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Dynamic Equations of the Economy: How to Trade Market and Make Money in "N" Dimensions

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Abstract
Econophysics, supported by mathematics, has proven to be a promising field for studying complex economic systems. However, the absence of a general theory limits its applicability. This work proposes a novel framework based on fluid dynamics and artificial intelligence to jointly model and predict dependent economic events. After the model "WAVES", the model termed "ABE" (Acceleration Balance Equation), defines a space with a potential field, where geodesics represent the trajectories of economic events (the "GT" Geodesic Trajectories model) as the metric is defined by the Hamiltonian action. Artificial intelligence is utilized to optimize the model and enhance its predictive accuracy based on existing projections, working with the model described as a neural network. Comprehensive tests are conducted to validate the proposed approach, both for individual events and sets of events. The results demonstrate the model's capability to capture underlying dynamics and predict future behaviour with minimal uncertainty. This method holds significant potential for real-time applications, as well as short, medium, and long-term analyses. By minimizing or removing human intervention and leveraging the laws of physics, particularly through transport and diffusion phenomena, the proposed approach offers a robust, targeted solution for the joint modeling and forecasting of economic events, built on the foundation of the so-called natural dynamics of the economy. Consequently, a range of useful tools are defined, providing economists with means to regulate the economy and intervene appropriately at critical moments, even creating new structures. These tools enable decision-makers to design and implement effective public policies that promote economic and social welfare, as well as optimal equilibrium. The 2024 Nobel laureate in Economics proposes the proper identification of institutions and the economic structure necessary to achieve sustainable growth [1]. This growth also requires effective cooperation among individuals, as well as the preservation of both individual and conditional freedom. This article offers a key tool to accomplish that goal.

Keywords: Econophysics, Navier Stokes, Forecasting, Economics, Black Scholes, Geodesies, Euler Lagrange, Potential, Mathematics, Trading, Econometrics, Action

Contributions
This study offers various contributions to economic science, including:
- The establishment of a set of rigorous criteria for the classification, identification, and selection of economic events, based on factors such as: the amount of information, the speed of information transmission, the degree of interdependence or influence between events, and the uncertainty in their predictions.
- Development of a mathematical model to jointly model and predict "n" economic events in both the short and long term.
- Design of procedures for selecting and quantifying time series with the highest possible interdependence; It is capable of identifying whether there is a dependency between phenomena and, if so, to what extent.

- The introduction of geometric concepts, such as phase spaces and metrics, has enabled both visual and quantitative representation of economic dynamics, making it easier to detect potential instabilities, critical points, and general states.
- Detection of the proximity or presence of instabilities (deep cracks or otherwise), with the aim of mitigating their effects.
- Understanding what would have happened if the instability had not occurred; in other words, exploring the dynamics of a group of events with and without instability.
- Intervention and control tools: Analytical instruments have been developed to allow governments to evaluate the impact of their economic policies and design strategies to achieve specific goals, whether they aim for equilibrium, growth, or evolution.
- Reducing the human factor in the modelling and forecasting of economic events, but maintaining individual freedom.
- This study has also provided methods to measure the cost, effort, and effectiveness of public interventions in the economy, depending on the changes required.
- To analyze and choose the best economic structure and political institutions to achieve optimal sustainable growth with the best possible dynamics with the minimal instabilities.
- Direct and non-iterative or recurrent calculation of the coefficients of the "ABE" model in the case of using it as a neural network.
- It provides a tool to ensure that investment decisions are the most appropriate, with a clear understanding of their consequences.

Introduction

Econometric Framework

Calculating the evolution of the stock market in "n" dimensions, known as highdimensional stock market forecasting, is a complex task that involves modelling the market's behaviour based on a large number of variables and features. While there is no single formula for accomplishing this, various approaches can be applied that combine statistical techniques with machine learning. The general steps in this process include: Data collection, preprocessing, dimensionality reduction, feature selection, and, based on these steps, the selection of an appropriate model. Among current models, we can highlight support vector machines (SVM), convolutional neural networks, random forests, model training and evaluation, interpretation, forecasting, continuous monitoring, and updating, among others. These are some of the existing methods; however, our goal is to go further and develop a general mathematical method that does not impose restrictions or conditions on the time series being analyzed. This challenge can be seen even in well-known methods like ARIMA and models such as ARCH, GARCH, and others [2,3]. Additionally, the use of neural networks and similar approaches, while often yielding satisfactory results, involves an extensive learning process [4].

Today, trading primarily relies on identifying repetitive patterns and accumulated experience. While this approach might seem "unscientific or lacking in rigor," it has some justification: many patterns tend to repeat, as we will see later, suggesting that pattern recognition in trading can provide valuable insights and offer a slight edge in predictions. For example, if this edge increases the probability of success to just 51%, consistent gains are likely over time. However, the aim of this article is to demonstrate how scientific methods can be applied to improve this probability more effectively. If not applied correctly, this advantage can easily diminish, making it unwise to act without a reasonable level of certainty. Relying solely on "pattern" and "repetition" detection, as is common in conventional trading, assumes that time series are static and that identified patterns will repeat indefinitely. This is similar to extrapolating a trend line without accounting for potential changes. A good illustration is the number "п", where certain sequences or patterns of digits may be found, only to disappear later. In our view, trend analysis remains one of the best predictive methods in trading today, especially when combined with backtesting, evaluating the relationship between trend duration and prediction accuracy, and using these insights to make more informed decisions. Therefore, there cannot be a universal trendline pattern that applies to all economic events: Each time series must be analyzed individually, with trends (in general, any pattern) evaluated and validated through backtesting. Traditional trading, which typically analyses a single time series, operates under the assumption that the series is independent of others. Otherwise, its behaviour would be purely stochastic, and the analysis would lose predictive value. For richer and more precise information, multiple time series need to be combined, as they can amplify, mitigate, or cancel each other out—much like waves.

And More

Attempting to detect patterns in an economic event and identify those same patterns in other events assumes that both events share the same dynamics; and that is not valid. Each event has its own dynamics and its unique way of evolving and changing. Current methods used to model economic events rely exclusively on the information provided by each individual time series, without incorporating any data beyond what is contained within the series itself. The economist must not, and cannot, invent data that doesn't come directly from the analyzed series. It is clear, however, that the fluctuations of a time series are influenced by other related series representing different economic events, and cannot be fully explained by the series alone. These interdependencies limit or condition the "random" movements of the series, creating uncertainty in analyses that focus solely on one data source. As a result, methods that analyse time series in isolation exhibit a high degree of uncertainty, they resemble geometric exercises that seek patterns without any guarantee of repetition. This is perfectly logical, since all such methods, by relying solely on the information within a single series, share similar levels of uncertainty in their predictions. Human-driven trading, on the other hand, can be seen as emotional speculation based on subjective beliefs applied to a single time series. This approach often leads to errors, inaccuracies, and false assumptions. While modern trading does take into account the combined analysis of multiple time series, it does so in a limited manner, such as with dependent pairs of assets. These are two or more assets that tend to move in the same direction, or in opposite directions, in a predictable way, reflecting a positive or negative correlation. This relationship can be influenced by various factors, such as:

- Commercial Relationship: Companies that compete with each other, suppliers and customers, or companies operating in the same sector.
- Macroeconomic Influences: Assets sensitive to the same economic factors, such as interest rates or oil prices.
- Historical Correlation: Assets that have shown a statistically significant relationship in the past. Nevertheless, there are methods for trading with dependent asset pairs:

Spread Trading

- **Concept:** Involves buying one asset and selling another that is expected to move in the opposite direction. The difference between the prices of the two assets is called the spread.
- **Strategy:** The goal is to profit from changes in the spread, whether it narrows or widens.
- **Example:** Buying shares of an oil company while selling shares of a renewable energy ETF.

Pair Trading

- **Concept:** Similar to spread trading, but instead of buying and selling, a long position is taken on one asset and a short position on another.
- **Strategy:** The aim is to take advantage of mean reversion, meaning that when the prices of the assets deviate from their historical relationship, they are expected to converge again.
- **Example:** Going long on a tech company's stock while shorting another tech company that has underperformed in the same sector.

Cointegration

- **Concept:** A statistical technique used to identify if two price series are cointegrated, meaning they tend to move together over the long term.
- **Strategy:** Cointegration models are used to build trading strategies based on short-term deviations from the long-term relationship.

Of course, it is also possible to attempt to identify patterns between two events and extend this approach to "n" events. However, this type of joint analysis is quite specific, as it assumes a particular dependence between the occurrences.

Indeed, the "VAR" model, widely used to model several dependent series jointly, assumes that the values depend on previous values, going back as far as the model is designed to consider; the coefficients are calculated using "AI," detecting patterns, neural networks, etc. The model generates a surface that interpolates all the points; its expression is as follows:

$$\begin{pmatrix} \mathrm{PIB}_t \\ \mathrm{Desempleo}_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \sum_{i=1}^{p} \begin{pmatrix} \phi_{11}^i & \phi_{12}^i \\ \phi_{21}^i & \phi_{22}^i \end{pmatrix} \begin{pmatrix} \mathrm{PIB}_{t-i} \\ \mathrm{Desempleo}_{t-i} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \quad (1)$$

Being "PIB" and "Desempleo", 2 economics events.

A background in fluid dynamics has provided a broad understanding across various branches of engineering, suggesting a potential interconnection between fluid mechanics and economic dynamics. While at first glance this comparison may seem artificial, the experience gained in different fields strengthens the notion of a deep connection between the two. To establish this analogy, it is essential to compare key concepts from fluid dynamics with those in economics:

- Waves: Fluids move through and by means of waves; in economics, prominent scholars have identified and studied what they referred to as "cycles" (or waves). One such example is Kondratieff's work [5]. There's a well-known saying in economics: "Nothing rises or falls forever", which is essentially a definition of a wave.
- Information Transport: This is a fundamental phenomenon in fluid dynamics; waves carry information that can be quantified. Similarly, in economics, information is also transmitted. We are all familiar with phenomena like "trends" or "fads"; word of mouth is a classic example of information transport in economics.
- Loss or Transformation of Information: A crucial phenomenon in fluid dynamics is the loss or transformation of information, which occurs due to diffusion. Over time, this process results in information being transformed into temperature (as seen in Kolmogorov scales) [6].

Throughout history, numerous attempts have been made to model economic phenomena using mathematical models derived from fluid dynamics. However, most of these efforts have focused on specific cases, such as applying turbulence dynamics to study stock market fluctuations or the velocity of money [7,8]. In economics, while we may not know what the most important equation is, the most famous one is undoubtedly the Black-Scholes-Merton equation, which earned a Nobel Prize [9]. This equation includes a critical component known as the diffusion term, essential for the proper functioning of the model. Diffusion is not only relevant in this context, but it also appears in other areas of economic structure, such as the analysis of the spread and evolution of pandemics or society's gradual acceptance of new taxes, among other examples. There are even authors, such as Victor Olkhov, who have attempted to model the dynamics of the economy using waves by leveraging the phenomenon of transport [10].

As will be shown when deriving the mathematical model, there are similarities with the Navier Stokes equations, and both share a notable property: the underlying geometry in fluid dynamics exhibits scale invariance, meaning that large-scale structures are repeated at smaller scales [11]. This feature makes the proposed method suitable for modelling and predicting time series over extended periods, such as 10, 20, or 30 years, as well as for analysing much shorter intervals, such as 24 hours. The main challenge in modelling and predicting an economic event, as mentioned earlier, is the lack of information. When analysing two dependent time series, "X" and "Y," together, the information obtained, "I," comes not only from each series individually, but also from their interaction. In other words:

$$I(X,Y) = I(X)+I(Y)+I(X \frown Y) \quad (2)$$

If the dependency between 'X' and 'Y' is strong, the information provided by the factor '(X ⌒ Y)' is significant; therefore, the goal of a good analysis is to work with time series or events that have the highest possible dependency. This makes modeling and prediction simpler and more useful. This increase in information, as will be analyzed in this article, arises from working with so-called relative masses or importances, as well as relative viscosities. The information used to model a phenomenon is the raw material that allows us to reduce uncertainty in any prediction. If the information is incorrect or insufficient, the level of uncertainty will be high. A simple example in the context of trading would be predicting what I will eat tomorrow: If, for the past 50 years, I've eaten seafood every day, the prediction might seem obvious. However, I will only eat seafood if it's available tomorrow: Meaning it depends on "another" time series. In other words, while it is relatively easy to trigger short term changes, sustaining those changes over time is much more complex.

Another crucial concept is the prediction of instabilities or "cracks" in the economy. These instabilities are indeed accounted for in the proposed model, and their prediction could have a significant impact, as it would allow us to anticipate their "proximity" and, consequently, mitigate or even alter their effects. In such cases, government intervention is absolutely essential and necessary to manage crises and their potential consequences. The goal of this article is to develop an extended model in "n" dimensions or events, based on the previously formulated one-dimensional model. These "n" events are interdependent, although if any of them are not, or are only marginally related, the model itself will identify and appropriately separate the independent events.

The classification criteria, whether natural, absolute, or relative, serve as essential tools for economists when deciding whether to include an event in the analysis, discard it, or replace it with another. This classification is based on three concepts: the amount of information, the uncertainty in prediction, and the capacity of one event to influence another. Additionally, it is possible to apply the so-called transfer function "TF" to a time series when some form of instability is detected [12-17]. This allows for the analysis of conditions both before and after the instability. Instability represents a radical change in the dynamics of the series, and the TF enables modelling how the series behaves in response to such a change [18]. Therefore, the transfer function also facilitates the classification of time series based on the velocity of change and its effects.

