

Volume 2, Issue 3

Research Article

Date of Submission: 07 August, 2026

Date of Acceptance: 03 September, 2026

Date of Publication: 10 September, 2026

Denominator-Controlled Return Closure and Equity-Premium Viability in Real-Estate Funds: Reconciling Exact-Date XIRR, Average-Equity Attribution, Financing Structure, DCF, and Economic Value Added

Hany Hassanien Badr*

Independent Researcher in Corporate Finance & Valuation Systems and Banker, Cairo, Egypt

***Corresponding Author:**

Hany Hassanien Badr, Independent Researcher in Corporate Finance & Valuation Systems and Banker, Cairo, Egypt.

Citation: Badr, H. H. (2026). Denominator-Controlled Return Closure and Equity-Premium Viability in Real-Estate Funds: Reconciling Exact-Date XIRR, Average-Equity Attribution, Financing Structure, DCF, and Economic Value Added. *Int Rev Bus Trade Econ*, 2(3), 01-22.

Abstract

This paper develops a denominator-controlled, closed-loop framework for assessing value creation and financing viability in real-estate funds. Exact-date XIRR, average-return attribution, discounted cash flow, and economic value added are assigned distinct roles rather than treated as interchangeable. The method traces dated equity, debt, and consolidated-fund cash flows; requires amount conservation before return aggregation; discloses the denominator and weights behind every average; and verifies EVA independently through capital-spread and equity-yield routes. DCF-EVA equivalence is examined in three separate control workbooks so that the real-estate cash-benefit bridge is not misrepresented as a direct DCF calculation.

Applied to a fund running from 30 June 2004 to 30 June 2011, the independently re-solved equity, debt, and fund XIRRs are 12.524602%, 5.491344%, and 8.553337%. A model-weighted claim-XIRR benchmark is 8.543440%, leaving a 0.989717-basis-point nonlinear aggregation residual. A separate period-grid NPV root of 8.563204% lies 0.986685 basis point above exact-date fund XIRR; the two differences therefore arise from different mechanisms. On the workbook-normalized exposure base, €336,346 of total benefit produces an 8.204640% fund return. The 12.939346% equity-benefit return decomposes exactly into recurring distribution, additional rental, and capital-gain components. Cumulative EVA is €95,722.461; the capital and equity routes agree in every period, while €92,716.191, or 96.8594%, arises at closure. Two weighting systems and two inverse reconstructions explain why nearby rates must not be called identical: Route A reproduces observed equity XIRR with zero residual, whereas Route B yields a counterfactual equity rate of 12.547409%. A 35/65 financing case fails the equity-premium test; a 20/80 case separates frozen-flow and two repriced regimes. The resulting architecture distinguishes exact closures, diagnostic residuals, denominator effects, and financing viability without conflating them.

Keywords: Real-Estate Funds, XIRR, Average Return, Economic Value Added, DCF, Capital Structure, Equity Premium, Model Audit, Return Attribution, Valuation Closure

JEL Classification: C63; G12; G31; G32; R30.

Introduction

Real-estate investment analysis frequently presents several plausible return numbers for the same fund: an equity IRR, a debt yield, a fund-level IRR, an average cash-on-cash return, an equity multiple, a DCF value, and an economic-profit estimate. Each number may be correctly calculated and yet still appear inconsistent with the others because it answers a different question, uses a different capital boundary, or weights cash flows differently. The analytical problem is

therefore not merely computational accuracy. It is structural attribution: determining which claim generated each cash flow, which capital base supported it, which dates govern discounting, and which identity must hold when the claims are recombined.

This paper formalizes a solution called nested return closure. "Nested" means that the analysis operates simultaneously at four levels: period operations, equity and debt claims, the consolidated fund, and total economic value. "Closure" means that each level must reconcile to the adjacent level through an explicit identity, with any remaining difference recorded as a diagnostic residual rather than suppressed by rounding or verbal explanation.

The study is computational and methodological. It does not claim that economic value added (EVA), discounted cash flow (DCF), or internal rate of return (IRR) is individually new. EVA and residual-income methods are established; the equivalence of DCF and residual-income valuation under consistent assumptions is also established; and the limitations of IRR have an extensive literature. The contribution is the integration of these ideas into a claim-level real-estate fund architecture that simultaneously tests: (i) cash-flow conservation, (ii) exact-date return closure, (iii) average-exposure attribution, (iv) equity-premium viability, and (v) DCF–EVA reconciliation.

The empirical object is a model archive rather than a statistical sample. The central dated case covers 30 June 2004 through 30 June 2011 and is expressed in euros. Two additional controlled financing cases are expressed in pounds sterling. They are analytical scenarios, not reported market observations. This status is an important strength when the research question is structural: it permits exact control over capital shares, payment timing, and repricing assumptions. It is also a limitation: the results demonstrate mathematical and computational validity, not population-level external validity.

Research Questions and Contributions

The paper addresses four questions:

- When can equity, debt, and fund XIRRs be reconciled without treating a weighted average of claim-level XIRRs as an identity?
- What does an average return add when XIRR already measures the time value of money?
- How can EVA and DCF be connected to distributions and capital gains without double counting?
- How should a model distinguish a frozen-cash-flow capital-structure counterfactual from a repricing-consistent financing case?

The answers yield seven contributions. First, the paper separates exact-date fund XIRR from both a linear claim-XIRR benchmark and a simplified period-grid NPV root, so that each displayed Difference has one defined subtraction order and one economic cause. Second, it defines an equity-premium viability rule from the spread between equity and debt XIRRs. Third, it proves that average-return component additivity is exact only when the components share the same normalized exposure denominator. Fourth, it establishes a fund-wide closure that reconciles rental income, debt interest, equity rental income, capital gain, total benefit, normalized balances, and claimant weights through zero-residual identities. Fifth, it identifies and bridges the two weighting systems used in the lower workbook panel, then reconstructs observed and counterfactual equity returns through Routes A and B. Sixth, it reproduces each dated EVA amount by both the capital-spread and equity-yield routes and connects cumulative EVA to distributions and equity benefit without double counting. Seventh, it keeps frozen-flow capital-structure counterfactuals separate from two competing repricing definitions.

Literature and Research Position

The paper sits at the intersection of investment appraisal, real-estate performance measurement, and value-based management. Shrieves and Wachowicz (2001) reconcile free cash flow, EVA, and NPV and show that valuation equivalence depends on consistent definitions [1]. Magni (2009, 2010, 2013, 2014, 2015, 2016, 2020, 2021) develops a broad architecture linking residual income, average rates of return, capital depreciation, investment and financing, and the logic of valuation [2-9]. Altshuler and Magni (2012) specifically introduce AIRR to the real-estate community and emphasize that conventional IRR should not automatically be interpreted as a period-by-period rate of return [10]. More recent work examines IRR equivalence and the connections among financial mathematics, accounting, and corporate finance [11,12].

Real-estate literature also recognizes the need to connect property decisions with corporate value and to distinguish operating performance from financing effects. Krumm and de Vries (2003) and related corporate-real-estate studies frame property resources as value-creation mechanisms rather than isolated assets [13]. The present paper is compatible with that direction but takes a narrower computational route: it asks whether a fund model can prove its own return and value statements through nested identities.

Positioning Discipline: The novelty claim is deliberately bounded. This paper does not claim the first use of EVA in real estate, the first use of XIRR, or the invention of average-return theory. It claims a new integrated diagnostic implementation and a documented case demonstration that combines those elements with claim-level conservation and financing-viability rules.

Source Architecture and Audit Protocol Workbook Hierarchy

The controlling source consists of three related real-estate workbooks. The first worksheet—titled “Assumption with limet informati” in the source workbook—is an intentionally hypothetical gateway that explores what could be inferred when complete cash-flow disclosure is unavailable. It is neither the evidential center nor a statement of actual historical disclosure. The paper therefore treats that sheet only as a boundary-condition generator. The proof chain begins after it:

Layer	Controlling worksheet	Scientific role
Operating source	PPM Funds Projection	Reconstructs rents, operating costs, EBITDA, interest, capitalized costs, dispositions, financing, and distributable cash.
Consolidated result	Final Result with Funds PPM	Recombines capital balances, claim cash flows, XIRRs, averages, weights, margins, and closing checks.
Economic proof	Economic Model	Computes ROIC, WACC, spread, invested-capital charge, dated-period value added, and the bridge to distributions.
Controlled case 2	Case Number Two / value Creation case 2	Tests a 35% equity / 65% debt structure and the minimum equity-premium rule.
Controlled case 3	Case Number Three / value Creation CASE 3	Tests a 20% equity / 80% debt structure under frozen-flow and repriced-flow regimes.

Cell-Level Verification

A post-assumption proof register links each material publication result to its workbook, worksheet, cell address, formula or input, cached value, number format, nearby label, and directly referenced precedents. The principal rates, amount closures, difference arrows, and EVA identities were also recomputed outside the source workbook. The register therefore functions as a reproducibility map: a reader can move from a reported result to its source cells and from those cells to the inputs and formulas that produce it.

Validation was performed at both formula and result levels. The three publication XIRRs in the original case were independently re-solved from their dated cash-flow vectors, the material amount and rate closures were recomputed, and no cached spreadsheet-error value was found in the controlling results. The two-case workbook contains external-link formulas whose cached values support the present analysis; a public replication copy should internalize those links before distribution.

Source-File Control and Reproducibility

The source workbooks, rather than screenshots or manually transcribed tables, are the controlling evidence. Screenshots serve only as visual maps of arrows, labels, and displayed relationships. The cell-provenance register in Appendix A identifies the principal formulas used in the paper; the complete register and file-level digests are retained in the replication archive for confidential editorial verification.

Audit Acceptance Rules

A numerical statement is accepted into the paper only when four conditions hold: the cell has a traceable source; the sign convention is explicit; the period and currency boundary are identified; and the result is independently recalculated or closed through a conservation identity. A result that is mathematically correct but economically mislabeled is not accepted as final evidence. This rule is particularly important for Case 3, where the displayed XIRR block freezes nominal claim cash flows while two other cells imply different repriced equity flows. Those three constructions cannot be represented as one forecast.

