) we had never speak about the constant $\hbar$: it can be replaced by any other constant and the 4-vector linked in will follow also.

For any corpuscle we introduce the 4- string-vector $\mathbf{l}$ as :

$$\mathbf{p} = a\mathbf{l} \quad (33)$$

And the components of the 4-string-vector are:

$$[l^i] = (l, \vec{l}) \quad (34)$$

With l : the string-length associated to the corpuscle

$\vec{l} = l\vec{n}$: string-vector in the 3-ordinary space

$\vec{n}$: direction of the propagation of the corpuscle (3-vector of one module).

Equating (34) and (15) we get:

$$\frac{E}{c} = al \quad (35)$$

$$\vec{p} = a\vec{l} \quad (36)$$

Let's search the vector $\frac{d\vec{l}}{d\omega}$ (considering the corpuscle as a packet of strings)?

$$\frac{d\vec{l}}{d\omega} = \frac{dl}{dv} \cdot \frac{dv}{d\omega} = \frac{1}{a} \frac{dp}{dv} \cdot \frac{dv}{d\omega} = \frac{1}{a} \frac{dp}{d\omega} = \frac{\hbar}{a} \frac{dk}{d\omega} = \frac{\hbar}{a} \cdot \frac{1}{v} = \frac{\hbar}{a} \cdot \frac{v_f}{c^2}$$

It comes that:

$$\vec{v}_f = \frac{ac^2}{\hbar} \frac{d\vec{l}}{d\omega} = \frac{ac^2}{\hbar} \frac{d\vec{l}}{dv} \quad (37)$$

From (35) & (16) we have:

$$E = acl = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (38)$$

From (36) & (16) we get:

$$a\vec{l} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \hbar\vec{k} \quad (39)$$

We have always:

$$\vec{p} = \frac{E}{c^2} \cdot \vec{v} = \frac{al}{c} \cdot \vec{v} = \frac{\hbar\omega}{c^2} \cdot \vec{v} \quad (40)$$

We can also define the 4-vector inertia ξ as:

$$\mathbf{p} = \xi c = m\mathbf{u} \quad (41)$$

The components of the 4-vector inertia are:

$$[\xi^i] = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \left(1, \frac{dx^1}{dx^0}, \frac{dx^2}{dx^0}, \frac{dx^3}{dx^0}\right) = \xi \cdot \left(1, \frac{dx^1}{dx^0}, \frac{dx^2}{dx^0}, \frac{dx^3}{dx^0}\right) \quad (42)$$

$$\text{With: } x^0 = ct \quad \& \quad \xi = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$$

In a system of units where $\hbar = c = a = 1$ equation (40) can be written in simply form as:

$$\vec{p} = \xi \vec{v} = l\vec{v} = \omega \vec{v} \quad (43)$$

Inertia can be a mass or a length or a frequency or anything else as time for example.

We define the inertial time as :

$$\zeta = \frac{\xi}{a} \quad (44)$$

It is evident if we associate to the corpuscle an inertial time at rest ζ_0 than according to equation (8) we have:

$$\zeta_0 = \sqrt{1 - \frac{v^2}{c^2}} \cdot \zeta \quad (45)$$

$$d\zeta = dt : \text{when the energy of the corpuscle change} \quad (46)$$

From (44) & (45) we get:

$$\zeta_0 = \frac{m}{a} \quad (47)$$

Of course we can also define the 4-vector inertial time ζ as:

$$\xi = a \zeta \quad (48)$$

From equations (17) & (25) we get:

$$h^2 v^2 - p^2 c^2 = m^2 c^4 \quad (49)$$

With: $E = h\nu$: the energy of the corpuscle

$\nu = \frac{\omega}{2\pi}$: reduced inertial frequency of the corpuscle;
 $h = 2\pi\hbar$

Differentiate (49) according to time:

$$h^2 v dv = c^2 p dp = \xi c^2 \vec{v} \cdot d\vec{p} = h v \vec{v} \cdot d\vec{p}$$

Which mean:

$$\frac{d\vec{p}}{dt} \cdot \vec{v} = h \frac{dv}{dt} = a c^2 \frac{d\zeta}{dt} = \vec{f} \cdot \vec{v} \quad (50)$$

With: $\vec{f} = \frac{d\vec{p}}{dt}$: the 3-dimensional force.