A space composed of events or occurrences is defined, within which a potential and a metric are established [19]. This allows for the calculation of the geodesics of the space, which represent the "natural" trajectories of events within the phase space [20-22]. These geodesics reflect the "normal" way in which events move, either in the absence of external forces acting on the system or under the influence of instabilities. The representation of measured data within this space also forms a geodesic, meaning that the observed dynamics follow these natural trajectories. In this article, the primary focus will be on locating and analysing the geodesics from any seed point [23]. This concept is crucial from a political perspective, as economic policy is based on exploring possible trajectories and measures to implement (seed points) to achieve specific goals. Understanding how the economy will evolve from a particular initial state provides valuable insights into possible actions to take and the changes that may occur. To calculate the geodesics, the ABE model is "solved" throughout the space, taking into account initial and boundary conditions, especially the pressure field. There are studies that address chaotic geometry, including attractors and repellers, in the context of economic or financial events [24,25].

However, these studies only confirm the existence of this geometry without providing concrete methods for applying it or predicting its dynamics. The goal of this article, while primarily focusing on modelling and predicting time series—both isolated and joint—with the least possible uncertainty, presents a method that can be used to model various dynamics, from the flow of a moving fluid to the behaviour of a group of pedestrians, as well as economic dynamics. Regarding pedestrian dynamics modelling, a notable example is the analysis of visitor flows at the Tokyo Aquarium [26].

This analysis is conducted by applying modified Navier Stokes equations, and the advantages of the proposed model are significant. It not only allows for the examination of small-scale dynamics within the facility but also predicts potential bottlenecks that could be dangerous in the event of an emergency evacuation. The improvements achieved by applying the method outlined in this article are remarkable compared to the previous study [26]. The developed model is based on specific definitions of fluids, which implies the need not only to extract information from the analyzed time series but also to parametrize that information using the required definitions, particularly: density, velocity, acceleration, pressure, and viscosity. Furthermore, this method is applicable to any time series without restrictions. Unlike current methods for modelling and predicting time series that require the series to meet a set of specific conditions, which limits their general applicability, the approach presented here is more flexible.

Defining and detecting the so-called equilibria of an event is fundamental. In the analysis of a single event, identifying this equilibrium can be complicated and involves high uncertainty, similar to the difficulties in prediction due to the scarcity of available information. The economy naturally tends toward equilibrium, although it is never complete; there will always be parts of the economic structure that do not achieve it. In other words, the equilibrium to which it tends is imperfect, but it satisfies the condition that the sum of tensions is minimal. The economist's role is to manage these tensions effectively by modifying certain elements, eliminating or replacing others, and even creating new ones that support or assist those that function poorly. This process is comparable to that of an automobile mechanic: detecting faults and correcting them, which requires a deep understanding of each component of the system.

The most problematic equilibrium that economic dynamics can tend toward is the socalled unstable equilibrium, characterized by being an apparent equilibrium. In economics, an unstable equilibrium refers to a situation in which the forces acting on a market or an economy are not strong enough to keep the system in its current state. This means that any small external disturbance can displace the system from equilibrium, leading it to a new state that may be better or worse than the previous one. The concept of unstable equilibrium is crucial in economics, as it helps explain a wide variety of economic phenomena, such as:

- **Economic Bubbles:** These occur when the price of an asset, such as stocks or real estate, rises rapidly above its fundamental value. This can be due to excessive speculation or other factors. In an unstable equilibrium, a small disturbance can cause the bubble to burst, leading to a sharp decline in prices and significant losses for investors.
- **Financial Crises:** These arise when there is a sudden loss of confidence in the financial system. This can result from a variety of factors, such as an increase in loan defaults, a banking crisis, or an economic recession. In an unstable equilibrium, a small disturbance can trigger a financial crisis, with severe consequences for the economy as a whole.
- **Sudden Price Changes:** These changes, such as stock market crashes or sharp increases in food and energy prices, can also result from an unstable equilibrium.

Examples of Unstable Equilibrium in Economics

- **The U.S. housing market in the 2000s:** This market was characterized by a rapid rise in housing prices, partly driven by high-risk mortgage lending. It was in an unstable equilibrium, and when the bubble burst in 2007, it triggered a global financial crisis.
- **The Euro Crisis:** This financial crisis began affecting the Eurozone in 2009 and was caused by a series of factors, including high debt levels in some countries and the lack of an adequate fiscal and banking union. The Eurozone was in an unstable equilibrium, and the crisis led to a recession in many of its member countries.

There are Three Types of Economic Instabilities

- Human-created instabilities.
- Those that can be defined as surprises, such as the fall of a meteorite.
- Instabilities inherent to a dependent system; in other words, those that arise from the very interaction and dependence of the analyzed events.

Instabilities are the "salt and pepper" of economic dynamics; without instabilities, the "natural" dynamics of the economy are quite dull... But: ¿ Is the Intentional Price Variation, a instability?: What Are Institutional Investors? Institutional investors are large entities that manage vast amounts of capital and invest on behalf of others.

Some Examples Include:

- Investment funds.
- Pension funds.
- Insurance companies.
- Investment banks.
- Hedge funds.
- Family (private offices of high-net-worth individuals).

These players manage millions or even billions of dollars, and therefore, their impact on the markets can be very significant. What are they after? Their ultimate goal is to generate returns, like any investor, but:

- They have more resources, information, tools, and access than retail investors.
- They aim for efficiency in capital management, but also seek power and influence in the market.
- Some (like hedge funds) may have much more aggressive goals: maximizing short-term profits, even if that means

causing sharp price movements.
- How do they do it? How do they influence prices? There are several strategies, some ethical, others quite questionable.

Here are the Most Common Ones

- **Coordinated Moves:** If several funds buy a stock at the same time, the price goes up. If they sell, it goes down. This can create false signals to attract or scare off other investors. Example: They buy large quantities of a cheap stock so the price rises so retail investors get excited and buy in so institutional investors sell at a higher price so the price falls so retail investors lose money.
- **Wash Trading and Spoofing (Illegal but They've Happened):** Spoofing: placing fake orders to create the illusion of interest (e.g., placing large buy orders and canceling them before execution). Wash trading: Buying and selling to themselves to simulate volume. Both practices are illegal, but there have been many cases, especially in crypto or less-regulated markets.
- **Psychological Manipulation (Market Sentiment):** Through the media, social networks, rumors, or even reports, they can influence public perception. Example: "Fund X has increased its position in Tesla" so media frenzy so the price goes up.
- **Institutionalized Pump and Dump:** This doesn't only happen in crypto. Sometimes, they inflate the price of an asset with buying activity, hype, or marketing, and then sell at the top. Many retail investors end up buying in too late.

Why do they do it? Because they can, and because the financial system, while regulated, has gray areas. Additionally:
- They have return targets they must meet.
- They compete with each other to be the best performers.
- In some cases, they aim to shake out "weak hands" (small investors) to enter positions more safely.

These investors don't coordinate with each other, but it often seems like they do: Just like birds in a flock, they don't explicitly agree to act in the market together, they simply respond to the movements and actions of others. This creates the appearance of coordinated behavior. This is why it's possible to model this group of large investors (just like a flock of birds), and their collective actions are not considered instabilities (a single isolated intervention, however, is considered an instability).In fact, modeling the movement of a single bird (isolated) and predicting its actions is not possible: The bird has freedom of action; but it is possible to model the dynamics of a group of birds; the bird isolated, as a investors, they respond to the actions or movements of the other birds. …. As already mentioned, freedom, within a group, becomes diluted...

The Instabilities Inherent to the Created Mathematical Model Can be of Two Types

- Instabilities caused by specific values belonging to dangerous intervals.
- Instabilities caused by wave resonance: Wave resonance is a physical phenomenon that can partially explain the formation of giant waves or rogue waves in the ocean. This phenomenon relates to the interaction of different waves in the water and how they can amplify under certain conditions. Wave resonance and the emergence of giant waves in the ocean can serve as a powerful analogy for understanding how major instabilities or crises arise in the economy. Both phenomena share fundamental principles of energy accumulation, dynamic synchronization, and nonlinear effects that amplify small disturbances into something much larger and disruptive:
- **Wave Resonance and Synchronization of Forces:** In the ocean, resonance occurs when waves interact in such a way that their frequencies and energies align, mutually amplifying. Similarly, in the economy, different "waves" (economic, financial, social, and political factors) can interact, synchronize, and produce disproportionate outcomes.
- **Waves in the Economy:** Factors such as interest rates, inflation, consumption cycles, currency fluctuations, and geopolitical tensions act as individual waves.
- **Economic Resonance:** If several economic trends or disturbances (like high levels of debt, speculative bubbles, or trade imbalances) align and reinforce each other, a significant economic crisis can emerge.
- **Example:** In an economy where consumption is sustained by rising debt, an increase in interest rates may coincide with a drop-in consumer confidence and a collapse in asset prices (such as real estate). The synchronized combination of these factors amplifies the negative impact.
- **Constructive Interference and Speculative Bubbles:** In the ocean, constructive interference allows the crests of different waves to combine, generating rogue waves. In the economy, this can be understood as the accumulation of speculative behaviours or financial risks that, instead of neutralizing each other, reinforce one another.
- **Economic bubbles:** Excessive speculation in a sector (such as housing in 2008 or dot-coms in 2000) can be seen as a growing wave. If credit policies support it (banks' lending massively) and investors act in sync, the bubble inflates to unsustainable proportions.
- **Catastrophic Collapse:** When opposing forces (such as credit contraction or changes in market expectations) come into play, the "giant waves" of instability collapse.
- **Opposing Currents and Economic Shocks:** In the ocean, waves can become higher and more dangerous if they encounter currents flowing in the opposite direction. In the economy, "opposing currents" can be disruptive factors that exacerbate existing problems.
- **Economic Example:** If a globalized economy faces a financial crisis (like in 2008) and simultaneously encounters trade tensions (such as a tariff war), these forces can amplify instability.

- Another opposing current could be political intervention, such as strict monetary controls that exacerbate a recession instead of alleviating it.
- **Nonlinear Chaos and Systemic Fragility:** In complex systems like the ocean or the economy, the interactions between elements are not always predictable. Small disturbances can generate significant effects due to structural fragility.
- **Global Economy:** Just as giant waves can arise from small variations, an economic crisis can originate from seemingly minor events, such as a debt default by a small company or country. If the global financial system is interconnected and in an unstable equilibrium, these disturbances can trigger a "financial tsunami".
- **Practical Analogy:** Normal waves: In the ocean, most waves are small or medium-sized, like normal economic cycles of expansion and contraction. These oscillations are usually manageable.
- Kolmogorov scales8Arise from the synchronization of multiple factors (resonance), much like major economic crises that are rare but devastating, such as the Great Depression of 1929 or the Financial Crisis of 2008.

A tremendously important economic concept underlying what has been analyzed so far is that of natural dynamics. It is defined as the dynamics primarily in the absence of "surprise" type instabilities. This logically means that the natural dynamics of the economy, as well as the model presented in this article, cannot predict unexpected instabilities and, therefore, cannot include them in their projections. However, it is possible to understand their consequences. Furthermore, we will analyse how to learn from these instabilities in general, to the point of predicting them and understanding their effects in order to modify them. The amount of instabilities in an economy directly depends on its structure.

We all know countries where the risk of instability is much greater than in others, due to the particularities of their economies.

In Galton's machine, each ball has a 0.5 probability of moving left or right as it chooses its path through the machine's labyrinthine structure [27]. However, the resulting geometric figure, resembling a normal probability distribution function, is always the same at the bottom. In economics, something similar occurs: its structure determines its dynamics and evolution. That is to say, each economy has a natural dynamic that depends on two factors: Structure and the will or decision-making style of individuals. It is important to clarify that this is not intended to carry a pejorative connotation nor to limit freedom of choice; rather, it emphasizes the importance of "educating" society appropriately:

The structure of Galton's machine is analogous to the power of human decision-making, though these decisions are optimal only when adequate knowledge is available. Having sound knowledge is crucial for making accurate decisions, as it provides a solid foundation of information to evaluate options and anticipate possible outcomes. The dynamics of a flock of birds can serve as an excellent analogy for understanding the economy, as both are characterized by complex behaviours and emergent patterns generated by individual decisions that, when interacting, create a larger structure:

Individual Actions and Collective Patterns (Emergence)

Flock: In a flock, each bird follows simple rules like avoiding collisions, staying close to the group, and aligning with the general direction of its neighbours. There's no leader directing the movement of the entire flock; rather, the structure emerges from the sum of individual decisions; that is: They respond to the actions or movements of the other birds. In fact, the flock, moves like a single bird.

Economy: In the economy, individual agents (people, businesses, governments) make decisions based on their interests (consumption, investment, production, etc.). While no single entity controls the entire economy, individual actions generate patterns like economic cycles, inflation, or market trends. These are emergent phenomena arising from the interactions of numerous individual choices without any central design.

Adaptation to External Changes

Flock: Flocks respond instantly to threats, environmental shifts, or predators. Birds don't communicate explicitly, yet any movement change by one bird impacts its neighbours, and this shift rapidly propagates across the group.

Economy: Similarly, the economy constantly adapts to external changes, such as technological advancements, regulatory shifts, or crises. If a company adopts an innovation or a country adjusts its monetary policy, these changes can ripple through and affect other economic agents, altering market dynamics, production, and consumption.

Benefits of Coordination

Flock: In a flock, coordination provides collective benefits like protection from predators and reduced energy expenditure. This unintentional cooperation makes the group safer and more efficient.

Economy: In the economy, coordination and interaction among agents enable growth, innovation, and stability. For instance, trade agreements, shared investments, and market-driven pricing allow for more efficient resource use, benefiting all participants.