Boundary and Sign Conventions

Object	Included	Excluded / separately tracked
Equity XIRR	Equity distributions and equity reimbursement at actual dates	Debt cash flows; accounting earnings not distributed
Debt XIRR	Interest and debt reimbursement at actual dates	Equity distributions and equity capital
Fund XIRR	The dated sum of equity and debt claim cash flows	No separately invented terminal plug

Average-return attribution	Mutually exclusive benefit components divided by one disclosed exposure base	Capital reimbursement and alternative denominator conventions
EVA bridge	Invested capital multiplied by the dated-period ROIC – WACC spread	Capital reimbursement; duplicated capital gain
DCF–EVA control	Beginning invested capital plus present value of economic profit	The real-estate benefit bridge, which is reported separately

Mathematical Framework

Claim Conservation

Let E_0 and D_0 denote initial equity and debt capital. Let $CF^E(t)$ and $CF^D(t)$ denote dated net cash flows to equity and debt claimants. At the consolidated-fund boundary:

$$I_0 = E_0 + D_0 \quad (1)$$

$$CF^f(t) = CF^E(t) + CF^D(t) \text{ for every dated event } t \quad (2)$$

Equations (1) and (2) are conservation conditions. They do not depend on a discount rate. If they fail, no return metric can repair the model because the claims do not recombine to the fund.

Exact-Date Internal Return

For claim $j \in \{E, D, F\}$, XIRR r_j solves:

$$\sum_i CF_j(t_i) / (1 + r_j)^{((t_i - t_0)/365)} = 0 \quad (3)$$

The use of actual dates matters because the fund includes broken first and final periods. Annual IRR applied to equally spaced periods would impose a different temporal architecture.

Weighted-Claim Return and Diagnostic Residual

Let $w^E(b)$ and $w^D(b)$ be claim weights calculated on a disclosed base b , with $w^E(b) + w^D(b) = 1$. The base may be initial capital, date-weighted outstanding capital, or a model-specific normalized net-claim-flow amount. Define the model-weighted claim benchmark:

$$r^w = w^E r^E + w^D r^D \quad (4)$$

Because XIRR is the root of a nonlinear present-value equation, it is not additive. Consequently, r^f generally differs from r^w even when claim cash flows conserve exactly. The benchmark must not be called “capital-weighted” unless b is demonstrably a capital base. The difference is retained as a diagnostic residual:

$$\varepsilon^x = r^f - r^w \quad (5)$$

A small ε^x supports internal coherence, but zero is not required as a universal theorem. The residual is interpreted only after the weighting base is disclosed. Initial-capital weights, average outstanding-capital weights, and terminal claim weights answer different questions.

Equity-Premium Viability

$$M^E = r^E - r^D \quad (6)$$

For a minimum hurdle $h \geq 0$, the equity structure is acceptable only if $M^E > h$. The special case $h = 0$ is the minimum logical condition that equity earns more than debt. A practical investor may require a strictly positive hurdle to compensate for subordination, volatility, illiquidity, and model risk.

Economic Value Added and DCF Closure

For period t , let IC_{t-1} be invested capital at the beginning of the period, $ROIC_t$ the realized operating return, and $WACC_t$ the required capital return. Then:

$$EVA_t = (ROIC_t - WACC_t) \times IC_{t-1} \quad (7)$$

$$V_0 = IC_0 + \sum_t EVA_t / (1 + WACC)^t = \sum_t FCF_t / (1 + WACC)^t \quad (8)$$

Equation (8) is a valuation equivalence, not an automatic spreadsheet property. It holds only when operating profit, capital investment, terminal value, and discount timing use consistent boundaries. In the real-estate model, the operational bridge is expressed as recurring distributions plus value added/capital gain, with capital reimbursements

tracked separately to prevent double counting.

Average-Return Attribution and Denominator Governance

Let $\bar{E}(b)$ be equity exposure measured under an explicitly stated denominator convention b , and let B_k be the total benefit attributed to component k across N normalized years. The component average return is:

$$\bar{a}_k = (B_k/N) / \bar{E} \quad (9)$$

$$\bar{a}^E = \sum_k \bar{a}_k \quad (10)$$

Equations (9)–(10) produce an exact attribution identity when all components use the same N and the same exposure denominator. They do not replace equation (3): the average return explains where economic benefit came from on a normalized exposure base, while XIRR measures the timing-aware return on the dated investor cash-flow vector. The sensitivity of the ratio to its denominator is mechanical and transparent:

$$\partial \bar{a}^E / \partial \bar{E} = -B / (N \bar{E}^2), \quad \text{so the elasticity of } \bar{a}^E \text{ with respect to } \bar{E} \text{ is } -1 \quad (10a)$$

Accordingly, a 1% increase in the chosen exposure denominator reduces the reported average-return ratio by approximately 1%, holding benefit and normalized years constant. Denominator selection is therefore part of the model specification, not a formatting choice.

Original Real-Estate Fund Case: Operating-To-Fund Closure

PPM Operating Reconstruction

The PPM projection begins with dated net rents and deducts ICI, insurance, property management, agency fees, and other costs to obtain net operating income. Deposit commissions, administrative and general costs, and fund-manager fees are then deducted to obtain EBITDA. Interest expense is deducted to obtain funds from operating activities. Project-management fees, extraordinary maintenance, and tenant improvements are capitalized and shown with their financing. Gross dispositions are reduced by disposition costs and debt reimbursements. The resulting annual amounts of cash available for distribution are €18,094, €47,714, €58,039, €54,382, €51,152, €46,131, €76,386, and €163,663. Their sum is €515,561.

PPM identity	Amount (€)	Workbook proof
Net rents	282,551	PPM Funds Projection!M4 = SUM(C4:J4)
Net operating income	256,287	PPM Funds Projection!M10 = SUM(M4:M9)
EBITDA	224,985	PPM Funds Projection!M14 = SUM(M10:M13)
Interest expense	(119,201)	PPM Funds Projection!M16 = SUM(C16:J16)
Funds from operations	105,784	PPM Funds Projection!M18 = SUM(C18:J18)
Capitalized costs	(39,195)	PPM Funds Projection!M23 = SUM(C23:J23)
Net asset dispositions	898,385	PPM Funds Projection!M27 = M25 + M26
Total financing activities	(449,413)	PPM Funds Projection!M30 = SUM(C30:J30)
Total cash available	515,561	PPM Funds Projection!M31 = SUM(C31:J31)

The closing allocation is exact at the workbook's displayed-unit precision: €515,561 of total cash available equals €217,145 of operating and capital-gain benefits plus €298,416 of equity capital reimbursement. There is no amount residual in this displayed closure. Capital reimbursement is kept outside the €217,145 benefit total so that return of capital is not counted as economic gain.

Claim-Level XIRR

Metric	Rate	Primary proof cell	Independent interpretation
Equity XIRR	12.524601817%	Final Result!K167; Economic Model! S27	Dated return on equity benefits plus reimbursement
Debt XIRR	5.491344035%	Final Result!K156	Dated return on interest and debt reimbursement
Fund XIRR	8.553337157%	Final Result!K174 / J100	Dated return on consolidated claim cash flows
Model-weighted claim benchmark	8.543439987%	Final Result!M174 = M156 + M167	Weighted claim XIRRs on normalized net-claim-flow bases
XIRR diagnostic residual	0.009897170 pp	Fund XIRR – model-weighted benchmark	0.989717 basis points

The dated vectors used in the three publication XIRRs are shown below. At every date, equity cash flow plus debt cash flow equals fund cash flow; the maximum absolute conservation residual is €0.00.

Date	Equity cash flow (€)	Debt cash flow (€)	Fund cash flow (€)
30 Jun 2004	-280,322	-438,124	-718,446
31 Dec 2005	47,713	53,777	101,490
31 Dec 2006	58,035	78,782	136,817
31 Dec 2007	54,392	86,965	141,357
31 Dec 2008	51,142	85,090	136,232
31 Dec 2009	46,134	76,248	122,382
31 Dec 2010	76,387	132,436	208,823
30 Jun 2011	163,664	44,027	207,691

Claim	Workbook XIRR	Independent XIRR	Absolute rate difference	XNPV at independent root
Equity	12.524601817%	12.524602024%	2.07×10^{-9}	-7.64×10^{-11}
Debt	5.491344035%	5.491343842%	1.93×10^{-9}	-4.37×10^{-11}
Fund	8.553337157%	8.553337418%	2.62×10^{-9}	-5.24×10^{-10}

The benchmark uses normalized net-claim-flow bases of €228,129.286 for equity and €297,572.429 for debt. These generate weights of 43.395195% and 56.604805%, respectively. They are not conventional invested-capital weights. The weighted contributions are 5.435075% for equity and 3.108365% for debt, closing to 8.543440%. The residual is small but nonzero because nonlinear roots are being compared through a linear benchmark. Treating 8.54344% as algebraically identical to 8.55334% would be incorrect; treating the difference as evidence of a broken model would also be incorrect. It is a disclosed diagnostic comparison after exact claim conservation.

The workbook also displays a second nearby rate, 8.563204008%, in Final Result with Funds PPM!L114. That value is the discount rate that reduces the workbook's period-grid NPV in K114 to approximately zero (€0.000000133 at cached precision), using exponents 0, 1.5, 2.5, ..., 7.0. It is not a second exact-date XIRR. The two arrows labelled Difference therefore test different propositions:

8.563204008% - 8.553337157% = 0.009866851 percentage point = 0.986685 basis point (period-grid timing residual) (8a)

8.553337157% - 8.543439987% = 0.009897170 percentage point = 0.989717 basis point (nonlinear aggregation residual) (8b)

The first difference arises from replacing actual dates with simplified period exponents; the second arises from comparing a consolidated nonlinear root with a linear combination of claim-level roots. Neither is an accounting imbalance, and neither may be used as evidence that the three rates are algebraically identical.

Arrow-direction and sign control. The workbook's green Difference arrow is displayed as fund XIRR minus the period-grid NPV rate, so its signed value is -0.009866851 percentage point (displayed approximately as -0.00987%). Equation (8a) reverses the subtraction only to report the positive magnitude of 0.986685 basis point. The next arrow is fund XIRR minus the claim-XIRR benchmark and is therefore +0.009897170 percentage point, or 0.989717 basis point. On a common direction, period-grid NPV minus the claim-XIRR benchmark is +0.019764021 percentage point, or 1.976402 basis points at the reported precision; it is the sum of the timing-grid and nonlinear-aggregation residual magnitudes. A sign is therefore never interpreted without naming the subtraction order.