If the energy of the corpuscle change then we have always:

$$\vec{f} \cdot \vec{v} = a c^2 \quad (51)$$

An important case is when the speed of the corpuscle is low compared to c . From equation (49) we get:

$$h\nu \approx m c^2 + \frac{1}{2} m \vec{v}^2 \quad (52)$$

$$\vec{p} \approx m \vec{v} \quad (53)$$

$$\vec{f} \approx m \frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \text{ with } \vec{p} = m \vec{v} \quad (54)$$

which mean:

$$h dv = m \vec{v} \cdot d\vec{v}$$

Than:

$$h \frac{dv}{dt} = \vec{f} \cdot \vec{v} \quad (55)$$

Equation (55) is the same equation as (50) even though it is obtained by approximation.

The square of (52) is :

$$h^2 v^2 = m^2 c^4 + m^2 c^2 \vec{v}^2 + \frac{1}{4} m^2 \vec{v}^4 \quad (56)$$

$$2 h^2 v \frac{dv}{dt} = m^2 c^2 2 \vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{1}{4} m^2 \frac{d\vec{v}^4}{dt}$$

$$h \frac{dv}{dt} = \frac{m c^2}{h v} \vec{v} \cdot \vec{f} + \frac{1}{4} m^2 \frac{d\vec{v}^4}{dt} \cdot \frac{1}{2 m c^2 + m \vec{v}^2} \approx \frac{m c^2}{h v} \vec{v} \cdot \vec{f} = \vec{f} \cdot \vec{v} \quad (57)$$

which mean : $h\nu \approx m c^2$ for low speed.

From equation(50):

$$h\nu^2 = \frac{m^2 c^4}{h} + \frac{p^2 c^2}{h}$$

$$\frac{d(h\nu^2)}{dt} = 2 \frac{c^2}{h} \vec{p} \cdot \frac{d\vec{p}}{dt} = 2 \frac{c^2}{h} \cdot \frac{h\nu}{c^2} \vec{v} \cdot \vec{f} = 2\nu a c^2 = 2h\nu \frac{dv}{dt} \quad (58)$$

Which mean:

$$a c^2 = h \frac{dv}{dt} \quad (59)$$

Than:

$\nu = \frac{a c^2}{h} t + \text{constant}$ when the energy of the corpuscle is changing.

But in this case we have: $dt = d\zeta$ so:

$$\nu = \frac{a c^2}{h} \zeta \text{ which is a trivial result (with } \text{constant} = 0) \quad (60)$$

Equation (51) signify that a corpuscle never can't be in rest otherwise the universal constant a will be equal to zero. Than absolute vacuum never exist in a frame : there is energy independent from the corpuscle and which don't let the corpuscle become in rest.

Wave-Corpuscle Duality:

Constant h was suggested and determined in 1900 by Max Planck in his model of black body radiation [2]. Planck model of black body was a cavity with many oscillators on his wall exchanging energy with radiation inner. Planck determine the mean energy of an oscillator and the radiation law at equilibrium in a temperature T of the cavity and get the values of constants h & k_B (Planck constant and Boltzmann constant) by a statistical manner and referring earlier to F.Kurlbaum & Wien measurements of black body radiation.

In his recent theory of heat radiation by 1911 Planck suggest that his oscillator should have enough time to absorb energy from radiation[3]. The Planck condition for this is that the frequency of the oscillator times time should be a great integer or in other manner:

$$\nu \cdot t \gg 1 \quad (61)$$

By 1923 De Broglie suggest the wave-corpuscle duality i.e. a corpuscle can have a waving behavior and vice versa: he propose the relation (27) and getting (28) & (29) by referring to Restraint Relativity proposed by A. Einstein in 1905.

In his suggestion De Broglie consider a corpuscle as a packet of waves moving in the same direction of the corpuscle and represented with a wave function having a constant amplitude. De Broglie consider a corpuscle as many monochromatic waves which reinforce each other in a limited region of spacetime and destroy each other away. Mathematically speaking this model can't be adopted because the waves will reinforce each other in many regions of spacetime in the same direction of the corpuscle motion and we can't see where is really the corpuscle. A replacement model of De Broglie one is that a corpuscle should be considered as a pulse and not as a packet of waves because in this case the corpuscle is more identified with the condition that the pulse should not spread so much in time. This condition implies that:

$$\nu \cdot t < 1 \quad (62)$$

The condition satisfying the two models (Planck & De Broglie) is:

$$\nu \cdot t \approx 1 \quad (63)$$

Which means that:

$$dt \approx -\frac{d\nu}{\nu^2} \quad (64)$$

We take always dt positive and so we can omit the sign minus in (64) or take its absolute value:

$$dt \approx \frac{d\nu}{\nu^2} \quad (65)$$

Replace (63) in (55) for low speed corpuscle we get:

$$h\nu^2 = \vec{f} \cdot \vec{v} \quad (66)$$

Replace (63) in (58) for high speed corpuscle we get:

$$h\nu^2 = \vec{f} \cdot \vec{v} \quad (67)$$

Equation (67) is the same equation as equation (66) so it is applicable for any kind of oscillator. It is the power absorbed or emitted by an oscillator in a black body.

Planck oscillator is a classic oscillator so equation (66) is the work of a force applied on a corpuscle by unit time i.e. for a corpuscle in motion in a straight line along the axis x for example:

$$\vec{f} \cdot \vec{v} = f v = h\nu^2$$

So the force acting on the corpuscle is:

$$f = \frac{1}{v} \cdot h\nu^2$$

Duality of wave-corpuscle implies that:

$$\frac{1}{v} = \frac{dk}{d\omega}$$

with $v = v_g$: the group speed of the packet of waves assimilated as a corpuscle;

k : wave-vector of the packet of waves

$\omega = 2\pi\nu$: the frequency of the packet of waves.

So:

$$f = h\nu^2 \cdot \frac{dk}{d\omega} = h\nu^2 \cdot \frac{dk}{2\pi d\nu} = \hbar\nu^2 \cdot \frac{dk}{d\nu} = \frac{d(\hbar k)}{dt}$$

With $\hbar = \frac{h}{2\pi}$: reduced Planck constant.

$\hbar k$: have the dimension of a moment. So:

$f = \frac{dp}{dt}$ with $p = mv$: is the moment of the corpuscle.

This relation is generalized as:

$$f = m\gamma \quad (68)$$

With: $\vec{\gamma} = \frac{d\vec{v}}{dt}$ the acceleration of the corpuscle

m : the mass of the corpuscle.

Equation (68) is the fundamental law of dynamics or the Newton first law.

Planck oscillator can have also very high speed and equation (67) will be applicable. As Planck suggest that his oscillators can absorb or emit energy with quanta as a multiple integer of $h\nu$ at the same time we extend this idea that Planck oscillators can absorb power or emit power with quanta as a multiple integer of $h\nu^2$.

Between the two models of oscillators (classic and relativist) there is a special frequency ν_0 in the border of our consideration to balance in one side or another and to deduce from Planck model of black body radiation even thought to apply this model for vacuum which can be considered as a black body at zero absolute temperature (no radiation) or approximatively near zero absolute. Replace (65) in (59) we get:

$$ac^2 \approx h\nu^2 \quad (69)$$

Which mean that the frequency ν is near ν_0 changing all time otherwise there is no constant.

In fact we have exactly:

$$a = \frac{h\nu_0^2}{c^2} \quad (70)$$

Even thought the constant ν_0 can be derived from classic approximations.

In general we have:

$$\frac{d\vec{p}}{dt} = \frac{d(a\vec{\gamma})}{dt} = a\vec{v} + \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} \frac{d\vec{v}}{dt} \approx a\vec{v} + m\vec{\gamma} \text{ for } \|\vec{v}\| \ll c$$

Than:

$$m\vec{\gamma} = \frac{d\vec{p}}{dt} - a\vec{v} = \vec{f} - a\vec{v} \quad (71)$$

All forces applied on the corpuscle are equal to $\vec{f} = \frac{d\vec{p}}{dt}$ plus a permanent force $a\vec{v}$ in the opposite side of its motion

which mean an external force which drive the motion of the corpuscle even thought it is in rest. The corpuscle never can't be in rest and will oscillate around its position.

Determination of Constant ν_0 :

Planck oscillators are classic oscillators. They are oscillating charged corpuscles. In a black body they are the electrons of the atoms of the wall of the black body cavity (oscillations of nucleus of atoms are neglected).

According to Bohr model of the atom, the electron in an hydrogen atom is moving in planetary motion (circular) as the speed of the electron is equal to αc where $\alpha = \frac{1}{137}$ the fine structure constant. The vacuum in atoms is the same

vacuum in the cosmos. Vacuum which is filled with energy has a certain mechanical impedance and a negative pressure so there is no friction in the motion of any corpuscle as it is given by General Relativity.