Simple Rules Generate Complex Structures

Flock: In a flock, simple rules (separation, alignment, cohesion) generate complex movements and visually impressive patterns without central planning.

Economy: Likewise, in the economy, basic principles like supply and demand or the pursuit of profit and efficiency lead to complex phenomena, such as globalization, economic cycles, or financial market behavior.

Synergy and Competition

Flock: Although all birds move together in collective action, each also has to protect its own space and avoid collisions, balancing synergy and competition.

Economy: The economy holds a similar balance between cooperation and competition. Companies and individuals seek to maximize personal benefits, but in doing so, they indirectly contribute to the overall system. Markets, for example, are arenas of competition but also promote efficiency and innovation, which benefit society as a whole.

Reaction to Local Signals

Flock: Birds don't react to the entire flock but to their nearest neighbours, basing flight decisions on what they see and sense locally.

Economy: Economic agents also make decisions based on locally accessible information, such as prices in their market, available resources, and nearby competition. These local adjustments can lead to larger shifts and trends, like the creation of industrial clusters or sectorial concentration in certain regions.

Cascade Effects and Vulnerability

Flock: A change in direction or speed by a few birds can trigger a cascade effect, impacting the entire flock. This is useful for escaping threats but can also lead to abrupt movements or group errors.

Economy: Cascade effects are also common in the economy, such as in financial markets where the behaviour of a few agents can trigger chain reactions. A shift in investor confidence, for example, can lead to financial crises, as seen in the dot-com bubble or the 2008 financial crisis.

Thus, a flock of birds and the economy function similarly in terms of collective dynamics and emergent patterns: Both are decentralized, self-regulating, and adaptive systems. This comparison illustrates how simple, local interactions among many individuals can create complex and resilient structures. The economy, like a flock, doesn't need a single central authority to coordinate all decisions; instead, individual choices and basic rules yield impressive collective outcomes, though these can sometimes be challenging to predict. Therefore, the economist's goal should focus on achieving the most stable equilibrium (state of minimum energy) possible, assisting and intervening in the economy to optimally and efficiently reach that state [28]. To do this, they may choose to appropriately modify the economic structure (by substituting, eliminating, or creating elements) or to influence how individuals make decisions. This does not imply that the economy is destined to follow an unavoidable path; rather, changes are complex and difficult to implement. A single bird has a lot of difficulty changing the direction of a flock, but when it joins others, the complexity is reduced, and the goal becomes achievable.

Similarly, a plumber can install an expansion tank or a pressure vessel to minimize sudden fluctuations in pressure; an electrician, in turn, can install a capacitor to stabilize voltage variations. Providing economists with mathematical tools to achieve such goals is another objective of this article. The concept of Natural Dynamics is therefore exceptionally important, and it can be related, perhaps redefined, with economic cycles: we know that there are various periods; some of them include: Kondratieff cycles - 54 years; Juglar cycles - between 7 and 11 years; Kuznets cycles - between 15 and 25 years; Kitchin cycles - between 3 and 4 years; Scheinkman cycles - between 11 and 12 years; Vanoli cycles - between 18 and 20 years [27,29]. If we were to sum all these cycles, we would obtain a graph that corresponds to the natural dynamics of the economy; this is the essence of the concept. The dynamics of this sum have been restored each time there has been an instability; and some of these instabilities have been significant throughout history. However, the dynamics have followed their trajectory and regained that natural quality, despite these instabilities in the economic system. Detecting, discerning, separating, and summing cycles is a good way to gain insights into the general dynamics of a group of economic events.

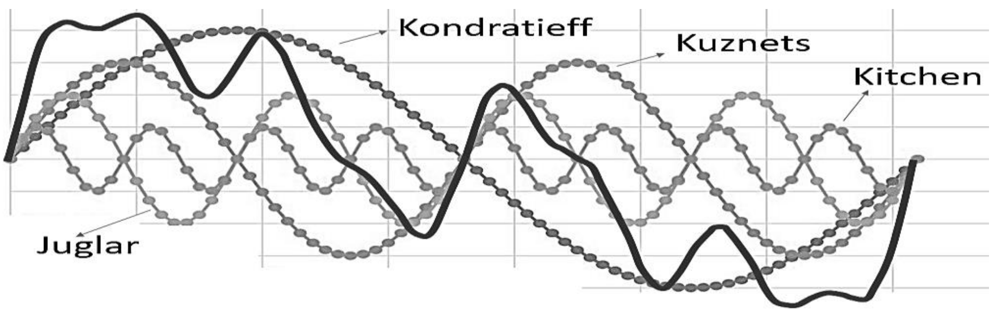


Figure 1:

The curve with the black line is the sum of the four named economic cycles. It is necessary to perform a Fourier analysis to determine the starting point of each wave. The use of waves or cycles to model the economy can be extended to the modeling of time series through their most significant harmonics. This could become a method for simplifying the series, thereby eliminating randomness and more. This will be analyzed further later on.

Note: The natural dynamics of the economy occur in the absence of instabilities. If we analyze a curve quantifying the rabbit population in an environment, we can predict its future evolution by observing the graph. However, this prediction will be limited. By incorporating another graph representing the fox population, we can combine both to achieve a more accurate prediction and joint dynamics of both populations. The mathematical model connecting these two dynamics is known as the Lotka-Volterra model. This dynamic, which the model can predict, represents the natural evolution in the absence of instabilities. However, if at a specific moment a virus begins to affect the rabbits, this natural state is disrupted. The virus becomes an unpredictable instability, although it provides an opportunity for learning. This is the essence of the concept of natural economic dynamics.

This article and the method it presents aim to be accessible from multiple perspectives. We believe that an overly complex explanation can alienate those interested in this novel procedure, thereby reducing the chances of it becoming a useful and widely adopted tool. Evsey Domar expressed this clearly: "My main connection, even if only occasional, with econometricians consists of the dollars I pay each year for their incomprehensible journal". An overly mathematical explanation, lacking clear foundations regarding the numerical models applied to economics, can lead to situations similar to those criticized by Wassily Leontief, Nobel Laureate in Economics in 1973, who stated that some economics departments "are preparing a generation of stupid scholars: geniuses of esoteric mathematics but true children in the realm of real economic life" (M. Szenberg, 1994). Can anyone imagine meteorologists stating that they "think" there will be bad weather tomorrow? The same principle should apply to economics: it is essential to use scientific methods to model and predict without bias, even in the absence of political influences. Like economics, meteorology relies on uncertain or random data, yet it continues to be considered a science. Therefore, it is crucial to incorporate human decision-making into a mathematical model for economics. Currently, there are numerous approaches, including Dynamic Stochastic General Equilibrium (DSGE) models, expected utility models, rational expectations models, decision-making under risk and uncertainty (Prospect Theory), stochastic models, Knightian uncertainty models, maximum entropy models, Bayesian models, and agent-based models (ABM) with random decisions, among others. The model presented in this article can integrate stochastic elements and decision-making, although this is not our primary objective.

In this paper, an application of the solution to the so-called sixth problem of David Hilbert is proposed; its statement, very briefly, is: "Axiomatize those parts of physics where mathematics is already successfully applied, in particular the theory of gases and mechanics [30]." In other words, Hilbert wanted the fundamental physical laws (such as those of gases or motion) to be derived from a clear set of mathematical axioms, just as is done in geometry or algebra. What does it mean to "axiomatize" something? Axiomatizing is like building a structure starting from logical foundations. For example: In geometry, Euclid's axioms allow us to deduce theorems. In physics, Hilbert wanted to find basic axioms from which mathematical laws, such as those of ideal gases or equations of motion, could be derived.

Sample: Density constant = Variation velocity constant = In 1 dimension: Variation in one step, is equal –variation in other step (extend this concept to any dimension): These axioms, create the concept Divergence of Velocity field = 0. What parts of physics are involved? The problem specifically focuses on two levels of the study of matter:

- **Microscopy (statistical mechanics):** How individual gas particles (atoms, molecules) behave, governed by Newton's laws.
- **Macroscopy (thermodynamics or kinetic theory of gases):** How gases behave as a whole, using variables like pressure, volume, and temperature.

The central question is: Can we derive macroscopic behavior (like the ideal gas law) from the laws of motion of individual particles? This connects with the idea that the large world (macro) should, in principle, emerge from the small world (micro).

- Why is it important? Because it attempts to do something fundamental and profound:
- Unite physics and mathematics into a single logical body.
- Show how the macroscopic world arises from microscopic rules.
- Provide a solid foundation for disciplines such as statistical mechanics, thermodynamics, or even quantum field theory.

Moreover, this Problem Anticipated Major Developments in 20th-Century Science, Such as

- The Boltzmann equation.
- Quantum mechanics and its relationship with statistics.
- Complex systems and chaos theory.

This work does not aim to include a mathematical proof of Hilbert's problem; it seeks, perhaps, to be a conceptual proof that the problem has a solution. In this way, through logical reasoning, a series of dynamic conditions are established to model the dynamics of time series jointly, and the economic dynamics in particular.

Prediction

We all know what the concept of prediction means, or at least, we think we do. In my professional experience in motorsport engineering, the use of "Lap Times" is widespread. This software takes the complete vehicle and circuit parameters and is able to calculate the minimum time the car will achieve on a lap. However, race engineers don't directly use the time data: What we are really interested in is knowing whether a particular change in the car setup (such as an adjustment to the suspension or engine curves, for example) is improving the lap time or not. We use this software to determine whether a change is beneficial. In other words, we work with the results from the "Lap Time" software from a qualitative rather than a quantitative perspective. While it is also important to know whether we are improving or worsening, whether significantly or slightly, this aspect can even be considered secondary.

In trading or investing, what really matters is knowing how to make the right decisions. That is, understanding whether an action or decision will be successful or not; this is the true concept of prediction that any economist or investor should consider. Throughout this work, several coefficients will be outlined that define and quantify the probability of making the right or wrong decision, no matter what. This analysis helps assess the suitability of any decision, based on all the time series considered, providing a key tool for achieving this. The concepts of prediction and decision should be equivalent.

Obviously: If the number of series analyzed together is large, the uncertainty in the prediction will be lower; this is precisely the definition of an increase in the precision of the interpolation concept.

Choice and Initial Treatment of the Time Series to Be Analyze

Introduction

What is the reason for the word 'initial'? Also, throughout the entire procedure, it is possible to appropriately choose and, based on the results and calculations obtained the events to be modeled. This work tries to solve the general problem of modeling and predicting a group of "n" events or economic assets, expressed through a group of "n" time series. There are 2 steps to be followed initially:

Choice:

The optimal choice of series is essential for good modeling and therefore for a prediction with the least uncertainty; for this it is necessary to work with the greatest amount of information; as we progress in this article, more and more selection criteria will be defined.

A time series that quantifies an event is a "2D" section, planar or not, of the universal phase space (composed of all existing and not yet existing events) in "n" dimensions of the event; these sections of the universal space can represent values specific to the event analyzed, but also others, so that, in a certain way, the values of the section can be contaminated by other series (even randomness can be a contamination); this can make the modeling is not simple and therefore, increase the uncertainty in its prediction. If this is found to be the case, it is advisable to change events or choose others that compensate for this lack of information. Therefore, the choice of an event may even entail the creation of a new event suitable for analysis: An event does not exist, until it is measured; in other words: There are events or indexes that may be appropriate for the analysis, but they correspond to "other" economic structures that have not yet been created. It is also possible to wonder about the interactions that trajectories describing attractors or repellers have with each other, in analogy to the world of fluids (Coanda effect, Magnus effect, etc). Therefore, the choice of those series suitable for our analysis can take place at any time, i.e.: When defining or proposing the problem to be solved, as we model or solve the problem or when we have classification criteria.

Treatment

We can Define an Action Protocol

- Try to eliminate or mitigate the stochastic signal or noise; this affects the variations of the time series, but "does not form" part of it. One of the ways to do this is through the so-called "Fast Fourier Transform" or "FFT"; thus, it is possible to detect the noise and eliminate it. If you are not sure if you can reduce it, it is better to do nothing.
- The application of the "FFT" also aims to detect signals / harmonics that are identifiable; in other words: Simplify the time series by eliminating the harmonics or events that are known, in order to clean the signal of contaminating events: Work with the purest possible signal. The main reason is that it will be much easier to calculate the viscosity, as we will see later. To perform this one-dimensional analysis, it is convenient not to choose indexes that depend on combinations between other values, since the "FFT" analysis will not work in an ideal way.
- **Missing Data:** They are due to poor quality of the measurement; in the case of missing data, it is recommended to average between the previous and next value. In other cases, try to replace the series.
- **Non-Typical or Extreme Data:** These are data generated by a bad measurement; they are usually extremely large peaks that "clearly" do not form part of the series; they are not instabilities per se; in this case, they are eliminated by replacing them with the mean between the previous and next value. In other cases, try to replace the series.
- **Choosing The Right Data:** It has to do with entropy which is defined below; it is a matter of reducing the number of values in the series in order to smooth it, preserving or perhaps increasing its information; for this, some subgroups of values are chosen (alternatively, 1 yes and 2 not, etc) calculating the entropy of the original series and

that of each subgroup; the ideal subgroup will be the one corresponding to the first minimum of the entropy curve. This is an exceptional method for the analysis of a single series, but not for several, since all series must have the same number of values.