Normalized-Exposure Attribution: Defining the Average-Return Object

The original workbook contains an unusually important second return layer. Its denominator is a workbook-normalized equity exposure: the sum of eight reported equity balances divided by the seven-year normalization horizon. Thus, €1,678,176 / 7 = €239,739.428571. This quantity is not the simple arithmetic mean of the eight observations and is not presented as a date-weighted exposure. Total equity benefits are €217,145, or €31,020.714286 per normalized year over seven years. Therefore:

$$€31,020.714286 / €239,739.428571 = 12.939346052\% \quad (11)$$

The same return decomposes exactly into three benefit channels:

Benefit channel	Total benefit (€)	Average annual benefit (€)	Return on average equity
Recurring dividend/cash distribution	121,422.539	17,346.077	7.235387647%
Additional rental distribution	41,927.461	5,989.637	2.498394745%
Capital gain / residual value add	53,795.000	7,685.000	3.205563660%
Total	217,145.000	31,020.714	12.939346052%

$$7.235387647\% + 2.498394745\% + 3.205563660\% = 12.939346052\% \quad (12)$$

This is not a decorative "average." It is an exact attribution proof under the workbook's stated normalization. The numerators are mutually exclusive benefit components whose totals add to the full equity benefit, and every component uses the same denominator. The ratio therefore explains the composition of benefit. XIRR remains the governing metric for the timing of the investor's dated cash flows.

The first panel contains two additional arrows that require separate labels. The 12.939346052% equity return can be regrouped as 7.235387647% recurring payout return plus 5.703958405% EVA return; the latter is exactly the sum of 2.498394745% additional rental return and 3.205563660% capital-gain return. It can also be read as 9.733782392% total rental return to equity plus 3.205563660% capital-gain return. These are alternative partitions of the same €217,145 benefit, not additional returns. By contrast, the displayed 8.214285714% 'Average Dividend Yield' is SUM(C59:J59)/7, a workbook-normalized average of eight period payout-yield observations over seven years. It is neither the 7.235387647% recurring-payout attribution rate nor the 9.733782392% equity rental-benefit rate, and it is not used in the closure equations.

Fund-Wide Conservation and Worksheet Reading Order

The three workbook panels are best read as one nested proof rather than as independent dashboards. The arrows move from the dated FCF-for-debt and FCF-for-equity source rows to their separately calculated claim XIRRs; the conserved sum of those two dated vectors produces the consolidated fund vector and fund XIRR. The adjacent average-interest and equity-benefit blocks then construct the normalized-exposure return, while the claim-XIRR weights construct the separate linear benchmark. That benchmark feeds Route A's like-for-like inverse closure; substituting the consolidated fund XIRR feeds Route B's controlled counterfactual. The lower comparison blocks finally reconcile the two denominator systems, decompose their contribution bridge, certify exact-zero reconstructions, and place the period-grid NPV root, fund XIRR, claim-XIRR benchmark, and normalized average in one difference ladder. The complete worksheet is the controlling evidence for every value and formula; the full screenshot and its enlarged crops are reader-facing maps of that worksheet, not independent numerical sources. Every amount-level arrow below closes with zero residual at the workbook's stored precision.

Rate	How the workbook obtains it	Plain meaning	Why it differs
8.204639586%	$\text{€}336,346 / (7 \times \text{€}585,637.285714)$	Economic benefit earned per normalized unit of total-fund exposure	Linear benefit ratio; it does not solve dated cash flows
8.543439987%	$56.604805\% \times \text{debt XIRR} + 43.395195\% \times \text{equity XIRR}$	A disclosed linear benchmark combining the two claim XIRRs	Uses normalized net-claim-flow weights, not exposure weights
8.553337157%	XIRR of the consolidated exact-date fund cash-flow vector	The fund's timing-aware money-weighted return	A nonlinear root; it is not the weighted average of claim roots
8.563204008%	Rate that makes the simplified period-grid NPV approximately zero	A timing-grid control rate	Uses exponents 0, 1.5, 2.5, ..., 7 rather than the exact calendar dates

Reader's interpretation. The four rates do not compete for one title. The first explains benefit relative to a declared normalized exposure; the second tests whether separately calculated claim XIRRs approximately recombine; the third measures the dated return of the whole fund; and the fourth checks the effect of simplifying the time grid. A small difference is acceptable only when its source is named and the underlying cash amounts still conserve exactly.

First, net rental income of €282,551 is allocated between €119,201 of debt interest and €163,350 of net rental income attributable to equity. The latter is itself split between €121,422.539 of recurring distributions and €41,927.461 of additional rental income. Adding the €53,795 capital gain produces €217,145 of total equity benefit. At fund level, €282,551 of rental income plus €53,795 of capital gain equals €336,346; the same total is obtained from €119,201 of debt return plus €217,145 of equity benefit. Finally, adding the €298,416 return of equity capital to the €217,145 benefit yields the €515,561 distributed cash total.

The resulting monetary ladder is:

$\text{€}282,551 - \text{€}119,201 = \text{€}163,350$	(12a)
$\text{€}121,422.539 + \text{€}41,927.461 = \text{€}163,350.000$	(12b)
$\text{€}163,350 + \text{€}53,795 = \text{€}217,145$	(12c)
$\text{€}282,551 + \text{€}53,795 = \text{€}119,201 + \text{€}217,145 = \text{€}336,346$	(12d)
$\text{€}41,927.461 + \text{€}53,795 = \text{€}95,722.461 \text{ EVA}$	(12e)
$\text{€}121,422.539 + \text{€}95,722.461 = \text{€}217,145.000$	(12f)
$\text{€}298,416 + \text{€}217,145 = \text{€}515,561$	(12g)

The same chain closes on the workbook's normalized exposure denominators. Debt and equity exposure are €345,897.857143 and €239,739.428571, so normalized total-fund exposure is €585,637.285714 and the corresponding exposure shares are 59.063496396% and 40.936503604%. The debt interest rate on average debt is 4.923047060%; the equity benefit return on normalized equity is 12.939346052%. Each exposure equals the sum of eight displayed balances divided by the common seven-year normalization horizon; neither is claimed to be a simple arithmetic mean or a date-weighted capital measure. Their workbook-normalized exposure-weighted sum is exactly the fund economic-benefit return:

$$(59.063496396\% \times 4.923047060\%) + (40.936503604\% \times 12.939346052\%) = 8.204639586\% \quad (12h)$$

$$\text{€}336,346 / (7 \times \text{€}585,637.285714) = 6.892393903\% \text{ rental return} + 1.312245683\% \text{ capital-gain return} = 8.204639586\% \quad (12i)$$

This 8.204639586% rate is an exact linear average-benefit closure. It must not be confused with either the 8.543439987% model-weighted XIRR benchmark or the 8.553337157% exact-date fund XIRR. The first uses workbook-normalized exposure weights and average benefit rates; the second uses normalized net-claim-flow weights and claim XIRRs; the third solves the consolidated dated cash-flow equation. The arrows are valid only when their numerator, denominator, timing convention, and aggregation rule remain attached to the displayed result.

The amount and normalized-exposure identities now establish the common base of the model. Section 6.6 follows the worksheet's lower arrows once, in their computational order, to decompose the alternative weighting system, certify the exact zeros, and interpret every named nonzero Difference cell.

A final terminology boundary is necessary. The workbook labels €336,346 as total NOPLAT, but this total includes €53,795 of capital gain in addition to €282,551 of rental income. The paper therefore refers to €336,346 as the workbook's total economic-benefit or NOPLAT layer; it does not claim that capital gain is part of conventional accounting NOPLAT outside this model.

Exposure convention	Exposure (€)	Benefit / exposure	Difference from equity XIRR
Workbook normalization: sum of 8 balances / 7 years	239,739.429	12.939346%	+41.4744 bp
Simple arithmetic mean: sum of 8 balances / 8 observations	209,772.000	14.787824%	+226.3222 bp
Date-weighted opening-balance exposure	242,080.253	12.814227%	+28.9626 bp
Date-weighted trapezoidal exposure	227,754.608	13.620236%	+109.5634 bp
Exact-date investor cash-flow solution	Not an average denominator	12.524602%	Reference: equity XIRR

The date-weighted opening-balance ratio is numerically closest to equity XIRR in this case, but proximity is not equivalence. XIRR is the unique solution to the dated present-value equation for this conventional one-sign-change equity stream. The four average ratios are descriptive return-on-exposure measures. The scientific conclusion is therefore precise: XIRR is the appropriate timing metric; the workbook-normalized 12.939346% ratio is an exact and useful benefit-attribution metric; and neither should be substituted for the other. Having fixed that measurement boundary, the next section returns to the workbook's own two denominator systems and follows their forward and inverse equations without changing the underlying cash flows.

Denominator-Controlled Inverse Closure and Equity-Gap Amplification

Denominator Foundations and Weight Conservation

Every arrow in the reconciliation layer begins with one of two denominator systems. The claim-XIRR system uses the workbook's debt and equity claim bases in L155 and L166, normalized by their total in L174. The average-capital system uses average debt in E132 and average equity in K118, normalized by average total funds in D132. Each pair of weights closes to 100%, but the two pairs are not interchangeable because their capital bases answer different questions.

Weighting system	Debt base	Equity base	Total base	Debt weight	Equity weight	Weight closure
Claim-XIRR	L155 = 297,572.428571	L166 = 228,129.285714	L174 = 525,701.714286	56.604804680%	43.395195320%	100.000000000%
Average-capital	E132 = 345,897.857143	K118 = 239,739.428571	D132 = 585,637.285714	59.063496396%	40.936503604%	100.000000000%

The source arrows therefore establish a necessary boundary rule: a rate may be multiplied only by the weight generated from its own denominator system. Mixing a claim-XIRR rate with an average-capital weight would not be a harmless approximation; it would change the mathematical object and invalidate a zero-closure claim.