Suppose that a corpuscle is in motion from a point A to a point B , its energy exchanged with vacuum is:

$$\varepsilon = \int_A^B a\vec{v} \cdot \vec{v} dt = \int_A^B a\vec{v}^2 dt = \int_A^B ac^2 \left(1 - \frac{\zeta_0^2}{\zeta^2}\right) d\zeta = ac^2(\zeta_B - \zeta_A) + ac^2\zeta_0^2 \left(\frac{1}{\zeta_B} - \frac{1}{\zeta_A}\right) \quad (72)$$

We have not do any approximation in equation (72). It is done in a 4-space dimensions frame .

Let's take an origin for the energy exchanged which is the point A where the corpuscle is in rest, than:

$$\begin{aligned} \varepsilon &= \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + ac^2\zeta_0^2 \left(\frac{\zeta_0 - \zeta}{\zeta_0\zeta}\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + mc^2 \left(\sqrt{1-\frac{v^2}{c^2}} - 1\right) = mc^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 + \right. \\ &\quad \left. \sqrt{1-\frac{v^2}{c^2}} - 1 \right] = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \left(1 - 2\sqrt{1-\frac{v^2}{c^2}} + \sqrt{1-\frac{v^2}{c^2}}\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \left(1 - \sqrt{1-\frac{v^2}{c^2}}\right)^2 \quad (73) \end{aligned}$$

For the electron of Bohr atom we get:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\alpha^2}} (1 - \sqrt{1-\alpha^2})^2 \approx \frac{1}{4} \alpha^4 mc^2 \quad (74)$$

With: $m = 9.1 \cdot 10^{-31} \text{ Kg}$ the mass of the electron.

If the electron is oscillating in a 4th space dimension than its oscillations are driven by an electromagnetic field in its fundamental state. The energy of an oscillator in its fundamental state is [4]:

$$E = \frac{1}{2} h\nu \quad (75)$$

Vacuum is defined as a degenerate state of the electromagnetic field near zero Kelvin. A Planck oscillator in this field have an infinite degree of states and only one degree of freedom and his energy can take any value multiple integer of $\frac{1}{2} h\nu$ and if the half of this integer is integer than it can be the source of spontaneous emission as proposed by Einstein

in his theory of laser radiation. The energy of the oscillator is also called zero-point energy and is protecting bound states in an atom [5].

We can modeling the mean energy of the oscillator as Planck did in his theory of black body radiation by considering the high states of the oscillator are not so probable and the low states are more probable by referencing to a characteristic energy $\frac{1}{2} h\nu_0$ of the vacuum ,which should be normally in the infrared.

Taking this model the mean energy U of the the electron is by statistical manner:

$$U = \frac{\frac{1}{2} h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \quad (76)$$

Equation (76) is obtained by statistical manner as for low frequencies the energy of the oscillator is:

$$E \approx \frac{1}{2} h\nu_0 \quad (77)$$

Considering the electron as 4-space dimension corpuscule than we get:

$$\frac{1}{2} h\nu_0 = 4x \frac{1}{4} \alpha^4 mc^2 = \alpha^4 mc^2 \quad (78)$$

Which mean:

$$\nu_0 = 2 \frac{\alpha^4 mc^2}{h} \approx 0.7 \cdot 10^{12} \text{ Hz} \quad (79)$$

What is a Corpuscule?:

Planck had determined his constant h from a thermodynamic system the black body which is a closed system. This constant is an adiabatic invariant used for linked systems (the oscillators) as an integral of action [6].

For open systems the action can vary with time and if we introduce an element of periodicity Planck constant will have all its generality using it for any system and this what Louis De Broglie had understand in earlier time [7].

To link Planck theory of the grain of energy $E = h\nu$ to the relativist theory of the corpuscule energy $E = mc^2$ De Broglie had associate to the corpuscule a wave with a phase speed as:

$$v_f \cdot v = c^2 \quad (80)$$

With : v_f : phase speed;

v : corpuscule speed.

He also associate to the corpuscule an internal frequency ν_{0i} as :

$$h\nu_{0i} = mc^2 \quad (81)$$

With m : the mass of the corpuscule in its rest frame S' (sometimes it is the mass of the electron for De Broglie).