- Normalize the series between "0" and "1" (divide by the maximum of the values (value /max) will be divided by the maximum of the absolute values of the series, in the case of series with negative values; they can also be shifted towards the positive part, to have all positive values) ("N1"). This will be done whenever you want to compare series with each other. If you work on a single event with no claim to compare it, it is possible to apply a complete normalization between "0" and "1" ((min-value)/(max-min)) ("N2"). Unless otherwise stated, we will work with the "N1" normalization. There will be cases in which it may be more appropriate to apply "N2":
- Very "constant or flat" series, without major alterations or variation; it is necessary to "highlight" the changes.
- To better visualize the variations, in the application of the "FFT" with the goal of detecting harmonics in the signal.
- Separation into sections. This number depends on the number of elements in the series.

This is discussed further later in this report.

Important

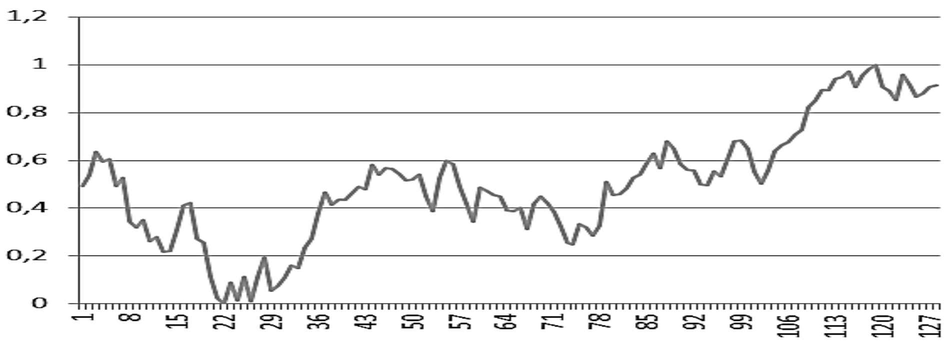
Using the "FFT" in this point, it is possible to do two things:

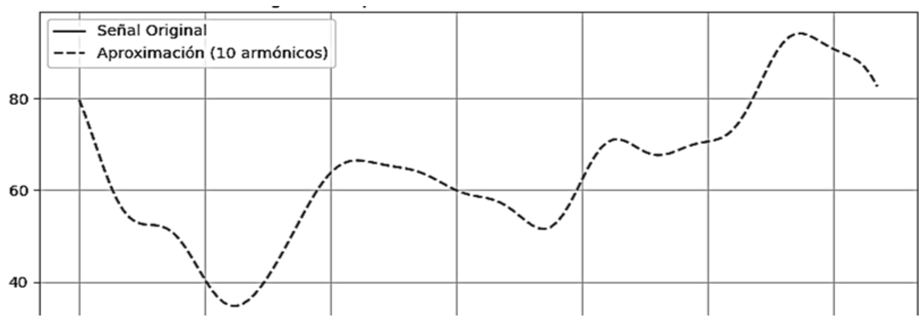
- Smooth the series (It is necessary to analyze the possibility of smoothing the series, and to compare modeling and prediction without smoothing them).
- Improving the joint analysis by incorporating other series (Extracting the "similar" harmonics and treating them as new series).
- It is also possible to predict the series: If we are able to discern sinusoidal functions in a curve, we can extrapolate them and, so, predict them. The problem with this method, as will be seen later, is that the oscillations or patterns will repeat in the future, since it does not work for nonlinear instabilities.

Let's look at Each of them

Smoothing and Simplification

If we extract the harmonics from the signal, we can choose some of them to smooth the initial signal. To do this, we select some of the most important harmonics:





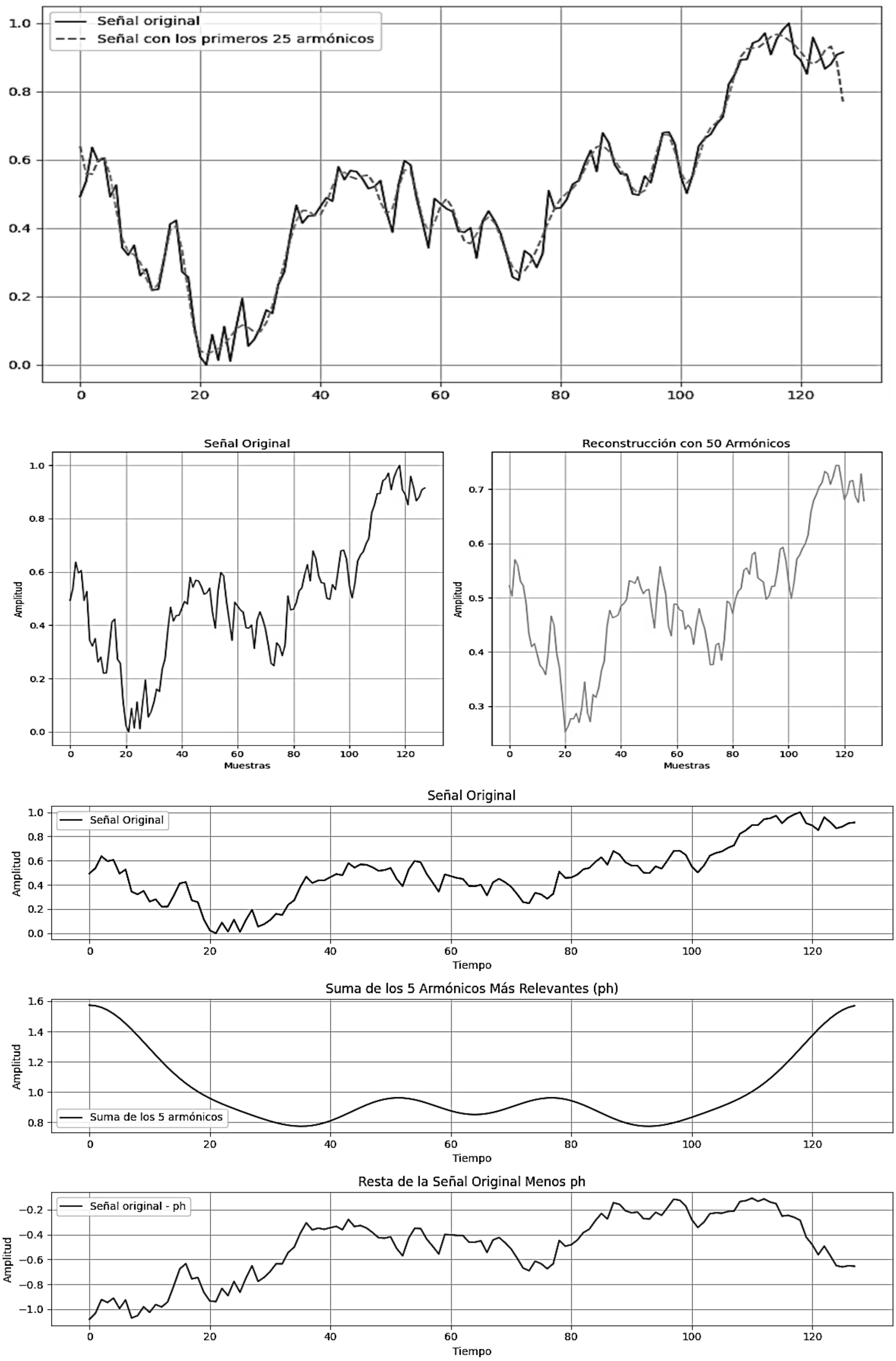


Figure 2: Example of Smoothing A Time Series, with Fourier

Analysis Optimization

Calculating the FFT and extracting the harmonics from the initial time series is equivalent to understanding the reasons behind the dynamics of the series. It is possible that the stochastic variations of the initial series are associated with variations of series (harmonics) that can be identified and understood. If this is the case, they can be removed, knowing which events they correspond to; that is: Removing the high frequency harmonics (chaos signal). Given a signal, we extract the most important harmonics (perhaps the first 5, for example, but it all depends on the analyzed series); these are low-frequency and high-amplitude signals that condition the signal and determine its overall or general dynamics;

they are like a quantification of the general trend.

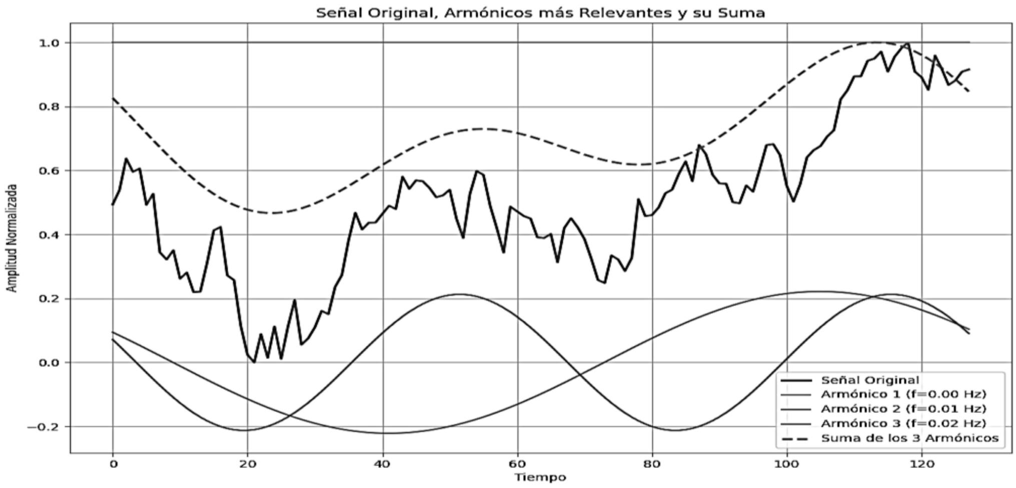


Figure 3

Representation of the 2 most relevant harmonics of the initial signal. The curves are shifted (up / down) to allow for clearer observation and comparison.

The importance of these harmonics is that they can repeat in other events, so it is crucial to detect them. If identified, they can be removed from each event and the harmonic can be treated as a new event, incorporating it into the joint analysis. If the harmonic repeats in several events, it indicates that it influences those events, implying a dependency between all of them, including the new event considered.

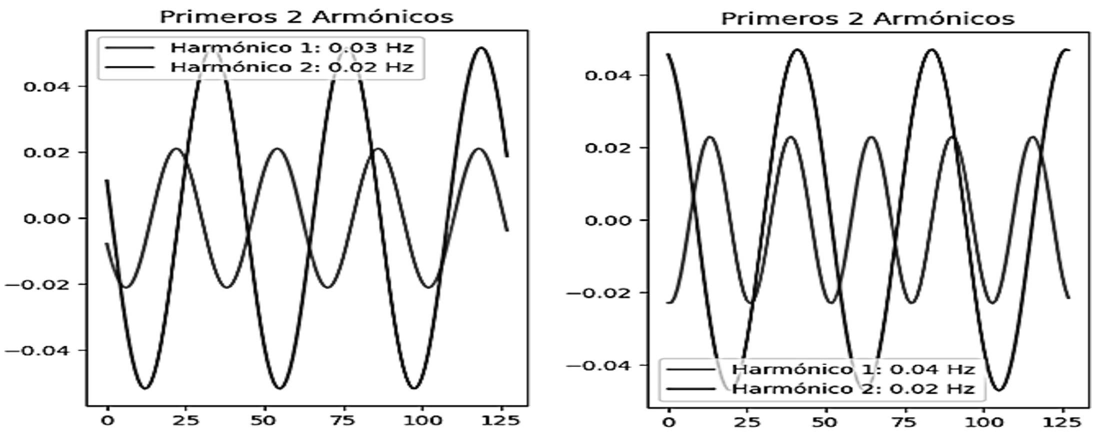


Figure 4: Harmonic Detection (0.02 Hz) in 2 Events

Conclusions

We can use this method to:

- Simplify or smooth the initial time series.
- Improve the analysis.
- Calculate the relative viscosities (we will see this special concept).
- Identify existing events that intervene in the analysis (for this, it is necessary to apply this procedure to all series in the analyzed event set), and if it is not possible, it may mean that it is necessary to "create" the measured and located event.

The process of "creating" events that, on the one hand, simplify the generation of the mathematical model and, on the other, assist in solving the problem, thereby improving its reliability and the accuracy of predictions, is very important. In order to create or generate these events, we understand the influence each one has within the set of events in the problem. To do this, it is necessary to apply the previously mentioned approach, using the "FFT" on each time series and calculating its harmonics. If the same harmonic is detected across all these analyses, we can conclude that there is an event (harmonic) that influences all the events. With all this knowledge of the influences, we can then consider how to generate this event.

"Waves" Model

Many economists have attempted to simulate the dynamics of the global economy based on periodicities or economic cycles, as we have already seen. However, combining all those cycles does not perfectly define the dynamics, as there are other cycles not included in this (usually summed) analysis; this is the fundamental reason why the method does not work optimally. To solve this problem, we have chosen an approach different from those used to date, but which can also be incorporated or added.

The process is as Follows

- A - Quantify the cycles mentioned above (Kondratieff, Juglar, etc.).
- B - Analyze the "n" series we will work with using Fourier (FFT). The most significant harmonics are calculated (those best defined or with greater amplitude).
- Compare the harmonics calculated in A with those in B. If any similarity between harmonics is detected, the harmonics from B are selected. This makes the cycles more realistic. Sum all detected periodicities, by analogy or otherwise. This sum will reflect the global dynamics of the economy by working with the "n" events.

Then, we will obtain an expression as a sum of sinus, with specific amplitudes and phase shifts for each term (dynamic curve for "n" events"):

$$f(t) = \sum_{i=1}^{n} A_i \sin\left(2\pi f t + \Phi_i\right) \tag{3}$$

Absolute Ranking Criteria: Choice of Time Series: Amount of Information From 1 Time Series

Introduction

It is of vital important to have criteria for classifying events and between events (absolute and relative). This undoubtedly makes it possible to choose those time series that provide the most information to the analysis and, therefore, to predict them with the least uncertainty. To begin with, we will analyze criteria related to the importance of a time series.