Forward Construction Under the Two Weighting Systems

The model's reconciliation layer does more than compare nearby percentages. It places two internally consistent weighting systems side by side. The first combines claim XIRRs using their denominator-derived claim weights; the second combines workbook-normalized benefit rates using normalized exposure weights. Because the rates and the weights both change, the two totals answer different questions and must not be merged.

Layer	Debt rate	Debt weight	Debt contribution	Equity rate	Equity weight	Equity contribution	Total
Claim-XIRR benchmark	5.491344035%	56.604804680 %	3.108364565%	12.524601817 %	43.395195320 %	5.435075422%	8.543439987%
Normalized-benefit layer	4.923047060%	59.063496396 %	2.907723722%	12.939346052 %	40.936503604 %	5.296915863%	8.204639586%
Contribution difference	--	--	0.200640842 pp	--	--	0.138159559 pp	0.338800401 pp

Contribution Bridge Between the Two Systems

Reading the table from left to right shows exactly where the 33.880040-basis-point bridge originates. Debt contributes 20.064084 basis points and equity contributes 13.815956 basis points to the difference. Those are contribution effects after changing both the underlying rate and its weight; they are not new standalone returns

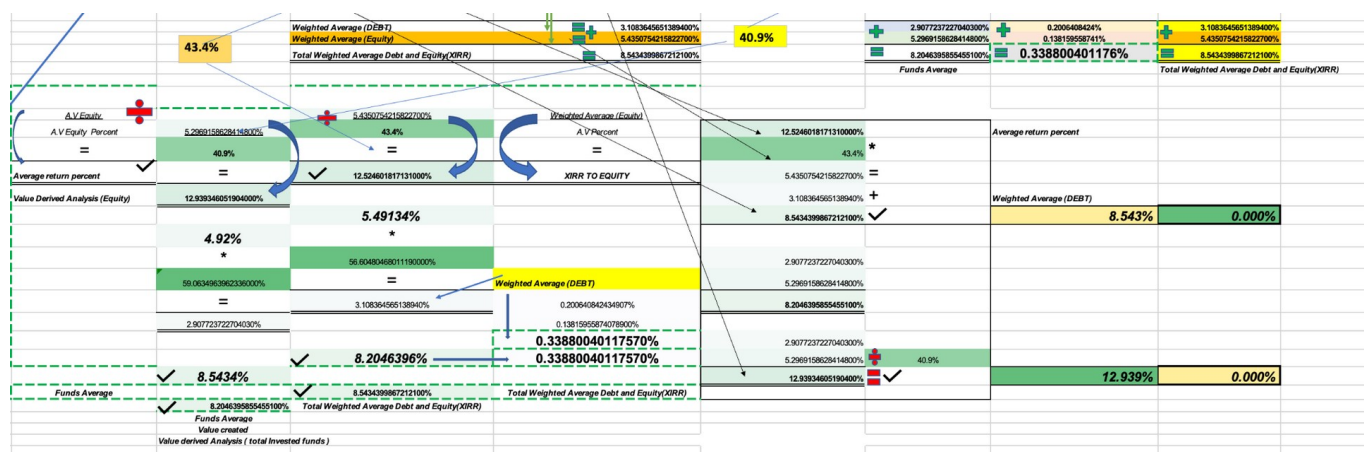


Figure 2: Denominator and Contribution Bridge. The Claim-XIRR Weights (56.604804680% Debt; 43.395195320% Equity) Construct 8.543439987%, While the Normalized-Exposure Weights (59.063496396% Debt; 40.936503604% Equity) Construct 8.204639586%. The 0.338800401-Percentage-Point Bridge is Fully Decomposed into Debt And Equity Contribution Effects. Source: Author's Original Excel Workbook; Controlling Cells are Listed in Appendix A

$$56.604804680\% \times 5.491344035\% + 43.395195320\% \times 12.524601817\% = 8.543439987\% \quad (12j)$$

The normalized-exposure construction is the identity already established in Equation (12h); it closes to 8.204639586% under its own disclosed weights and denominator.

$$(3.108364565\% - 2.907723722\%) + (5.435075422\% - 5.296915863\%) = 0.338800401 \text{ percentage point} \quad (12k)$$

Source-Cell Zero-Residual Proof

The decisive reconciliation test is not visual proximity after rounding. It is the stored source-cell residual. The following reconstructions use the workbook's own formulas and cached values. A zero is reported only when the reconstructed quantity and its comparator are the same mathematical object under the same rate, weight, denominator, and timing definition.

Workbook proof	Source-cell reconstruction	Comparator	Stored residual	What the result proves
Claim-XIRR contribution closure	$O209 = O207 + O208 = 3.10836456513894\% + 5.43507542158227\% = 8.54343998672121\%$	$Q209 = M174 = 8.54343998672121\%$	$R209 = 0.00000000000000\%$	Debt and equity claim-XIRR contributions recombine exactly to the disclosed linear benchmark.
Equity-benefit reverse closure	$O217 = O216 / P216 = 12.93934605190397\%$	$Q217 = M164 = 12.93934605190397\%$	$R217 = 0.00000000000000\%$	The average-capital reverse division reconstructs the workbook-normalized equity-benefit return exactly.
Route A equity-XIRR reconstruction	$L185 = L183 + L184 = 12.524601817131\%$	$L186 = K167 = 12.524601817131\%$	$L187 = 0.00000000000000\%$	Inverting the claim-XIRR benchmark under its own 43.395195320% equity weight returns observed equity XIRR.
Route B substitution test	$N185 = N183 + N184 = 12.547408881320\%$	$N186 = K167 = 12.524601817131\%$	$N187 = 0.022807064189 \text{ pp} = 2.280706418867 \text{ bp}$	Replacing the linear benchmark with consolidated exact-date fund XIRR produces a controlled, nonzero implied-equity gap.

These three zeros are independent logical controls, not repeated decoration. R209 closes the weighted claim contributions; R217 closes the reverse reconstruction of the normalized equity-benefit rate; and L187 closes Route A's inverse recovery of observed equity XIRR. In particular, the workbook's visible '= 0' annotation beside the 8.543439987% weighted benchmark belongs to this like-for-like recombination control; it does not assert that the benchmark equals the 8.553337157% consolidated fund XIRR. Their separately displayed difference is $K174 - M174 = 0.009897170$ percentage point, or 0.989717 basis point. Route B is intentionally different: N187 is not expected to be zero because its starting rate is a consolidated nonlinear XIRR rather than the linear claim-XIRR benchmark.

A/B Differentiation and Equity-Gap Amplification

The red A and B routes in the workbook then invert the weighting equation. Route A starts from the linear claim-XIRR benchmark and must recover the observed equity XIRR exactly. Route B starts from the consolidated fund XIRR and asks what equity XIRR would be required if the debt XIRR and the same 43.395195320% equity weight were held fixed.

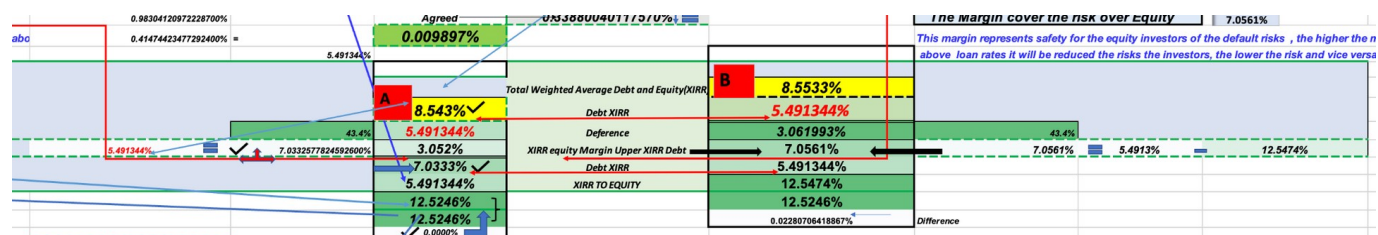


Figure 3: Routes A and B in the Source Worksheet. Route A Begins with the 8.543439987% Linear Claim-XIRR Benchmark and Recovers Observed Equity XIRR with Zero Residual. Route B Substitutes the 8.55337157% Consolidated Fund XIRR and Produces a 12.547408881% Implied Equity Rate, 2.280706 Basis Points Above Observed Equity XIRR. Source: Author's Original Excel Workbook, Final Result with Funds PPM

Step	Route A: internal linear reconstruction	Route B: consolidated-fund counterfactual
Starting fund rate	8.543439987% claim-XIRR benchmark	8.55337157% exact-date fund XIRR
Less debt XIRR	$8.543439987\% - 5.491344035\% = 3.052095952\%$	$8.55337157\% - 5.491344035\% = 3.061993122\%$
Divide by equity weight	$3.052095952\% / 43.395195320\% = 7.033257782\%$	$3.061993122\% / 43.395195320\% = 7.056064847\%$
Add debt XIRR	$7.033257782\% + 5.491344035\% = 12.524601817\%$	$7.056064847\% + 5.491344035\% = 12.547408881\%$
Compare with observed equity XIRR	$12.524601817\% - 12.524601817\% = 0$	$12.547408881\% - 12.524601817\% = 2.280706 \text{ bp}$
Interpretation	Exact internal reconstruction	Implied equity rate; not an observed return

The zero in Route A proves only that the disclosed linear weighting equation has been reconstructed correctly. It does not prove that the linear benchmark equals the consolidated fund XIRR. Route B deliberately exposes that nonlinear aggregation gap in equity-rate terms. The workbook's phrase 'margin cover the risk over equity' is therefore interpreted here as an equity-return spread or financing-viability headroom above debt XIRR, not as a measured probability-of-default protection ratio; no PD, LGD, or credit-loss model is present.