If the corpuscule is in motion from a fixed observer in an inertial frame S with a fixed speed v than according to Lorentz transformations of time we have:

$$t' = t \sqrt{1 - \frac{v^2}{c^2}} \quad (82)$$

t' : time measured with a fixed clock in the frame S'

t : time measured by the observer with a fixed clock in the frame S .

The internal frequency associated to the corpuscule becomes for the observer as the inverse of transformation (82) of time:

$$\nu_i = \nu_{0i} \sqrt{1 - \frac{v^2}{c^2}} = \frac{mc^2}{h} \sqrt{1 - \frac{v^2}{c^2}} \quad (83)$$

And the length of the wave associated to the corpuscule is:

$$\lambda_i = \frac{v_f}{\nu_i} = \frac{h}{mv \sqrt{1 - \frac{v^2}{c^2}}} \quad (84)$$

But for relativity the energy of the corpuscule is :

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (85)$$

And the grain of energy according to Planck theory should have a frequency ν as:

$$\nu = \frac{mc^2}{h\sqrt{1-\frac{v^2}{c^2}}} \quad (86)$$

And this grain of energy should have an associated wave as :

$$\lambda = \frac{v_f}{\nu} = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{mv} \quad (87)$$

The two associated waves are not the same. De Broglie had showed that they have the same phase which he had called the harmony of phases. So we have according to this harmony:

$$\frac{2\pi}{\lambda_i} x - 2\pi\nu_i t = \frac{2\pi}{\lambda} x - 2\pi\nu t \quad \text{for every } (x, t) \quad (88)$$

It is very easy to deduce from equation (88) that:

$$\nu = 0 \quad (89)$$

This is not what we should expect. Instead of equation (89) we can relax it as:

$$\nu \approx 0 \quad (90)$$

The corpuscle is an oscillating resonator with a mean speed equal to zero ($\bar{q} = 0$).

For De Broglie "the energy of an electron is spread over all space with a strong concentration in a very small region". Speaking about a mass in a point have no sense and this mass which characterize the corpuscle only because it is not undividable and constitute a unit as De Broglie postulate.

The formulae of the energy of a corpuscle as given by Restraint Relativity $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ is in fact not an exact formulae: it should be revisited. From Restraint Relativity we can deduce such a conclusion.

Let's consider the scalar product of the 4-vector force and the 4-vector speed as[8]:

$$\mathbf{f} \cdot \mathbf{u} = \frac{d\mathbf{p}}{d\tau} \cdot \mathbf{u} = \left(\frac{dm}{d\tau} \mathbf{u} + m \frac{d\mathbf{u}}{d\tau} \right) \cdot \mathbf{u} = c^2 \frac{dm}{d\tau} + m\mathbf{u} \cdot \frac{d\mathbf{u}}{d\tau} = c^2 \frac{dm}{d\tau} \quad (91)$$

Where we have used that in (14) the fact that $\mathbf{u} \cdot \mathbf{u} = c^2$.

If $m = \text{Constant}$ than $\mathbf{f} \cdot \mathbf{u} = 0$ so the 4-force doesn't bring any mass to the corpuscle. We can relax this assertion by considering that if the parcel of energy mc^2 does not spread much away in his rest frame than the 4-force $\mathbf{f}$ is a pure force.

If the energy of the corpuscle is not exactly determined than we can consider it is as oscillating around which we consider its position and it have some spatial extension as:

$$\mathbf{p} = a\mathbf{l} \quad (92)$$

With $\mathbf{l}$: 4-vector string associated to the corpuscle

The corpuscle is oscillating because we need $\dot{\bar{q}} = 0$ to be conform with De Broglie phase wave. Such a corpuscle will influence by its packet of waves all the other corpuscles around it and vice versa because there is overlap of waves incoming from many sources. As a string when it is stretched by an increasing force it will vibrate more and more sharply, the overlap of waves will create an interaction force between corpuscles and this is universal independent from the nature of corpuscles: this is *gravitation*.