Entropy

A time series is a sequence of measured observations of a variable over time; being able to know the amount of "information" it possesses is absolutely essential: With this data, it is possible, among other things, to classify series or even to know whether they should be used or not; for this, we will use the so-called Shannon entropy. This entropy, also called information entropy, is a mathematical measure of the uncertainty associated with a source of information. In other words, it tells us how predictable the content of a source is. The higher the entropy, the greater the uncertainty and the less information it possesses; it is like measuring the degree of "surprise" of a signal: The more surprise a variation of a signal provides, the more information it will be contributing to the analysis.

This Entropy is analogous to the concept of Volatility, so widespread in economics (Volatility, just like the other definitions analyzed in this paper, can be used as a classification criterion for economic events; even as a characteristic, inherent, and differentiated curve for the event itself. It can be used as a curve (as all definitions and more), but also by extracting the maximum value, the minimum value, the average, the discrete integral or the sum, etc..). Let xi / i=1,...,n values of a time series "X"; "pi" is the probability of occurrence of "xi"; thus, the Shannon entropy "H" of "X" is defined. To eliminate the influence on the Entropy of the variable "number or quantity of values" of "X", or "length" of "X" and thus be able to classify events, "H" is divided by the length "n" = long(X):

$$H(X) = \frac{\sum_{i=1}^{n} p_i \log_2\left(\frac{1}{p_i}\right)}{n} \tag{4}$$

Therefore, the optimal choice of time series to be analyzed should also be based on a "quantity of information" criterion. That is, the time series that provide the most information to the study will be selected (later, the criterion of greater or lesser dependency will be analyzed to determine whether a series should be chosen). Hence, those with fewer variations (non-constant) will be chosen. It is necessary to smooth out the random segments as they do not provide information and, moreover, obscure information.

Group "E1" and "E" of Time Series

Let "E1" be the group of events formed by the Ibex-35 (Spanish Index), the price of Gold and Banco de Santander; composed of 200 data each, with daily frequency and ranging from May 13, 2019 to May 11, 2020 (data from Yahoo Finances).

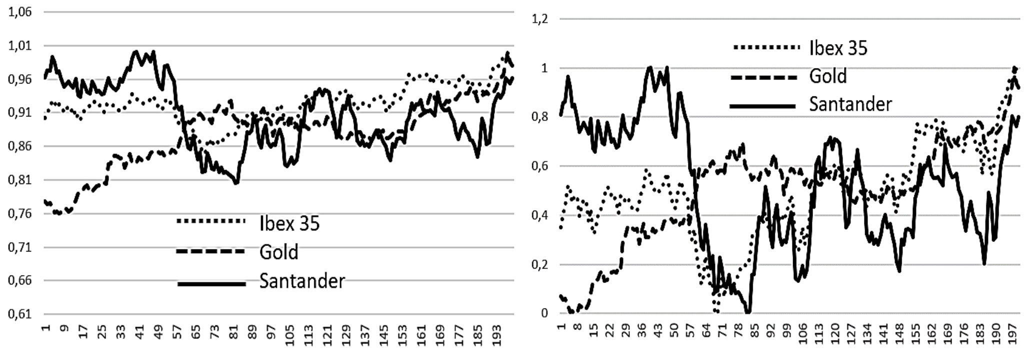


Figure 5

Graphs of the 3 events of group "E1", with normalization "N1" and "N2".
Let there be another group of 3 events "E2" formed by the Price of a barrel of Brent oil, Dow Jones Index and Nasdaq Index; composed of 200 data each, with daily frequency and ranging from February 9, 2023 to November 24, 2023.

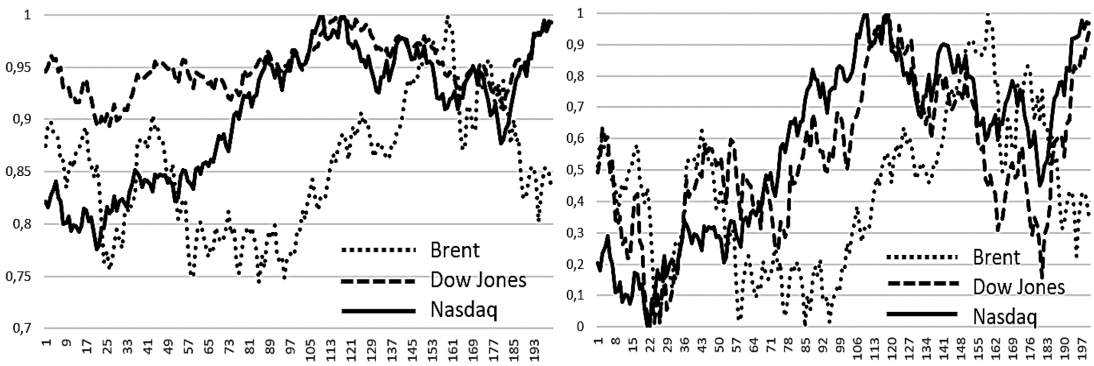


Figure 6

Graphs of the 3 events of group "E2", with normalization "N1" and "N2".

Examples: Calculation of Entropies
Global Entropy
Events studied: Group "E1" and "E2":

Table 1
Global "H" entropies for "E1" and "E2"
Global Entropies (H) - E1

Global Entropies (H) - E1	
IBEX-35 (Spain Index)	0.0621
Gold Price	0.0560
Santander Bank	0.0464

Global Entropies (H) - E2	
Dow Jones	0.0668
Brent	0.0435
Nasdaq	0.0467

Entropy by Sections

Entropy curve in each of the sections, containing 50 values, of group "E1" and "E2":

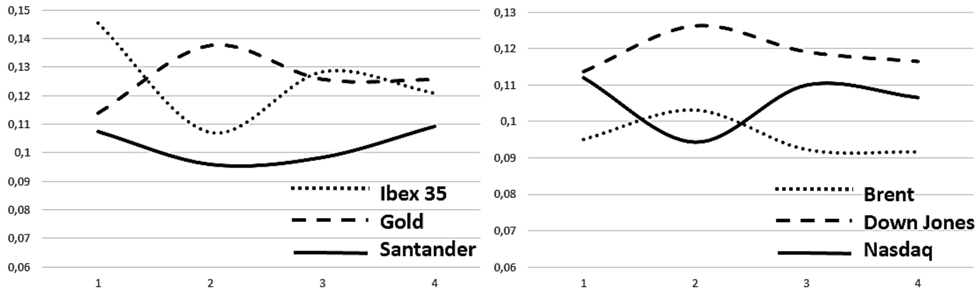


Figure 7: Entropy Curves for "E1" and "E2"

It is possible to calculate on the curves generated by sections, different indexes: Maximum, minimum, average, integral (discrete) of the curve, etc. These indexes can also be considered as classification and selection criteria.

Important: It seems that they vary at the same moment, harmoniously... It is possible that the state of the global economy affects all events to a greater or lesser extent. The magnitude of entropy is, therefore, an inherent property of the event being analyzed; for this reason, it can become an important classification criterion.

FFT

It is possible to apply the "FFT" to a series "X", in such a way that the more "relevant" harmonics detected, the more information it will have; the term "relevant" refers to the fact that the harmonics considered must have a magnitude, intensity or energy greater than a certain "IX". The magnitude "Mag" of a harmonic is as important as its own existence (with a greater quantity and magnitude of harmonics, greater dependency); therefore, to calculate the information provided by the series from its relevant harmonics, it is necessary to multiply the sum of the magnitudes by the number of relevant harmonics "Na" ("A_i" is the harmonic "i"; Mag(A_i) is the magnitude of the harmonic "A_i"); in this way, "AV" is obtained for event "X":

$$AV(X) = \frac{Na}{n} \sum_{i=1}^{Na} Mag\left(A_i\right) \qquad (5)$$

If the "FFT" is applied in sections, harmonics can be observed that otherwise would not be possible; but it is complicated by the "scarcity" of values (0's are added at the end to reach the necessary amount). The fact of detecting harmonics such as seasonalities and others, may imply that such harmonics are inherent to the analyzed event itself; that is: Regardless of when the event is analyzed again, those harmonics will be there, although they may not be detectable.

Example

Calculation of Global "AV" Using the "FFT"

Remember that in order to apply the FFT, the series must have a certain number of elements multiples of 2 (..., 64, 128, 256, 512, etc). As an illustrative example, the 3 time series of group "E1" are analyzed, obtaining the following plots of the harmonics (intensity versus frequency):

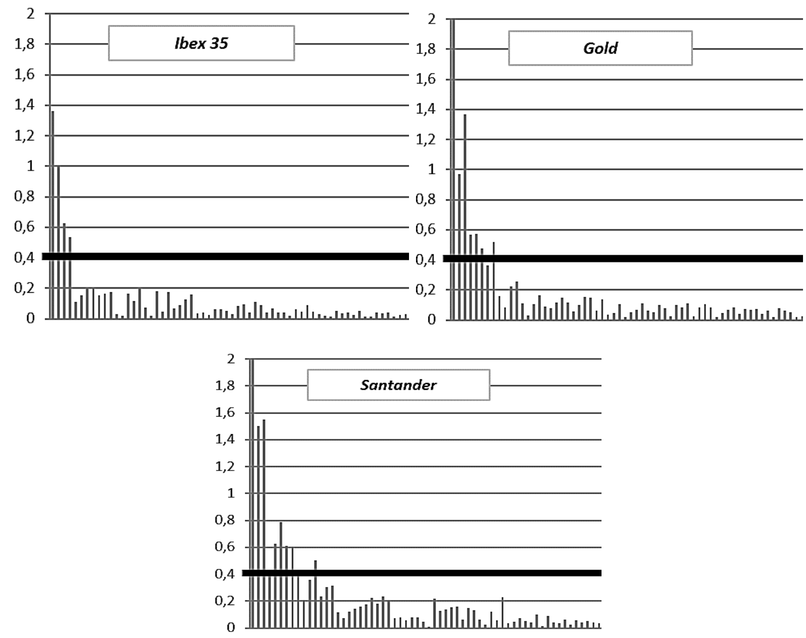


Figure 8: Magnitude of Each harmonic Detected, Applying the FFT

The limit "0.4" (horizontal black line) has been chosen as the relevant harmonic selection criterion, but it is necessary to analyze other possibilities (procedures or values). The

"AV" values are:

Table 2
"AV" global entropies

"AV" global entropies

	AV
IBEX-35	0.1099
Gold Price	0.45446
Santander	0.87726

Groups could be formed with fewer elements than the total number; for example, in a group of 256 elements, two groups of 128 elements each, or in a group of 192 elements, two groups of 128 and 64 elements, respectively. However, the information being worked on would be reduced.

Spectral Entropy

It tries to evaluate if the "FFT" analysis performed on an event "X", has more or less information: For this, the entropy of the frequency spectrum obtained by the FFT is calculated. A higher spectral entropy would indicate a more uniform distribution of energy in the frequencies, which "could be interpreted" as more information, contrary to what is known as entropy. In simpler terms, spectral entropy provides a measure of the complexity of a time series from a frequency perspective, indicating how much energy is in different frequency bands and quantifying the richness of the series in terms of spectral information. This measure is therefore useful to evaluate whether the frequency analysis performed by "FFT" has significant information or not. This value is called "EV(X)".

Example

Calculation Spectral Entropy

We worked on the 3 events of the "E1" group:

Table 3

EV Values	
IBEX-35	0.0404
Gold Price	0.0548
Santander	0.0473

Total Mass – 1 (m1)
We define "m1" as the classification criterion; let "X" be the event:

$$m1(X) = \frac{A(X) * EV(X)}{H(X)} \quad (6)$$

Thus, we obtain a "global" classificatory value based on the amount of information which we will call "m1" or mass of the event "X": The larger it is, the more information the event will have.

Example: Calculation "m1" Global

We worked on the 3 events of the "E1" group:

Table 4

Values m1	
IBEX-35	0.1689
Gold Price	0.4644
Santander	0.8605

Extraction and Parametrization of Information From 1 Time Series

Introduction

A series of definitions are necessary, which are developed from the kinetic theory of perfect gases and its similarities with wave dynamics [31]. All these definitions are essential to explain the transport and diffusion of information and the possible transformation into temperature or velocity (Kolmogorov scales)[32]. In turn, they can be used as classification criteria. The fact of being able to extract the information and use it, is equivalent to increase the scientific quality of the analysis and therefore, to model adequately reducing the uncertainty in the prediction.

Nodes

First, the time series "X" is separated into "mt" sections of equal length (T1, T2,…, Tm); if this cannot be done, values are eliminated. The number of sections must not be less than 8 and the number of values in each section must not be less than 25. Let "Xs" be a subset of the time series "X"; for k>0; "xi" is defined as the "i-th" node of "X" or the "i-th" element of "Xs", if $|x_i x_{i-1}| > k$. A value of "k" is taken through 4 procedures:

- Graphically; that is: The representation of the nodes of the series is equivalent to a smoothing of the initial series, since the nodes reflect a certain variation in the values. Therefore, such smoothing should reflect the main or remarkable variations of the original series and be based on them, to generate the curve composed of nodes. That is: The nodes are the values in the time series with the most 1/information.
- For each of the "k" the density curve is calculated (to be defined later) as a function of various "mt": mt = 25, 20, 10. The average of all the densities (we will see) is calculated to obtain the final density curve. The fact of averaging all the densities calculated from "k" and "mt" is equivalent to give importance to the much accentuated variations ("k" large) and also to the smooth ones ("k" small); this allows to have a very good idea in terms of density, of the dynamics of the series.
- The selection of nodes is incredibly important; therefore, a protocol is needed that minimizes dependence on the scientist's subjectivity, or ideally eliminates it altogether. To achieve this, we can select the nodes of the analyzed series as the subset of points in the series that, being nodes, contribute the maximum possible entropy.