Algebraic Proof of Exact Closure and Gap Amplification

The A/B construction can be stated as one exact model identity. Let r_d and r_e denote the disclosed debt and equity XIRRs; let w_d and w_e denote their claim weights, with $w_d + w_e = 1$; and let C denote the linear claim-XIRR benchmark. The benchmark is $C = w_d r_d + w_e r_e$. For any candidate aggregate rate g , define the equity-recovery operator $\phi(g) = r_d + (g - r_d) / w_e$. This operator asks which equity rate is implied when the debt XIRR and the disclosed equity weight are held fixed.

$$\phi(C) - r_e = [C - (w_d r_d + w_e r_e)] / w_e = 0. \quad (12l)$$

Equation (12l) explains the zero in L187. It is not a coincidence created by display rounding. Route A begins with the same linear benchmark generated by the same rates, weights, denominator, and timing convention; its inverse must therefore return the observed equity XIRR exactly. The zero is a validity certificate for the internal attribution system.

$$\phi(F) - r_e = (F - C) / w_e, \text{ where } F \text{ is the consolidated exact-date fund XIRR.} \quad (12m)$$

Equation (12m) explains the controlled nonzero result in N187. The fund-level aggregation gap is $8.55337157\% - 8.543439987\% = 0.009931583\%$ percentage point, or 0.9931583 basis point. Dividing that gap by the 43.395195320% equity weight gives 0.022807064 percentage point, or 2.280706 basis points, exactly matching the difference between the Route B implied equity XIRR of 12.547408881% and the observed equity XIRR of 12.524601817%. The workbook therefore traces, rather than merely reports, how a fund-level aggregation difference is transmitted and amplified in equity-rate terms.

0.989717005 basis point / 0.43395195320 = 2.280706419 basis points.

(12n)

The multiplier $1 / 0.43395195320 = 2.304402579$ is the exact sensitivity of the inverse equity-recovery operator to a change in the aggregate rate. It is an algebraic attribution multiplier, not a probability of default, loss-given-default estimate, or independently measured financial-leverage coefficient. This result does not establish that an average return is universally more accurate than XIRR. It establishes a task-specific distinction. The denominator-controlled weighted average is exact for linear attribution, decomposition, and inverse reconstruction; exact-date XIRR is exact for the nonlinear, timing-sensitive return of the consolidated dated cash-flow vector. Earlier AIRR research develops average-return alternatives and documents limitations of conventional IRR [3,4]. The contribution here is the workbook-level dual-route protocol: an exact zero closure certifies like-for-like reconstruction, while a named nonzero residual maps the change of metric into an economically interpretable implied-equity difference, with every stage tied to source cells.

Difference Ladder and Its Exact Interpretation

The nonzero Difference cells form a second proof layer. They do not contradict the zero closures above; they measure what changes when the model deliberately moves from one metric or denominator to another. Their signs, magnitudes, and additivity are fixed by the source formulas below.

Very advanced Analysis compared 3 cases			
XIRR for total invested funds		8.553337%	
NPV roles for total invested funds		8.56320%	
XIRR for total invested funds < NPV	→	-0.00987%	Difference
XIRR for total invested funds		8.5533%	
Total Weighted Average Debt and Equity(XIRR)	✓	8.543%	✓ = 0
XIRR for Total Inv Funds > Total Witted Average D + E	→	0.009897%	Difference
NPV - Total Weighted Average Debt + Equity D+E	→	0.0198%	Difference

Figure 4: Workbook Difference Ladder. The Simplified Period-Grid NPV Root, Consolidated Exact-Date Fund XIRR, and Linear Claim-XIRR Benchmark are Deliberately Distinct. The Combined 1.976402-Basis-Point NPV-to-Benchmark Difference Equals the 0.986685-Basis-Point Timing-Grid Effect Plus the 0.989717-Basis-Point Nonlinear-Aggregation Effect. Source: Author's Original Excel Workbook, Controlling Cells L109, L114, L115, K174, M174, L176, and E184

Source cell	Exact workbook operation	Result	Controlled cause
L115	L114 - L109 = 8.563204008% - 8.553337157%	0.009866851 pp = 0.986685 bp	Simplified period grid versus exact calendar dates
L176	K174 - M174 = 8.553337157% - 8.543439987%	0.009897170 pp = 0.989717 bp	Nonlinear consolidated XIRR versus linear claim-XIRR aggregation
E184	L114 - M174 = 8.563204008% - 8.543439987%	0.019764021 pp = 1.976402 bp	Combined timing-grid and aggregation effects
M176	M174 - M171 = 8.543439987% - 8.204639586%	0.338800401 pp = 33.880040 bp	Both rate definitions and weighting bases change
N187	N185 - N186 = 12.547408881% - 12.524601817%	0.022807064 pp = 2.280706 bp	Route B implied-equity sensitivity

The ladder itself reconciles: 0.986685 basis point + 0.989717 basis point = 1.976402 basis points at the reported precision. The scientific conclusion is therefore exact. Zero residuals establish like-for-like algebraic closure; named nonzero residuals establish controlled differentiation between unlike metrics. Treating every nearby rate as identical would destroy the proof, while treating every nonzero Difference cell as an error would misread the model's principal diagnostic contribution.

Integrated Proof Chain and Acceptance Logic

Read as a single system, the workbook arrows form the following closed proof chain. Each stage consumes the verified output of the preceding stage, and no stage is allowed to borrow a weight or denominator from the other system without reporting the resulting Difference cell.

Stage	Source cells	Verified result	Logical role
1. Denominator formation	L155, L166, L174; E132, K118, D132	Two weight pairs, each summing to 100%	Defines the boundary of each mathematical object
2. Forward claim construction	M156 + M167 -> M174	3.108364565% + 5.435075422% = 8.543439987%	Builds the linear claim-XIRR benchmark
3. Forward average construction	P198 + P199 -> P200	2.907723722% + 5.296915863% = 8.204639586%	Builds the normalized average-capital return
4. Cross-system contribution bridge	Q198 + Q199 -> Q200; M176	0.200640842 pp + 0.138159559 pp = 0.338800401 pp	Explains the entire change between the two weighted totals
5. Inverse reconstruction	R209, R217, L187	Three independent exact-zero residuals	Certifies like-for-like reversibility
6. Metric substitution	L176, N187	0.989717 bp at fund level -> 2.280706 bp at equity level	Maps nonlinear fund-XIRR differentiation into implied equity terms
7. Timing-and-metric ladder	L115 + L176 -> E184	0.986685 bp + 0.989717 bp = 1.976402 bp	Separates calendar-grid and aggregation effects

The proof is therefore closed in both directions. Forward equations construct each benchmark from disclosed rates and denominator-derived weights; inverse equations recover the originating component; zero residuals certify internal identity; and nonzero residuals disclose the precise consequence of changing timing, metric, or denominator. The arrows are not decoration: together they are the workbook's acceptance protocol. With the return architecture now closed, Section 6.7 tests whether the same equity benefit is independently explained by the workbook's dated EVA and DCF/ value-reconciliation layers.

EVA and DCF Closure: Dated-Period Value Creation and Equity Reconciliation

The Economic Model computes dated-period ROIC, WACC, their spread, and beginning invested capital. Multiplying the spread by beginning invested capital produces period-level value added. The resulting sequence is €3,249.272; €61.357; -€131.755; -€55.524; -€53.282; -€66.790; €2.992; and €92,716.191. The signs identify local creation or destruction. The total is €95,722.461.

Period	Beginning invested capital (€)	ROIC	WACC	Spread	EVA (€)
30 Jun 2004	755,895	3.301649%	2.871791%	0.429858%	3,249.272
31 Dec 2005	743,403	7.039923%	7.031670%	0.008254%	61.357
31 Dec 2006	694,248	6.950542%	6.969520%	-0.018978%	-131.755
31 Dec 2007	605,685	5.848254%	5.857422%	-0.009167%	-55.524
31 Dec 2008	499,750	6.014807%	6.025469%	-0.010662%	-53.282
31 Dec 2009	393,577	6.815693%	6.832663%	-0.016970%	-66.790
31 Dec 2010	298,020	6.605597%	6.604593%	0.001004%	2.992
30 Jun 2011	108,883	90.746949%	5.594821%	85.152128%	92,716.191
Total					95,722.461

$$€95,722.461 \text{ EVA} + €121,422.539 \text{ recurring distributions} = €217,145.000 \quad (13)$$

Equation (13) is the central value-closure result. It prevents the capital gain from being treated as an unexplained residual: the gain is derived from dated-period ROIC-WACC spreads applied to invested capital. It also prevents double counting: capital reimbursement is kept outside the benefit total, while recurring distribution and EVA together explain the economic benefit received by equity. Within the wider arrow trace, equation (13) is one interior equality rather than the entire closure: equations (12a)-(12n) show how the same benefit is conserved across operating, claimant, equity-attribution, EVA, and workbook-normalized exposure representations.

Dual-Path EVA Proof AND Equity Cash-Flow Construction

The Economic Model sheet contains a stronger test than the period-by-period EVA column alone suggests. For every dated period, value added is calculated twice. The capital route multiplies the ROIC-WACC spread by beginning invested capital. The equity route subtracts the recurring dividend yield from the total cash-distribution yield and multiplies the resulting equity spread by beginning equity. Agreement between the two routes connects operating performance, financing cost, distributions, and the equity claim without inserting a balancing plug.

$$\text{EVA}_t = (\text{ROIC}_t - \text{WACC}_t) \times \text{Invested Capital}_{(t-1)} = (\text{Cash Distribution Yield}_t - \text{Dividend Yield}_t) \times \text{Equity}_{(t-1)} \quad (13a)$$

Period	Beginning equity (€)	Cash-distribution yield	Recurring dividend yield	Equity spread	Equity-route EVA (€)	Capital-route EVA (€)
30 Jun 2004	298,416	4.388840%	3.300000%	1.088840%	3,249.272	3,249.272
31 Dec 2005	293,419	9.720911%	9.700000%	0.020911%	61.357	61.357
31 Dec 2006	274,229	9.451954%	9.500000%	-0.048046%	-131.755	-131.755
31 Dec 2007	242,114	6.577067%	6.600000%	-0.022933%	-55.524	-55.524
31 Dec 2008	203,646	6.673836%	6.700000%	-0.026164%	-53.282	-53.282
31 Dec 2009	166,095	8.159788%	8.200000%	-0.040212%	-66.790	-66.790
31 Dec 2010	133,514	7.202241%	7.200000%	0.002241%	2.992	2.992
30 Jun 2011	66,743	145.215229%	6.300000%	138.915229%	92,716.191	92,716.191
Total	--	--	--	--	95,722.461	95,722.461

The equal columns are a genuine cross-check because their inputs come from different parts of the model. A negative value in 2006-2009 is not a spreadsheet error: in those periods ROIC is marginally below WACC and the total distribution yield is correspondingly below the recurring dividend yield. The final period reverses that pattern because the fund closes and the terminal economic benefit is released.

$$(90.746949\% - 5.594821\%) \times €108,883 = (145.215229\% - 6.300000\%) \times €66,743 = €92,716.191 \quad (13b)$$

The terminal amount is €92,716.191, or 96.8594% of the €95,722.461 total EVA; the preceding seven periods contribute only €3,006.270 net. This concentration is economically important. The 5.703958405% normalized EVA return is a full-horizon attribution measure dominated by the closing period, not evidence of a stable annual operating spread in every year.