If there is a wave associated to the corpuscle we expect that this wave move in the same direction of the corpuscle and it is represented by the wave function [9]:

$$\psi(x, t) = A \exp(ikx - i\omega t) \quad (93)$$

This wave function describe a plane wave which itself and the same phase planes move with the phase speed:

$$v_f = \frac{\omega}{k} \quad (94)$$

The group speed is the speed at which a quantity of energy is transmitted in the space and the corpuscle can be considered as a packet of energy which move at the speed:

$$\frac{1}{v} = \frac{dk}{d\omega} = \frac{dk}{dv} \cdot \frac{dv}{d\omega} \quad (95)$$

If the relation $E = \hbar\omega$ is valid for photons and still continue to be valid for corpuscles than we have:

$$\hbar\omega = E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (96)$$

and from (95) & (96) we deduce that:

$$\hbar k = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = p \quad (97)$$

If equations (96) & (97) are valid in a frame S than they should be valid in any inertial frame S' which is in constant motion by reference to S . If the frame S' is the rest frame of the corpuscle than and by convenience that the phase $kx - \omega t$ is invariant than we should have:

$$kx - \omega t = -\frac{mc^2}{\hbar} t' \quad (98)$$

The time t' is given by Lorentz transformations as:

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (99)$$

And so we get from (98) the same relations (96) & (97).

If we neglect totally the quadratic term $\frac{v^2}{c^2}$ we get De Broglie wave function without the notion of internal vibration: it signify only that the position of the corpuscle (the electron) is very well defined as in the classic way.

Let's take the dynamic state of the corpuscle described by the initial wave function $\psi(x, 0)$. What is the position and the moment of the corpuscle at the time $t = 0$?

Photons propagate as train of waves . We don't know exactly the position of the photon. If Δx is the uncertainty on the position of the corpuscle than we can take the length of the train of material waves as an approximate measure of the uncertainty on the position. If the train contain n periods than we have:

$$\Delta x \approx n\lambda = \frac{nh}{p} \quad (98)$$

Where : λ : the wave length

It is evident that the wave length is more defined when the number of oscillations in the train is great. As a rough measurement of the relative uncertainty of the wave length we can take the quantity:

$$\frac{1}{n} \approx \frac{\Delta\lambda}{\lambda} = \frac{\Delta p}{p} \quad (99)$$

Where : Δp : the uncertainty on the moment

Combining equations (98) & (99) we get:

$$\Delta x \Delta p \approx h \quad (100)$$

The length l associated to the corpuscle can be considered as the uncertainty on the position of the corpuscle. We have:

$$l = \Delta x = n\lambda \quad (101)$$

Taking in consideration that :

$$p = \frac{h}{\lambda} \quad (102)$$

It is evident that if l is so great than the moment p of the corpuscle becomes well defined and vice versa. We deduce from (102) that:

$$\lambda^2 = \frac{h}{na} \quad (103)$$

With n : integer

If $n \rightarrow \infty$ than $\lambda \rightarrow 0$: the position of the corpuscle is very bad defined

If $n \rightarrow 1$ than $\lambda \rightarrow \sqrt{\frac{h}{a}}$: it is a length to get a compromise between the corpuscle behavior and the wave behavior.

It depends on the circumstances of the experiment and how precise the position or the moment of the corpuscle we want to measure.

If $\lambda \rightarrow \infty$ than $n \rightarrow 0$: the position of the corpuscle is very well defined: it exist in the all Universe .

In fact associate to the corpuscle an internal frequency when it is in rest , signify normally to De Broglie that the position of the corpuscle is not so precise even thought it is in rest: we should take it like this.

Conclusion

By introducing the notion of 4-vector string associated to a corpuscle we had in fact incorporate the principle of uncertainty in Restraint Relativity.

References

1. M. P. Hobson, G. P. Efstathiou and A. N. Lasenby 2006 "General Relativity" page5 Cambridge University.
2. Planck, M. (1901). On the law of distribution of energy in the normal spectrum. Annalen der physik, 4(553), 1.
3. Planck, M. (1900). The theory of heat radiation. Entropie, 144(190), 164.
4. E.Chpolski "Physique atomique" page 491 -MIR Moscow 1977
5. Matt Visser « Lecture 1 of 4 : Zero point energy » Victoria University of Wellington2016
6. G. LOCHAK « Sur quelques interprétations de la constante de Planck
7. Louis De Broglie On the theory of quanta
8. M. P. Hobson, G. P. Efstathiou and A. N. Lasenby 2006 "General Relativity" page122,Cambridge University.
9. Eyvind H.Wichmann « Berkley cours de physique volume 4 » Physique Quantique, Chapitres 5 &6, Paris 1974