Example: Original Against Nodes

We work with Brent, of "E2" group of events:

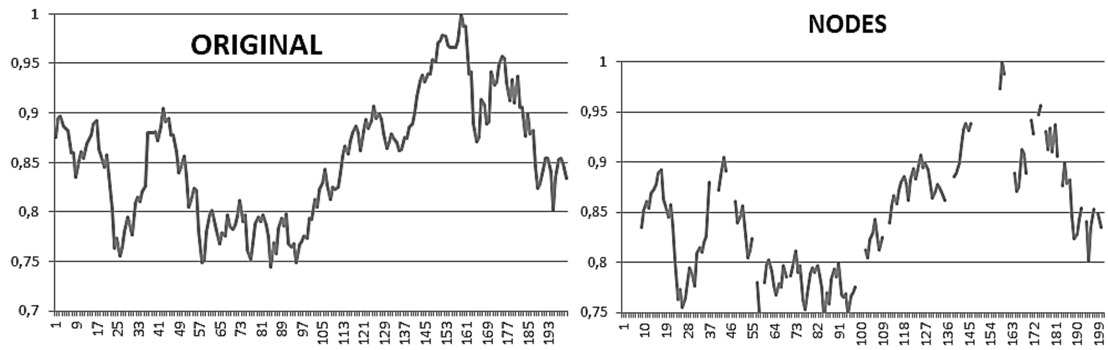


Figure 9

Graphical comparison between original data's from temporal series and Nodes: The nodes are, the points more important.

Density

Let "NTi" be the number of nodes in the section "Ti"; the density in each span is defined as:

$$\rho T_i = \frac{NT_i}{\max(NT_i)} \qquad (7)$$

Velocity

Firstly, "$x_i T_j$" is defined as the node "i" in the section "T_j"; velocity in the section "Ti" is defined as "VT_i"; "n" is the quantity of nodes in "T_j":

$$VT_j = \rho T_j * sign\left(\sum_{i=1}^{n}\left(x_i T_j - x_{i-1} T_j\right)\right) * \frac{\sum_{i=1}^{n}\left|x_i T_j - x_{i-1} T_j\right|}{n} \qquad (8)$$

This definition corresponds to the analogous concept of velocity as the product of frequency and wave amplitude.

Volatility

It is one of the most widely used definitions of a time series in economics in general; it could be defined as a curve that indicates the variability of the series to which it is applied. For its calculation, the logarithm of the quotient between a value and its previous one is first determined; the volatility at a value "P", corresponds to the standard deviation of the 25 values previous to "P". We denote this definition as Volatility "A". Let's look at volatility in another way: The volatility of a "T" leg of the time series, is the sum of the absolute values of the differences between a value and its previous one, interval by interval (The number of intervals into which we divide the time series can be arbitrary; if the number is large, the resolution will be higher). We denote Volatility "B" to this new definition ("n" number of elements of "T"):

$$x_i \in T$$

$$Volatility"B"(T) = \sum_{i=1}^{n} \left| x_{i-1}T - x_i T \right| \quad (9)$$

Volatility "B" is a smoothing of Volatility "A". The mathematical relationship between both definitions of volatility is evident (Volatility "B" is included in Volatility "A").

In general, it has been observed that there is a positive relationship between volatility and entropy, meaning that more volatile time series tend to have higher entropy. This is because entropy is a measure of the information (within order) or uncertainty in a system, and volatile time series are more unpredictable and therefore with less information. In economics, volatility is used to describe the fluctuation of financial asset prices. It has been shown that the volatility of financial asset prices can be related to information entropy. The rate of change of financial asset prices can also be related to volatility. Therefore, the definitions made here on density with velocity are analogous to the definition of volatility, since they work on the variations of values by weighting them in importance from the nodes.

Important

It´s possible so, to define velocity as a product between volatility "B" by density. It´s a type of Volatility. In fact, to select the nodes of a time series, in addition to the criterion already described above, the volatility curve must be as similar as possible to the velocity curve.

Example: Calculation Volatility "A" and "B"

We calculate both volatilities for the events in group "E2":

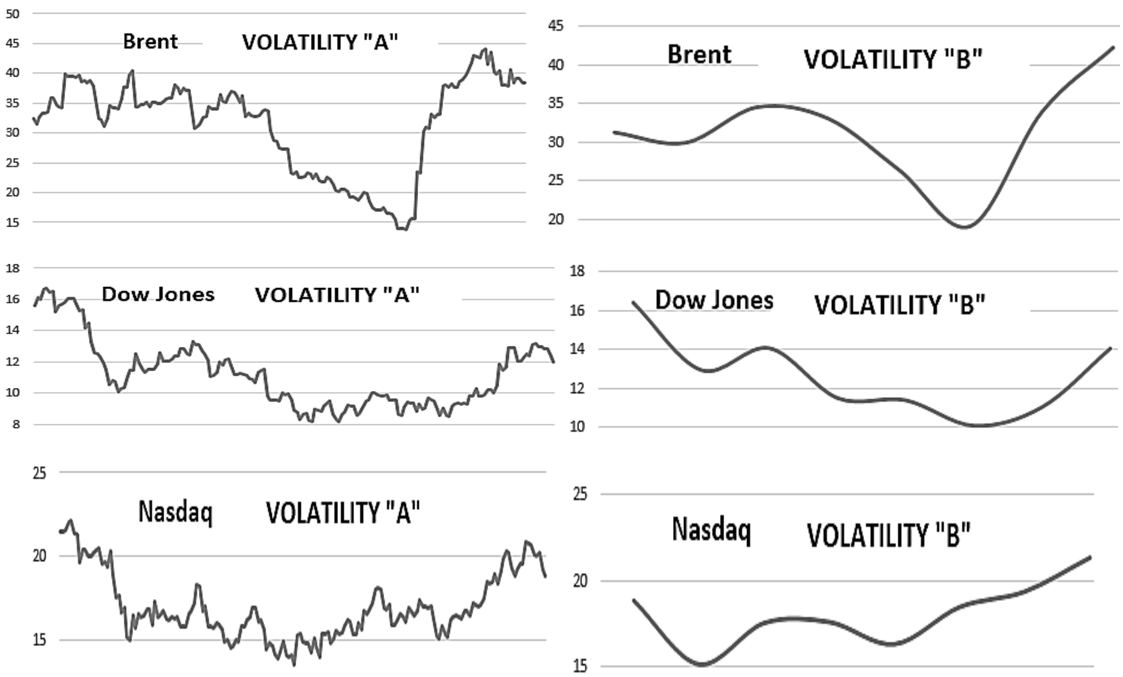


Figure 10

The excellent correlation (even in values) between "A" and "B" volatilities of both columns is perfectly visible so evident.

Generation "nodes": other procedure: volatility "c"

To ensure a valid and accurate node distribution, it is essential that the volatility function closely approximates the behavior of the velocity function. As previously demonstrated, both quantities are analytically defined in terms of variations, rendering their expressions nearly identical. The only component missing from the volatility expression is a term that captures the direction (positive or negative) of the variation. This can be addressed by incorporating a sign factor, as is done in the definition of velocity. The resulting formulation is referred to as Volatility "C". It is therefore necessary to evaluate the feasibility of using Volatility "C" as a surrogate for velocity, either including or excluding the density term.

Example

Velocity Against Average Serie We worked with the Brent event and the Dow Jones from the "E2" group; we calculated, on one hand, the curve of the points as averages of each segment and the velocity curve; we observed the similarity between these curves and the volatility curve, both "A" and "B".

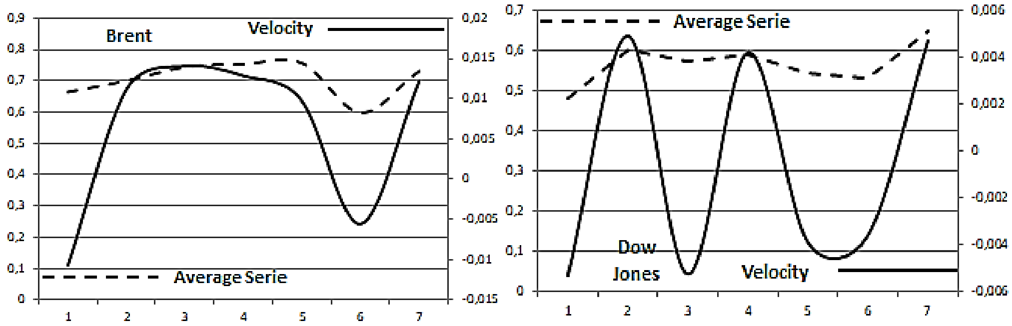


Figure 11

Curves of the average of each segment and velocity.

Pressure and Temperature

Let "Vol" be a volume of particles in 3D, "ρ" be the density in that volume; from the kinetic theory of gases, "P" is proportional to: "m" as total mass of the particles, "1/Vol", "u" average velocity of the particles, impulse (momentum of movement m*u) and "N" number of particles; thus: (we denote by "►", the dependence as a function) [31]:

$$P \triangleright \frac{mN u^2}{Vol} = \rho * u^2 * N \triangleright \rho u^2 \tag{10}$$

If in the Navier Stokes equation, with zero viscosity and in the absence of forces outside the system, the "P" is replaced by this expression, it follows that the proportionality factor must be "1/2" [33]:

$$P = \frac{1}{2}\rho u^2 \tag{11}$$

"P" is also called in astrophysics "Ram Pressure" with "c" velocity of light, "m" mass, "E" energy; the following expression establishes the analogy between pressure and energy:

$$E = m c^2 = \frac{m}{Vol} c^2 Vol \triangleright \rho * Vol \quad \rightarrow \quad E \triangleright P * Vol \tag{12}$$

Therefore, the larger the volume with the same density or pressure, the higher the Energy will be: "It is a measure of the existing and available energy in the volume". In fact, the same relation can be found, again in the kinetic theory of gases, which says ("K" Boltzmann's constant, "T" temperature, "N" number of particles) →E=(3/2)NKT; in terms of that postulate, one can calculate the pressure for 1 mole of molecules in a volume "Vol". Nomenclature: "N_A" the Avogadro number, "M" the molecular mass, "R_g" is the universal constant of gases (in this case it depends on the velocity "u", the density "ρ", the pressure "P", etc), "u" the average of the velocities of the molecules and "T" the temperature ("K" constant of proportionality): The higher the average velocity of the molecules, the hotter the fluid will be; on the other hand, heat transfer, always tends towards the lower temperature zone; therefore, this increase in velocity implies a greater facility to transmit heat:

$$P * Vol \triangleright m u^2 N_A \triangleright Rg * T \quad \rightarrow \quad T \triangleright M * u^2 \triangleright u^2 \tag{13}$$

The heat equation contains the so-called Laplacian "Δ" which is the second order derivative ("t" the time, "x" the spatial coordinate, "k" is the heat transfer coefficient or conductivity). Discretizing the heat equation, we obtain:

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} = k\Delta T \quad \rightarrow \quad T_x^{n+1} = T_x^n - k\frac{\Delta t}{\Delta x^2}\left(T_{x+1}^n - 2T_x^n + T_{x-1}^n\right) \tag{14}$$

This means that the rate of exchange over time is given by the average rate of change in space (turmoil is therefore, a very important factor). If ∂T/∂t is the rate of change of temperature, then it can be seen that there is more heat exchange in regions where the temperature is highly variable and less heat exchange when the temperature varies slightly. Note that viscosity is transformed into temperature, due to the Kolmogorov scales; in other words: It is transformed into velocity. Greater turbulence therefore implies greater and more efficient temperature transfer; this temperature can be considered information, so a high variation (volatility) can imply a greater and efficient transfer of information; this is something tremendously important, since it makes turbulence a powerful tool not only for the economist, but also for the sociologist, psychologist, etc.

(15) $$P \triangleright \rho T \rightarrow P = K\rho T \quad \rightarrow \quad T \triangleright \frac{P}{\rho}\frac{1}{K} \triangleright \frac{P}{\rho}$$

Therefore:

(16) $$E = \frac{3}{2}NKT \triangleright \frac{3}{2}P * Vol$$

The pressure "P" and the velocity "u" are inversely proportional, therefore; moreover: To maintain the relationship between pressure and velocity the "constant" or function of proportionality is precisely the density; if the velocity is large, it means that the pressure is small; this drop (difference) of pressure produces a suction (higher pressure towards lower pressure) on the particles around it. Let "Pe" be the quantity of motion or momentum named above: Pe=m*u; we work with only 1 dimension "x"; "m" constant, but "u" depends at each point; thus:

(17) $$\int Pe\, dx \cong \frac{1}{2} m u^2 = E$$

Pressure is a measure of energy, as is the quantity of motion (=mu); in this case, an attempt is made to "normalize" the measure of mass, transforming it into mass per unit volume "Vol": Density:

(18) $$P = \frac{Pe}{m} \rho u = Pe \frac{u}{Vol}$$

The acceleration "a" produced by a pressure difference "P" over an event of density "ρ" is:

(19) $$a = \frac{\Delta P}{\rho} \qquad a = \frac{\Delta P}{\rho} = \frac{F}{m} = \frac{F}{Vol}\frac{Vol}{m} = \frac{F}{Vol} \qquad \Delta P = \frac{F}{Vol}$$

It is obtained again that Pressure is a type of Energy; in fact, in the absence of external forces to the system and if there is no pressure difference, the particle does not vary its acceleration; in other words: "P" is the energy used by the particle to move. The pressure in the section "T_i" → "PT_i", is defined as:

(20) $$PT_{i-1} - PT_i = \\ = \rho T_i * VT_i^2 * (-sign(VT_{i-1} - VT_i))$$

The pressure variation, as thus defined, is negative when the velocity variation is positive and vice versa, since it must fulfill this same relationship that is seen when applying the Navier Stokes equations; it is a property that fulfills the dynamics of any fluid. The relationship between the rate of change of temperature and entropy (or the transmission of information) is related to the concept of thermodynamics and information theory. Temperature and entropy are linked in several ways:

- **Entropy and Disorder:** Entropy is a measure of disorder or lack of information in a system. If the temperature increases, thermal energy is distributed more chaotically among the particles in the system. This leads to an increase in entropy, as there are more possible ways in which particles can be distributed in a high-temperature state. Therefore, higher temperatures are generally associated with higher entropy.
- **Phase Changes:** When a phase change occurs, such as melting or vaporization, the temperature remains constant during the phase transition. During this process, entropy increases considerably. For example, when ice melts at a constant temperature, the entropy of water increases significantly as water molecules move from an ordered structure in ice to a more chaotic structure in liquid water.
- **Information Theory:** In the context of information theory, entropy is used to measure uncertainty or the amount of information in a system. A highly ordered system has low entropy, while a highly disordered system has high entropy. The relationship between temperature and entropy is reflected in the way temperature affects the uncertainty and the amount of information in a system. As temperature increases, entropy and uncertainty generally increase, which can affect the transmission of information in physical systems and in communication systems.