Aggregate Leverage Bridge

The aggregate block links the same €95,722.461 to both the average capital base and average equity. Beginning invested-capital balances sum to seven times €585,637.285714, while beginning-equity balances sum to seven times €239,739.428571. Their ratio is the workbook-normalized average leverage of 2.442807548. The invested-capital-weighted average ROIC-WACC spread is 2.335001138%; multiplying this spread by leverage converts the fund-level spread into the equity-normalized EVA return.

$$\text{Average leverage} = €585,637.285714 / €239,739.428571 = 2.442807548 \quad (13c)$$

$$2.335001138\% \times 2.442807548 = €95,722.461 / (7 \times €239,739.428571) = 5.703958405\% \quad (13d)$$

$$7.235387647\% \text{ recurring-payout return} + 5.703958405\% \text{ EVA return} = 12.939346052\% \text{ total equity-benefit return} \quad (13e)$$

The sheet also displays end-of-period equity plus the current period's EVA. That column is a book-equity adjustment and an internal bridge; by itself it is not a complete market-value or DCF estimate because it does not capitalize all future residual incomes. The paper therefore uses it as a reconciliation aid and relies on the separate DCF-EVA controls for valuation equivalence.

From EVA and Capital Reimbursement to Equity XIRR

The right-hand cash-flow block converts the accounting and value-added layers into the investor's dated equity cash-flow vector. The first observation deducts the €298,416 initial equity contribution and adds the first period's €13,097 benefit and €4,997 capital reimbursement. Each later observation adds that period's benefit and equity reimbursement. This prevents distributions from being confused with return of capital.

Date	Initial equity (€)	Benefit: payout + EVA (€)	Equity reimbursed (€)	Equity cash flow (€)
30 Jun 2004	-298,416	13,097	4,997	-280,322
31 Dec 2005	--	28,523	19,190	47,713
31 Dec 2006	--	25,920	32,115	58,035
31 Dec 2007	--	15,924	38,468	54,392
31 Dec 2008	--	13,591	37,551	51,142
31 Dec 2009	--	13,553	32,581	46,134
31 Dec 2010	--	9,616	66,771	76,387
30 Jun 2011	--	96,921	66,743	163,664
Totals	-298,416	217,145	298,416	217,145 net

Applying XIRR to these exact dates and cash flows produces 12.524601817%. The amount total of €217,145 is not the XIRR numerator; XIRR is the nonlinear rate that sets the dated present value of the entire vector to zero. The amount identities establish conservation, while XIRR supplies the timing-aware investor return.

$XIRR(\{-280,322; 47,713; 58,035; 54,392; 51,142; 46,134; 76,387; 163,664\}, \text{exact dates}) = 12.524601817\% \quad (13f)$

Date control. The PPM horizon is deliberately broken-date: it begins on 30 June 2004, continues through the successive 31 December observations, and ends on 30 June 2011. The dates reported here are taken from the controlling exact-date vectors in Economic Model!O18:O25 and Final Result with Funds PPM!C152:J152. These full dates, rather than the abbreviated year display, govern the XIRR calculation and the interpretation of the two half-year boundary periods.

Controlled Financing Case 2: The Rejection Boundary

Case 2 is intentionally synthetic and limited-information. Total capital is £27,075,200, financed by £9,476,320 of equity (35%) and £17,598,880 of debt (65%). The model derives annual debt service of £1,159,766.192 and annual net equity distribution of £578,276.85224 after tax and fees. The corresponding net expected equity return is 6.102335635%, below the 6.59% nominal debt rate by 0.487664365 percentage point.

Test	Equity	Debt	Fund / decision
Capital	£9,476,320 (35%)	£17,598,880 (65%)	£27,075,200
Annual cash amount	£578,276.852	£1,159,766.192	£1,738,043.044 before closing reimbursement
XIRR	6.388300955%	6.924998462%	6.736694276%
Equity premium	-0.536697507 pp		Not accepted
Weighted-XIRR result			6.737442792%
Weighted vs fund residual			0.074852 bp

The 6.102336% net expected equity return is a periodic income-to-equity ratio after tax and fees; the 6.388301% equity XIRR is the exact-date solution including terminal reimbursement. They are not competing calculations. The case's model-weighted claim benchmark uses benefit-inclusive bases of £9,765,458.426 for equity and £18,178,763.096 for debt, producing weights of 34.946253% and 65.053747%. These weights generate a 6.737443% linear benchmark, only 0.074852 basis point above the 6.736694% fund XIRR. The benchmark's proximity does not cure the negative equity premium.

The rejection is not an Excel error or an aesthetic judgment. It is the output of equation (6): equity fails to earn a positive premium over debt. The case is particularly useful because the fund-level XIRR is close to the model-weighted

claim benchmark; nevertheless, the structure is unattractive to equity. Aggregate return closure cannot replace claimant-level viability. The workbook's red "Not accepted" annotation is therefore preserved as a decision result, not treated as decoration.

Controlled Financing Case 3: Frozen Flow and Two Repricing Definitions

Case 3 changes the capital mix to 20% equity (£5,415,040) and 80% debt (£21,660,160). The workbook's published XIRR block holds the Case 2 cash-flow amounts fixed while changing the claim capital bases. This produces an equity XIRR of 11.589228511%, debt XIRR of 5.572939217%, and the unchanged total-fund XIRR of 6.736694276%. These figures are mathematically valid only as a frozen-cash-flow allocation sensitivity: they ask how the same nominal claim cash-flow pool would look when divided over different principals.

A cell-level review also identifies two distinct repricing definitions inside Case 3. "Case Number Three"!F15 reports annual net equity distribution of £373,427.91968 after tax and fees; "value Creation CASE 3"!D16 instead applies its mixed-rate construction in C16 to equity capital and reports £377,940.052709. Both use annual debt service of £1,427,404.544. Neither amount is used by the workbook's published XIRR block, which retains £578,276.85224 annualized equity flow and £1,159,766.192 annualized debt flow from Case 2. The paper therefore does not suppress the inconsistency: it solves both repricing definitions independently on the same eighteen-date grid and labels their source cells.

Regime	Equity XIRR	Debt XIRR	Fund XIRR	Equity premium	Interpretation
Workbook frozen-flow block	11.589229%	5.572939%	6.736694%	+6.016289 pp	Displayed workbook sensitivity; Case 2 claim cash amounts remain frozen.
Net-distribution repricing	7.263977%	6.924999%	6.992666%	+0.338979 pp	Uses Case Number Three!F15 and F11 after tax/fees.
Mixed-rate repricing	7.356525%	6.924999%	7.011096%	+0.431526 pp	Uses value Creation CASE 3!D16 and Case Number Three!F11.

The three results must not be merged. The workbook's green "accepted" annotation attaches to the frozen-flow displayed layer. It cannot, by itself, validate either repriced forecast. Both independently reconstructed repricing variants nevertheless preserve a positive equity premium, but the margin changes from 0.338979 to 0.431526 percentage point depending on which workbook equity-flow definition is selected. The net-distribution variant is the cleaner decision case because it directly subtracts the stated tax and fee amounts from the Case Number Three pre-tax equity return; the mixed-rate variant is retained as a documented sensitivity rather than silently discarded.

Workbook Annotations as Decision Rules

The workbook's side notes articulate an economic decision rule: the margin above the loan rate is intended to provide a safety cushion to equity investors against default risk; a higher margin reduces risk, while a lower margin does the opposite. This narrative is consistent with equation (6), but it does not determine a sufficient commercial hurdle. Case 2's "Not accepted" label corresponds to a negative XIRR equity premium of -0.536698 percentage point. Case 3's "accepted" label corresponds to the frozen-flow premium of +6.016289 percentage points. For publication, these labels are treated as workbook decisions conditional on their cash-flow regime, not as universally valid investment recommendations.

Propositions and Computational Proofs

- Proposition 1: Claim Conservation is Prior to Return Aggregation**

If equations (1) and (2) hold for every date, the total-fund cash-flow vector is exactly the sum of equity and debt vectors. This guarantees cash conservation but does not imply $r^f = w^{Er}E + w^{Dr}D$ because each XIRR is the root of a different nonlinear equation. The observed original-case residual of 0.989717 basis points is therefore compatible with exact conservation.

- Proposition 2: Average Return and XIRR are Complementary**

The workbook-normalized equity return closes exactly by equation (12), while equity XIRR is 12.524602%. The 41.4744-basis-point difference is not an error. The average ratio uses normalized annual benefit divided by the workbook-defined exposure base. XIRR uses the full dated cash-flow vector, including the broken start and end dates. The former is an attribution statistic; the latter is a money-weighted temporal solution.

- Proposition 3: A Positive Fund Return Does Not Establish Equity Viability**

Case 2 has a positive fund XIRR of 6.736694% yet fails the equity-premium test because equity XIRR is 53.669751 basis points below debt XIRR. A consolidated return can therefore coexist with an unacceptable claimant hierarchy. This is a financing-absorption result: the project cash pool is positive, but debt absorbs enough return that subordinate equity is not adequately compensated.

- Proposition 4: EVA Localizes Value Creation that DCF Aggregates**

The original fund creates €95,722.461 of cumulative EVA, but four intermediate periods have negative spreads. DCF

and total XIRR compress those local episodes into aggregate values. EVA preserves the path and identifies the exact periods in which operating return falls below the cost of capital. The large final-period value creation does not erase the intermediate destruction; it offsets it in the aggregate.

- **Proposition 5: Capital-Structure Experiments Require a Flow-Regime Label**

Changing capital weights while freezing nominal claim cash flows measures redistribution over fixed flows. Changing both capital and claim cash amounts measures repricing. If a workbook supplies competing repriced claim amounts, each definition must be solved and identified separately. All are legitimate controlled experiments when transparently labeled, but they estimate different objects. The regime and source-cell labels are therefore part of the mathematical specification, not narrative footnotes.