In fact, if one considers the rate of transmission of 'information' to be 'V', one has the rate of transmission of heat; so:

(21) $$Velocity(heat) \triangleright \sqrt{T}$$

(22) $$Velocity(heat) \triangleright \sqrt{T} \triangleright V$$

The Pressure in the Economy as a Concept, it´s Very Important

The analogy between pressure in a fluid and the economy can be useful for understanding how certain factors influence economic stability. In fluid dynamics, when pressure drops, turbulence tends to form more easily because the fluid particles have more freedom to move chaotically. Conversely, as pressure increases, the flow tends to be more stable and laminar, which reduces turbulence. A similar situation can occur in the economy, with factors such as fiscal and monetary policy playing key roles. When a government "increases pressure" on the economy through measures like increasing public spending, controlling inflation, or raising interest rates, it may aim to reduce economic turbulence (such as crises or extreme fluctuations) and foster a more stable economy. Here, "pressure" refers to the policies that stabilize the economy.

Example: The 2008 Financial Crisis and Government Measures

When the 2008 financial crisis erupted, many global economies faced a collapse in confidence and severe turbulence, such as the collapse of banks and stock markets. To mitigate these turbulences, governments, primarily through their central banks, took steps to increase the "pressure" on the economy. For example:

- **Expansionary Monetary Policy:** Central banks lowered interest rates to encourage credit and investment. This functioned similarly to increasing the "pressure" in the financial system to prevent a deep economic slowdown.
- **Fiscal Stimulus:** Governments implemented economic stimulus packages, such as increased public spending on infrastructure or subsidies, to inject money into the economy and promote economic activity.

In this case, fiscal and monetary "pressure" (public spending and low interest rates) worked like increasing the pressure in a fluid, stabilizing the economy and reducing the "turbulence" that would have occurred if these measures hadn't been implemented. Just as a fluid under higher pressure flows more steadily, the economy remained more stable due to these governmental interventions. However, it's important to note that, just like with fluids, excessive "pressure" can also be dangerous. If a government excessively increases public debt or makes unsustainable interventions, it could create other issues, such as inflation or economic bubbles. It is a delicate balance. In this context, "pressure" in the economy could be interpreted as the policies and measures governments implement to maintain economic stability and avoid "turbulence," such as financial crises, runaway inflation, or recessions. In the economy, greater government control or intervention (through fiscal, monetary, or regulatory policies) can help maintain stability by reducing the likelihood of crises or imbalances.

Example

A clear example of how governments increase "pressure" to avoid economic turbulence is the monetary policy of central banks. During periods of uncertainty or crisis, central banks typically raise "pressure" through measures like:

- **Raising Interest Rates:** When there is a risk of high inflation, central banks (such as the U.S. Federal Reserve or the European Central Bank) may increase interest rates to cool down the economy and reduce inflationary pressure. This acts as a "brake" to prevent the economy from overheating and causing turbulence.
- **Injecting Liquidity:** In crisis situations, such as the 2008 financial crisis or the COVID19 pandemic in 2020, central banks reduced interest rates and injected large amounts of money into the economy to maintain credit flow and avoid a deep recession. This is similar to increasing pressure in a system to keep it stable.
- **Financial Regulations:** After the 2008 crisis, many governments implemented stricter regulations in the financial sector (such as the Basel III agreement) to prevent banks from taking excessive risks that could lead to another crisis. These regulations act as a "pressure" that keeps the financial system more stable.
- Excessive (as always) pressure or intervention, can have negative effects, such as stifling economic growth or distorting markets. Therefore, balance is key.
- In economic terms, increasing pressure on the economy means implementing policies or measures that promote economic stability, reduce volatility, and control factors that could lead to turbulence or crises. Some ways to "increase pressure" on the economy include:
- **Restrictive Monetary Policy:** Central banks can raise interest rates. By doing so, credit becomes more expensive, which reduces consumption and investment, slowing down economic expansion and preventing overheating. This also helps control inflation and ensures that growth does not get out of hand.
- **Example:** During periods of high inflation, the central bank may raise interest rates to cool down the economy and prevent excessive price increases.
- **Reduction in Public Spending:** Governments can reduce public spending. By doing this, they decrease the amount of money circulating in the economy, which can reduce aggregate demand and lower inflation. It can also prevent fiscal deficits, helping to maintain long-term economic stability.
- **Example:** A government may reduce spending on social programs or infrastructure projects to control public spending and reduce debt.
- **Increase in Taxes:** Raising taxes (especially indirect taxes like VAT or corporate taxes) reduces the purchasing power of consumers and businesses, decreasing consumption and investment. This can help control inflation and reduce the fiscal deficit.
- **Example:** During times of high inflation or when there is a high level of public debt, governments may increase taxes to curb spending and control indebtedness.
- **Tightening Financial Regulation:** Increasing regulation in the financial sector can also be a way to "increase pressure." This can include measures such as requiring higher capital reserves for banks or implementing stricter

regulations to prevent excessive risk in financial markets.

- **Example:** After the 2008 financial crisis, many governments implemented stricter regulations to prevent banks from taking excessive risks, such as the Basel III agreement.
- **Control of the Money Supply:** Central banks can reduce the amount of money in circulation through policies like selling bonds or increasing bank reserves. This also serves as a measure to control inflation and prevent an oversupply of money that could create economic bubbles.
- **Example:** If the economy is experiencing very high inflation, the central bank may sell bonds in the markets to withdraw money from circulation and curb inflation.
- **Credit or Borrowing Restrictions:** Limiting access to credit or increasing the conditions for obtaining financing is another way to increase pressure. This reduces spending by consumers and businesses, as more expensive borrowing reduces investment and consumption.
- **Example:** In situations of economic overheating, central banks may impose credit restrictions, such as raising collateral requirements or setting loan limits.
- **Price Controls:** At times, the government may intervene directly in markets through price controls or wage controls to prevent prices from rising too quickly, which is a way to control inflation and maintain economic stability.

Example

During inflationary crises, a government may intervene in markets to control the prices of essential products such as food or fuel.

Therefore, increasing pressure on the economy is a way to intervene in the demand and supply of goods, services, and money with the goal of preventing imbalances, uncontrolled inflation, or economic bubbles. Restrictive fiscal and monetary policies are the main tools to achieve this goal, but they must always be applied with caution to avoid negative effects such as economic slowdown or recession.

Overall Measurements and by Tranches

The definitions made can be measured globally or as a whole; in other words: Applied over the full series obtaining a value, or by sections or intervals obtaining a curve and sometimes from the curve, also general values.

Turbulent Kinetic Energy

"TKE" is a measurement that represents the absolute or intrinsic energy of a particle. Let be a particle with (2D) velocity component (u´,v´) (the "check mark" above each component indicates that they are its standard deviations) and let be "N" be the number of samples (the bar above "u" indicates the mean):

(23) $$u^{'} = \sqrt{\frac{\sum_{i=1}^{N}\left(u_i - \bar{u}\right)}{N-1}} \rightarrow TKE \triangleright u^{'2} + v^{'2}$$

Such a proportionality function can be "m" and "ρ":

(24) $$TKE = m\left(u^{'2} + v^{'2}\right) \rightarrow TKE = \rho\left(u^{'2} + v^{'2}\right)$$

"TKE" can also be called generically, kinetic energy: Its expression even perfectly recalls the "traditional" expression for this energy ($0.5*m*v^2$).

Work

When a particle moves along 2 points "1" and "2", it does so using work; that is: A Force
"F" for each unit of length "d" ("K_1", "K_2" kinetic energies of points "1" and "2") and a distance "d" between both points (which we will define later as "space metric

(25) $$J = F*d = K_2 - K_1 \rightarrow F = \frac{K_2 - K_1}{d}$$

On the other hand, it is possible to obtain some important relations where the work defined here intervenes, knowing from a thermodynamic point of view, that: The variation of energy is proportional to the variation of mass and temperature:

(26) $$J = Fd \triangleright Pd \rightarrow \begin{matrix} P = \rho T \rightarrow J \triangleright \rho T d \\ J \triangleright energy * d \end{matrix}$$

Working with the "Action" (to be defined and analyzed later) and being "a" the acceleration and "d" the "Action": The work that a particle does to move between 2 positions, is:

$$J = F * d = m * a * d \quad (27)$$

This expression is very revealing, since it relates 2 types of Energy (Work and Action) with the importance and velocity of an event.

Power and Energy

There are 2 concepts applied to the information provided by the time series. Let "V" be the velocity, "Po" be the power, "f" the frequency, "x(t)" the parameterized expression of the time series itself with respect to time:

$$Po = \frac{V^2}{\mu} = \lim_{t \to \infty} \frac{\int_{-t/2}^{t/2} |x(t)|^2 dt}{t} \qquad E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df \quad (28)$$

The energy spectral density = S(f):

$$S(f) = |X(f)|^2 \quad (29)$$

Note

Detecting an instability or crisis is extremely important; a series of concepts to achieve this are defined below. Note: A turbulent state may be necessary to achieve certain objectives.

Similarity Number "S"

Introduction

We try to define a dimensionless number that reflects the state (laminar or turbulent) of the dynamics of a group of economic particles (nodes) in motion, analogous to the Reynolds number "Re" used in problems related to fluids. The Reynolds number "RE", is defined as follows; let "V" be the velocity, "ρ" the density, "μ" the viscosity and "A" the characteristic dimension; the objective is to extend this definition to economics, but in economic terms, it is necessary to find and analyze an equivalence with "A"; working for example in pedestrian dynamics, "A" can be considered the space that a person needs to move; thus:

$$\mathrm{Re} = \frac{\rho V A}{\mu} \quad (30)$$

"S" Number

Therefore, "S", applied to an economic time series, is defined as:

$$S = \frac{\rho V}{\mu} \quad (31)$$

The applications of this value of "S" are very varied: pedestrian dynamics (with it, for example, rapid evacuation systems of sports stadiums, hotels, etc), vehicle dynamics, economics, etc [34]. This value can be measured absolutely, in other words, as a single value applied to the whole series, or calculated in sections. Various values directly and indirectly related to "S" can therefore be defined: "S", "ρV", integral of "S", average of the absolute values of "S", etc. In the case of the calculation of "S" by sections, a curve is obtained, on which, its variations or trends are analyzed with the aim of trying to know the possible proximity of instabilities. The viscosity "μ" will be defined later.

In Economic Dynamics; Laminar and Turbulent Economy

The Similarity number "S" is the ratio of inertia forces to viscous forces and is a useful parameter for predicting whether a flow will be laminar or turbulent. It can be interpreted that when viscous forces are dominant (low "S") they are sufficient to keep all fluid particles in line (laminar flow); but: Even an excessively low "S" can indicate viscous drag motion; then, inertia effects are practically negligible or negligible. When inertia forces dominate over viscous forces, in other words, when "S" is larger, the flow can easily become turbulent, much more likely than otherwise. Some considerations related to economics:

- If the reaction time of an event to the movements of another event increases, "S" also increases; this means that the Economy is less active, reacts more slowly, less agile, possesses less potential or reserve energy.

- With a high "S", Turbulence is more likely to occur. An agile, fast-responding economy is unlikely to enter a turbulent dynamic.
- A high-velocity economy can amplify instabilities or turbulence.
- On the other hand, Pressure "P" relates velocity to density; thus, substituting it in the expression of "S", we obtain:

$$S = \frac{P}{V\mu} A \qquad (32)$$

In the above expression for "S", the relation already known for Temperature "T" can be substituted:

$$T \triangleright \frac{P}{\rho} \rightarrow T \triangleright V^2 \rightarrow T \triangleright P \rightarrow S = \frac{T\rho}{V\mu} A \qquad (33)$$

• High viscosity can mitigate instabilities; hence, turbulent entry.

Number "J"

Definition

Turbulence Index "J" is defined as the value from which turbulence or instability occurs; it is an attempt to improve the prediction of "S"; this is: To know:

- Transition from laminar to turbulent dynamics.
- Formation of traffic jams in a dynamic of pedestrians or vehicles dynamic.
- Etc.