- **Proposition 6: Average-Return Additivity is Exact but Denominator-Dependent**

If benefit components are mutually exclusive and exhaustive and use the same exposure denominator, their component ratios add exactly to the total average-return ratio. However, the level of that ratio changes inversely with the selected exposure base. The decomposition can therefore be exact without being denominator-invariant. This resolves the apparent conflict between the workbook's 12.939346% attribution ratio and its 12.524602% equity XIRR: both are correct within different mathematical questions.

Robustness and Model-Family Triangulation

The DCF–EVA identity was also tested in three non-real-estate control workbooks designed to generate cash, remain approximately value-neutral, and consume cash. These models are not pooled with the real-estate case; they serve as structural triangulation for equation (8).

Control state	Continuing ROIC	WACC	PV of economic profit	DCF value	DCF–EVA residual
Generating cash	54.771280%	18.000000%	+3,420,034.840	5,052,034.840	0.00
Neutral in aggregate	14.237988%	14.898531%	+0.000000310	1,632,000.000	2.10×10^{-9}
Consuming cash	14.237988%	18.000000%	−1,130,960.721	501,039.279	1.63×10^{-9}

For each control state, value of operations from DCF equals beginning invested capital plus the present value of economic profit to within a maximum absolute numerical residual of 2.10×10^{-9} . The neutral case is especially informative: neutrality does not require ROIC to equal WACC in every period. Positive and negative economic-profit contributions can offset over the complete valuation horizon. This reinforces the paper's insistence on pathwise evidence rather than reliance on a single terminal spread.

Managerial and Institutional Implications

For bank credit and project-finance teams, the framework separates project cash sufficiency from claimant viability. A fund can service debt and still fail to compensate equity. Conversely, a high equity XIRR created by a reduced equity denominator may not represent improved project economics if cash flows were held artificially fixed. The financing committee should therefore require both a fund-level conservation check and a claim-level premium check.

For valuers, the average-return proof provides a transparent bridge between recurring yield and capital gain. This is useful when stakeholders ask why a headline XIRR differs from an observed average distribution ratio. The denominator-sensitivity table makes the response auditable: the analyst must name the exposure convention before interpreting the percentage. Recurring yield, additional rental distribution, and value add are measured separately, then recombined on the same base.

For fund governance, the dated-period EVA sequence turns an aggregate success statement into an auditable performance path. Negative spread periods can be investigated for operational deterioration, financing cost, extraordinary capital needs, or disposal timing. The final capital-gain event can then be assessed as a genuine economic recovery rather than an unexplained closing plug.

For model-risk management, the proof register supplies an operational standard. Every reported result should have a source cell, formula, sign convention, period boundary, independent recomputation, and residual tolerance. External links should be internalized before publication or regulatory use. Frozen-flow scenarios should be labeled, and rounded display totals should be distinguished from underlying precise values.

Limitations and Future Research

The case archive is deterministic and does not model probability distributions, liquidity shocks, refinancing options, tax-law variation, or stochastic exit values. The core real-estate case is based on a single dated project architecture. Case 3 also contains two internally different repriced equity-flow definitions, which this paper discloses and solves separately rather than resolving by assumption. Accordingly, the results establish computational validity and analytical usefulness, not universal empirical superiority.

The term “average return” in this paper is a model-specific attribution measure on a workbook-normalized equity exposure. The denominator equals the sum of eight reported balances divided by seven normalized years; it is neither the simple mean of eight observations nor a continuous-time exposure integral. It should not be represented as identical to the formal AIRR construct without mapping the underlying capital stream to the precise AIRR definitions in the literature. The paper uses AIRR scholarship to position the distinction between average-based and IRR-based evaluation, but preserves the author’s workbook measure under its documented definition. The workbook’s label ‘Total (NOPLAT)’ is also model-specific in the closing period because the €336,346 total contains €282,551 of rental income and €53,795 of capital gain. This paper therefore interprets it as the workbook’s total economic-benefit/NOPLAT layer and does not generalize the inclusion of capital gain to conventional accounting NOPLAT.

Future work should test the diagnostic residual under simulated timing changes, floating interest rates, refinancing events, waterfall distributions, and multiple property exits. A further paper can embed uncertainty and estimate a tolerance surface for ε^x . Another extension can compare initial-capital weights, time-weighted average exposure, and benefit-inclusive weights to determine which residual has the strongest monitoring properties for real-estate funds.

Conclusion

The paper shows that the value of the workbook architecture lies not in producing another headline return, but in forcing different measures to account for one another. XIRR solves the exact-date investor problem. The period-grid NPV root isolates the effect of a simplified time grid. The model-weighted claim rate provides a disclosed linear benchmark for claim XIRRs. The workbook-normalized fund return closes operating benefit to claimant returns on a shared seven-year exposure convention. The workbook-normalized equity return explains the composition of equity benefit. EVA identifies the dated periods and amounts of value creation or destruction. Its capital-spread and equity-yield routes independently reproduce the same period values, while the lower-panel A and B reconstructions separate exact internal weighting identities from the nonlinear fund-XIRR residual. The three control models establish the DCF-EVA valuation boundary. Claim conservation and the equity-premium rule connect these layers to financing viability.

In the original case, the numerical system closes at multiple levels. €282,551 of rental income equals €119,201 of debt interest plus €163,350 of equity rental income; that equity rental income equals €121,422.539 of recurring distributions plus €41,927.461 of additional rental income. Adding the €53,795 capital gain yields €217,145 of equity benefit, while €119,201 plus €217,145 closes to €336,346 of fund economic benefit. On workbook-normalized exposures, the same €336,346 produces an exact 8.204639586% fund return, equal to the normalized-exposure weighted debt and equity benefit rates. Separately, €515,561 of distributable cash equals €298,416 of capital reimbursement plus €217,145 of benefit; €217,145 also equals €121,422.539 of recurring distributions plus €95,722.461 of EVA. The 12.939346% equity average return is the exact sum of three mutually exclusive components. The terminal €92,716.191 contributes 96.8594% of total EVA, so the normalized EVA return is a horizon attribution dominated by the closing period rather than a stable annual operating spread. The lower-panel proof then closes in both directions: R209 closes the claim-XIRR contributions, R217 reverses the normalized equity-benefit calculation, and Route A cell L187 recovers the observed 12.524601817% equity XIRR with an exact stored residual of zero. Route B deliberately changes only the aggregate starting rate; its 2.280706-basis-point implied-equity residual is exactly the 0.989717-basis-point fund aggregation gap divided by the 43.395195320% equity weight. Thus the zeros certify like-for-like reversibility, whereas the named nonzero residual identifies the precise consequence of metric substitution. Exact-date fund XIRR lies 0.989717 basis point above the model-weighted claim-XIRR benchmark and 0.986685 basis point below the simplified period-grid NPV root; these are different diagnostic residuals and not failed closures. The denominator test demonstrates that exact component additivity is not denominator-invariant. In Case 2, return closure coexists with rejection because equity underperforms debt. In Case 3, separating the frozen-flow block from both repricing definitions prevents a capital-structure sensitivity from being misrepresented as a unique forecast.

The resulting contribution is a disciplined, auditable system for real-estate valuation and financing analysis. It replaces metric competition with metric cooperation: each measure is retained, but each is assigned a precise role, boundary, and proof obligation.

Appendix A. Core Cell-Provenance Register

The table below maps each principal publication output to its controlling workbook location, formula or closure identity, and verified value. It is a reader-facing route into the model rather than a record of the editorial review process.

Output	Workbook / worksheet / cell	Formula or closure	Verified value
Total distributable cash	Real Estate Fuinds Model / PPM Funds Projection / M31	=SUM(C31:J31)	€515,561
Equity XIRR	... / Final Result with Funds PPM / K167	=XIRR(C167:J167,C152:J152)	12.524601817%
Debt XIRR	... / Final Result with Funds PPM / K156	=XIRR(C156:J156,C152:J152)	5.491344035%
Fund XIRR	... / Final Result with Funds PPM / K174	=XIRR(C174:J174,C152:J152)	8.553337157%
Average equity	... / Final Result with Funds PPM / M29	=SUM(C29:J29)/7	€239,739.428571

Equity benefit	... / Final Result with Funds PPM / K31	=SUM(C31:J31)	€217,145
EVA total	... / Economic Model / K14	=SUM(K6:K13)	€95,722.461
Recurring distributions	... / Economic Model / J14	=SUM(J6:J13)	€121,422.539
Economic benefit closure	... / Economic Model / L14	=SUM(L6:L13)	€217,145
Independent equity XIRR	... / Economic Model / S27	=XIRR(S18:S25,O18:O25)	12.524601817%
Case 2 equity XIRR	Two-case workbook / value Creation case 2 / W26	=XIRR(E26:V26,E23:V23)	6.388300955%
Case 2 debt XIRR	... / value Creation case 2 / W29	=XIRR(E29:V29,E28:V28)	6.924998462%
Case 2 fund XIRR	... / value Creation case 2 / W33	=XIRR(E33:V33,E32:V32)	6.736694276%
Case 3 frozen equity XIRR	... / value Creation CASE 3 / W27	=XIRR(E27:V27,E24:V24)	11.589228511%
Case 3 frozen debt XIRR	... / value Creation CASE 3 / W30	=XIRR(E30:V30,E29:V29)	5.572939217%
Case 3 frozen fund XIRR	... / value Creation CASE 3 / W34	=XIRR(E34:V34,E33:V33)	6.736694276%
Average fund exposure	Original case / Final Result with Funds PPM / L173	=L154+L165	€585,637.285714
Fund economic benefit	Original case / Final Result with Funds PPM / K171	=SUM(C171:J171)	€336,346
Average fund return	Original case / Final Result with Funds PPM / M171	=(K171/L173)/7	8.204639586%
Period-grid NPV rate	Original case / Final Result with Funds PPM / L114	PV residual K114 $\approx$ 0 on exponents 0,1.5,...,7	8.563204008%
Period-grid timing residual	Original case / Final Result with Funds PPM / L115	=L114-L109	0.986685 bp
Workbook average dividend yield	Original case / Final Result with Funds PPM / K59	=SUM(C59:J59)/7	8.214285714%
EVA return on normalized equity	Original case / Final Result with Funds PPM / O51	=(K51/M29)/7	5.703958405%
Period EVA: capital route	Original case / Economic Model / E6:F13 -> K6:K13	=(ROIC-WACC) x beginning invested capital	€95,722.461 total
Period EVA: equity route	Original case / Economic Model / Q6:T13 -> U6:U13	=(cash-distribution yield-dividend yield) x beginning equity	€95,722.461 total
Weighted spread	Original case / Economic Model / E14	=SUMPRODUCT(period spread, beginning invested capital)/SUM(beginning invested capital)	2.335001138%
Average leverage	Original case / Economic Model / E16	=F14/M14	2.442807548
Equity XIRR vector	Original case / Economic Model / O18:S27	=XIRR(S18:S25,O18:O25)	12.524601817%
Claim-XIRR denominator system	Original case / Final Result with Funds PPM / L155:L174	L155/L174 and L166/L174	56.604804680% debt; 43.395195320% equity
Average-capital denominator system	Original case / Final Result with Funds PPM / E132,K118,D132	E132/D132 and K118/D132	59.063496396% debt; 40.936503604% equity
Debt bridge contribution	Original case / Final Result with Funds PPM / Q191	=L191*M191-N191*P191	0.200640842 pp
Equity bridge contribution	Original case / Final Result with Funds PPM / Q192	=L192*M192-N192*P192	0.138159559 pp
Claim-XIRR contribution closure	Original case / Final Result with Funds PPM / O209:Q209 -> R209	=O209-Q209	0.00000000000000%; exact zero residual
Equity-benefit reverse closure	Original case / Final Result with Funds PPM / O216:Q217 -> R217	=O217-Q217	0.00000000000000%; exact zero residual
Route A implied equity	Original case / Final Result with Funds PPM / L183:L187	=(benchmark-debt XIRR)/equity weight + debt XIRR	12.524601817%; zero residual
Route B implied equity	Original case / Final Result with Funds PPM / N183:N187	=(fund XIRR-debt XIRR)/equity weight + debt XIRR	12.547408881%; 2.280706 bp residual
XIRR-versus-average bridge	Original case / Final Result with Funds PPM / M176	=M174-M171	33.880040 bp
NPV-versus-XIRR benchmark	Original case / Final Result with Funds PPM / E184	=D180-D184	1.976402 bp
Implied-equity sensitivity	Original case / Final Result with Funds PPM / N187	=N185-N186	2.280706 bp

Case 3 source-cell control: the net-distribution equity amount is £373,427.91968 in Case Number Three!F15; the alternative mixed-rate equity amount is £377,940.052709 in value Creation CASE 3!D16; annual debt service is £1,427,404.544 in Case Number Three!F11. The repriced XIRRs reported in Section 8 are independent reconstructions from these amounts, the eighteen dated events in value Creation CASE 3!E24:V24, and terminal principal reimbursements.

Appendix B. Numerical Closure Tests

Test	Calculation	Residual / result
Average-return decomposition	$7.235387647\% + 2.498394745\% + 3.205563660\%$	12.939346052%; zero decomposition residual
Economic-benefit bridge	$€121,422.539 + €95,722.461$	€217,145.000; zero amount residual
Original weighted-claim closure	$8.553337157\% - 8.543439987\%$	0.009897170 pp = 0.989717 bp
Case 2 weighted-claim closure	$6.737442792\% - 6.736694276\%$	0.000748516 pp = 0.074852 bp
Case 2 viability	$6.388300955\% - 6.924998462\%$	-0.536697507 pp; reject
Case 3 repriced viability	$7.263977444\% - 6.924998525\%$	+0.338978919 pp; positive but narrow
Case 3 mixed-rate sensitivity	$7.356524767\% - 6.924998525\%$	+0.431526242 pp; alternative source definition
XIRR numerical audit	60 dated XIRR roots	maximum absolute rate difference $\approx 5.96 \times 10^{-9}$
Equity rental allocation	$€282,551 - €119,201$	€163,350; zero amount residual
Equity rental composition	$€121,422.539 + €41,927.461$	€163,350; zero amount residual
Fund operating/gain closure	$€282,551 + €53,795$	€336,346; zero amount residual
Fund claimant closure	$€119,201 + €217,145$	€336,346; zero amount residual
Average-fund decomposition	$6.892393903\% + 1.312245683\%$	8.204639586%; zero rate residual
Claim-weight closure	$56.604804680\% + 43.395195320\%$	100.000000000%; denominator-consistent
Average-capital weight closure	$59.063496396\% + 40.936503604\%$	100.000000000%; denominator-consistent
Test	Calculation	Residual / result
Normalized-exposure weighting	$59.063496396\% \times 4.923047060\% + 40.936503604\% \times 12.939346052\%$	8.204639586%; zero at displayed precision
Claim-XIRR contribution reconstruction	$3.10836456513894\% + 5.43507542158227\%$	8.54343998672121%; R209 = exact zero residual
Equity-benefit reverse reconstruction	$12.93934605190397\% - 12.93934605190397\%$	R217 = exact zero residual
Period-grid timing comparison	$8.563204008\% - 8.553337157\%$	0.986685 basis point; timing convention
Difference-ladder additivity	$0.986685 \text{ bp} + 0.989717 \text{ bp}$	1.976402 bp at reported precision
XIRR-versus-average bridge	$8.543439987\% - 8.204639586\%$	33.880040 basis points; rate/weight-basis change
NPV-versus-XIRR benchmark	$8.563204008\% - 8.543439987\%$	1.976402 basis points
Implied-equity sensitivity	$12.547408881\% - 12.524601817\%$	2.280706 basis points; counterfactual only
Dual-path period EVA	(ROIC-WACC) x invested capital versus (distribution yield-dividend yield) x equity	All eight periods agree; €95,722.461 total
Terminal EVA concentration	$€92,716.191 / €95,722.461$	96.8594%; closing-period dominated
Leverage-spread bridge	$2.335001138\% \times 2.442807548$	5.703958405%; equals normalized EVA/equity return
Route A reconstruction	$(8.543439987\% - 5.491344035\%) / 43.395195320\% + 5.491344035\%$	12.524601817%; zero residual
Route B counterfactual	$(8.553337157\% - 5.491344035\%) / 43.395195320\% + 5.491344035\%$	12.547408881%; 2.280706 bp above observed equity XIRR

Appendix C. Reproducibility Checklist

- Use the original Excel workbooks as the controlling sources; use screenshots only as visual explanations.
- Preserve actual dates and the workbook's 365-day XIRR exponent convention.
- Keep equity, debt, and fund cash-flow vectors separate before consolidation.
- State whether claim weights are initial, average-outstanding, terminal, or benefit-inclusive.
- Keep recurring distribution, additional rental benefit, value add, and capital reimbursement mutually exclusive.
- Recompute EVA from ROIC, WACC, and beginning invested capital for every period.
- Do not merge frozen-flow and repricing-consistent capital-structure scenarios.
- Where the workbook supplies two repricing definitions, preserve both source cells and identify the selected decision convention.
- Internalize the 84 external-link formulas before public distribution of a replication workbook.

Author Statement

The author conceived and developed the models, assumptions, formulas, and analytical architecture and prepared the manuscript from his original source workbooks. AI-based tools supported only independent computational checks and English-language editing under the author's supervision; they did not originate the theory, models, data, assumptions, mathematical relationships, or conclusions. The author retains full responsibility for all research claims, numerical results, interpretations, and publication. No external funding was received, and no conflict of interest is declared. The proprietary source workbooks may be made available for confidential editorial verification under appropriate intellectual-property protections.

Hany Hassanien Badr is an Egyptian banker and independent researcher in corporate finance and valuation systems. A Cairo University economics graduate, he began his career at the Central Bank of Egypt in 1974 and has more than fifty years of experience in banking, credit, treasury, investment, corporate governance, and valuation.

References

1. Shrieves, R. E., & Wachowicz Jr, J. M. (2001). Free cash flow (FCF), economic value added (EVA™), and net present value (NPV): a reconciliation of variations of discounted-cash-flow (DCF) valuation. *The engineering economist*, 46(1), 33-52.
2. Magni, C. A. (2009). Splitting up value: A critical review of residual income theories. *European Journal of Operational Research*, 198(1), 1-22.
3. Magni, C. A. (2010). Average internal rate of return and investment decisions: a new perspective. *The Engineering Economist*, 55(2), 150-180.
4. Magni, C. A. (2013). The internal rate of return approach and the AIRR paradigm: a refutation and a corroboration. *The Engineering Economist*, 58(2), 73-111.
5. Magni, C. A. (2014). Mathematical analysis of average rates of return and investment decisions: The missing link. *The Engineering Economist*, 59(3), 175-206.
6. Magni, C. A. (2015). Investment, financing and the role of ROA and WACC in value creation. *European Journal of Operational Research*, 244(3), 855-866.
7. Magni, C. A. (2016). Capital depreciation and the underdetermination of rate of return: A unifying perspective. *Journal of Mathematical Economics*, 67, 54-79.
8. Magni, C. A. (2020). Investment decisions and the logic of valuation. Linking finance, accounting, and engineering.
9. Magni, C. A. (2021). Economic profitability and (non) additivity of residual income. *Annals of Finance*, 17(4), 471-499.
10. Altshuler, D., & Magni, C. A. (2012). Why IRR is not the rate of return for your investment: Introducing AIRR to the real estate community. *Journal of Real Estate Portfolio Management*, 18(2), 219-230.
11. Favero, G., & Piacitelli, G. (2024). Irr and equivalence of cash-flow streams, loans, and portfolios of bonds: G. Favero, G. Piacitelli. *Decisions in Economics and Finance*, 47(2), 379-399.
12. Magni, C. A. (2026). Financial mathematics and its unexpected connections to accounting and corporate finance: CA Magni. *Decisions in Economics and Finance*, 1-48.
13. Krumm, P. J., & de Vries, J. (2003). Value creation through the management of corporate real estate. *Journal of Property Investment & Finance*, 21(1), 61-72.