By observing and comparing the value of "S" in several time series with instabilities or turbulences, it is possible to find patterns on values of "S" and to know the closeness of such instabilities.

In Economics Dynamics

Attributing to the economy an upcoming turbulent state, does not imply that: Either a crash is going to happen or an exceptionally large downturn or upswing (recession/boom) is going to happen necessarily: Being Turbulent, it is more likely that both, will happen. The "ABE" model, as we shall see, provides a dynamic simulation tool for "n" variables or events. But is it possible to use it to obtain an "instantaneous" value? It is possible to "solve" the model by knowing the state of the values before an instability. This point is transcendental since before and after it, we know the conditions under which the turbulent regime has originated and evolved to become, perhaps, a dangerous instability. Knowing this is essential to foresee, mitigate or change the adverse effects of instabilities.

Fractal Index

The concept on which the definition of this index is built is, in a certain way, similar to the "J" index; mainly in terms of its generation, determination or calculation. What is intended in giving the qualification of fractal to geometry is to calculate its dimension or its distribution of dimensions. But it is possible to have "hints" about its fractality, if a certain degree of turbulence or at least, little "laminarity" is appreciated. In fact, if the particle flow is turbulent, it is more likely to be fractal geometry and this is the essence of this index. We assume, therefore, that we have several time series of data in which turbulence appears in different time periods or moments (backtesting); from here, we proceed as follows (calculating the fractality means calculating the fractal dimension or the distribution of fractal dimensions):

- Calculate the fractality of the zone before instability.
- From all these calculations, look for a pattern in the dimensions and use it in other analyses. In other words: The next time such a "pattern" is detected, it can be concluded that the series will most likely enter turbulence: Learning from the past is the goal. This pattern or number is denoted as "IF".

Benford Index

The economy tends to follow a natural dynamic, imposed by its own nature. Only if it has been modified by some instability, as for example a human intervention, the naturalness is lost. This unnaturalness of the economic dynamics implies that it does not follow Benford's Law; and this unnaturalness may also imply: either a turbulent state, or the possibility of entering shortly into a turbulent state or phase, or simply an unnatural dynamic necessary to correct imbalances. In any of the cases, it is very useful to analyze the fulfillment of Benford's Law or not, to know both the current and the future state. Suppose we analyze the dynamics of an SP500 company; in particular, the behaviour of the company's opening value in the early morning for 1 year is to be analyzed. A list of the first figure of each security is therefore drawn up; objective: to find out whether the opening values correspond to a "natural" dynamic or not. For this purpose, the frequency curve of occurrence of the values in the generated list is plotted: The Benford Index can be defined as the difference between the theoretical Benford curve and the generated frequency curve. The "naturalness" of the analyzed values will be greater the closer the Benford index is to "0" and vice versa. This naturalness of the dynamics of these values implies, in the vast majority of cases, that the dynamics is in a non-turbulent state or not intervened by man; if it is intervened, it means that there is a risk of entering a turbulent phase or it is already in one. In any case, with this Benford index, we want to go further; in other words: to use it to know the probability or proximity of a possible entry into a turbulent period.

This procedure is based on prior learning (backtesting): A series of data series will be analyzed in which it is positively known when they enter the turbulent zone, studying and comparing the appropriate periods, in order to, based on these values and calculating the corresponding Benford indexes, be able to affirm that the entry into the turbulent zone is probable in a short period of time; in this way, it is possible to recognize future periods of turbulence, identifying the patterns of the Benford indexes calculated by backtesting.

Example

Parametrization of Information

Series analyzed: Event group "E2"; the series are divided into 8 sections of 25 elements each.

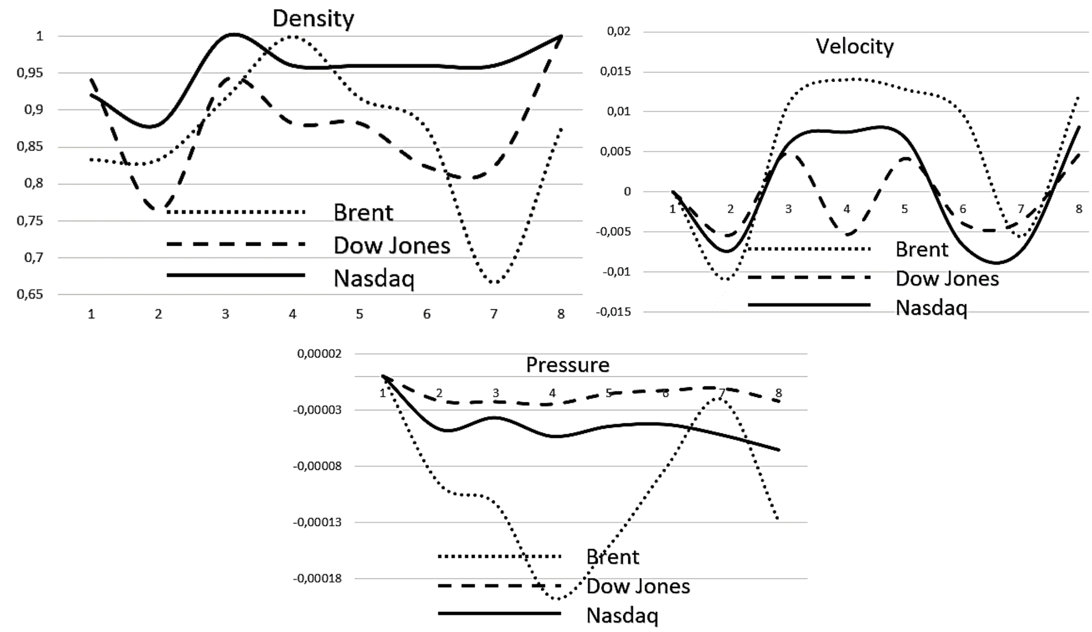


Figure 12

Parameterization of known time series information: Density, velocity and pressure.

Example:

Calculation of "S"

Let's calculate "S" and some of its variations, over 2 events of group "E2" (5 and 4 last intervals):

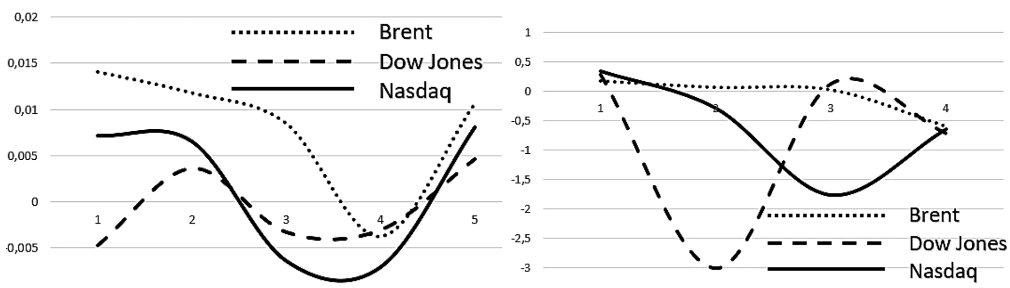


Figure 13

Curves that quantify the possibility of entering a turbulent zone or its presence (V*ρ and V*ρ/μ).

Similarly, we can calculate the integral of each "S" curve, which is nothing more than the sum of the absolute values (discrete integral), in order to obtain a global criterion from a curve:

Table 5

V*ρ	Integral of "S"	Average ABS(S)
Brent	0,04866576	0,00973315
Dow Jones	0,01926964	0,00385393
Nasdaq	0,03530881	0,00706176
V*ρ/μ	Integral of "S"	Average ABS(S)
Brent	0,691800514	0,172950128
Dow Jones	3,3180074	0,82950185
Nasdaq	2,42869879	0,6071747

Acceleration Balance: 1D Model

Potential

The potential "V" of a phenomenon is defined as the function that quantifies the energy available for that phenomenon: It is like a reserve, store or reservoir of energy; for an event "A" located at "P1" to move to "P2" with an acceleration "a", it is necessary that:

- There is a pressure difference (ΔP) between "P1" and "P2". The greater the pressure difference, the greater the variation of "a". The direction of "a" is always from higher to lower pressure.
- The higher the density of the particle, the lower "a" will be.

Therefore:

$$V = \frac{\Delta P}{\rho} \rightarrow \frac{P}{\rho} \quad (34)$$

In the case of "n" dimensions and supposing that in each dimension there is a pressure difference that sucks or pushes the particle "A", this suction depends on the importance of each potential; therefore, the potential in dimension "x" is:

$$V_x = m_x \frac{\Delta P_x}{\rho_x} \quad (35)$$

Introduction

"ABE" model: "Acceleration Balance Equations": Let there be a system of particles; let "a" be the acceleration of one of them and "m" be its absolute mass; their product is equal to the sum of all the forces that affect the particle to produce that "a".

$$m * a = \sum_{i=1}^{n} F_i \quad (36)$$

Let a function "u" be Real ($\Re \rightarrow \Re$) be defined in a space in 2 dimensions ("x" spatial dimension, "t" temporal dimension, "a" acceleration, "u" velocity of " x"); "D" is called the total or complete derivative:

$$a = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \quad (37)$$

The second term is defined as the nonlinear transport equation, since, when applied to a wave, it makes it move; in economic terms, transport must be understood as a transport of information. On the other hand, the "logistic" equation is used to model population dynamics; it has an analogous expression to the so-called nonlinear transport equation ("x_n" is the population of a certain species at an instant "n"):

$$x_{n+1} = x_n(a - b\,x_n) \quad (38)$$

There is a relationship between this equation and the nonlinear transport equation of "ABE": In order to appreciate this relationship, a change of variable is made:

(39)
$$y_{n+1} = y_n(a - b\,y_n) \rightarrow y = \frac{xa}{b} \rightarrow x_{n+1} = a\,x_n(1 - x_n)$$

By discretizing the nonlinear transport equation, we obtain the so-called CourantFriedrich-Levy (CFL) number; "CFL" must be less than 1 for the iteration to converge:

(40)
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \qquad u = u(n,t) = u_n^t$$

$$u_n^{t+1} - u_n^t = u_n^t \frac{\Delta t}{\Delta x}(u_{n-1}^t - u_n^t)$$

Note: The word logistics, which we traditionally identify with "transportation" and "distribution" of goods, comes from here. Notation: "x" is the values of the time serie, "u" is the velocity of the series.

The other essential piece is the phenomenon of diffusion: Suppose 2 periodic series with different frequencies; if the series are modeled in isolation, it is not possible to know whether these periodicities will disappear or not; this is the reason why, in any joint modeling of events, the participation of diffusion is necessary since it provides a possibility for the frequencies of both events to vary, interacting with each other and even cancelling each other out. Therefore, a model is needed that includes the phenomena of transport (logistics) and diffusion; both are essential. It is like analyzing independently and dependently the variation curves of prey and predators in an ecosystem (Lotka Volterra joint-full model) [35].

Incompressible Model

By means of a balance of accelerations ("a(viscous)(x)" is the acceleration due to viscosity (loss due to friction or diffusion), "Ex" are the accelerations caused by forces external to the system, "aP(x)" is the acceleration due to potential, "ρ" is the density, "μ" is the viscosity which will be defined later, but to say simply now, that it is a value indicating the laziness of the series to vary before an excitation), we obtain:

(41)
$$\frac{\partial u}{\partial t} = a_P - u\frac{\partial u}{\partial x} - a(viscous) + Ex$$

$\frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2}$ → a(viscous) to a particle of density "ρ". Since we know what units this term should have, this expression can be derived from Buckingham's Pi theorem [36]. Henceforth, any partial derivative is discretized by finite differences, which can, in order to increase accuracy by using more known values, be used of orders greater than 1.

Compressible Model

The main difference with the previous point lies in the fact that the density is not constant; continuity equation:

(42)
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}u + \frac{\partial u}{\partial x}\rho = 0$$

On the other hand, from the expression of the Pressure "P" in 1 dimension "x", "u" the velocity and "ρ" the density:

(43)
$$\frac{\partial P}{\partial x} = \frac{1}{2}\frac{\partial \rho}{\partial x}u^2 + \rho u\frac{\partial u}{\partial x}$$

Therefore:

(44)
$$\frac{\partial \rho}{\partial x}\frac{1}{\rho}u^2 \triangleright \frac{\partial P}{\partial x} \qquad \frac{\partial \rho}{\partial x}\frac{1}{\rho}u^2 \triangleright friction \qquad V \triangleright \frac{P}{\rho} \triangleright m\frac{P}{\rho}$$

Thus:

(45)
$$\frac{\partial Potential}{\partial x}\frac{1}{m} = \frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right) = a_P(x)$$

The final expression for an event is obtained

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} + \frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right) - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} \quad (46)$$

Nomenclature: "x" is the value of the time serie; "ρ", "P", "μ" and "u" with the density, pressure, viscosity and velocity of the time series of the economic event analyzed. The last term of the above equations is called the viscous or diffusive term.

Transmission Velocity of Information (absolute)

The transmission of information is essential in economics; therefore, to be able to quantify this information and to know its velocity, are 2 important tasks; the first one we have already achieved; with the second one, it is necessary to define the "ABE" model to be able to quantify it. We are based on 1 dimension "x"; from the acceleration of a particle located in a field of pressures, obtaining:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} \rightarrow \partial P = (-\rho \partial u)\frac{\partial x}{\partial t} = (u\partial\rho)u \quad (47)$$

$$\rightarrow u \equiv c = \sqrt{\frac{\partial P}{\partial \rho}}$$

$$\rightarrow u = \sqrt{abs\left(\frac{\partial P}{\partial \rho}\right)}$$

If this value is considered solely with the quotient, that is: abs (pressure variation / density variation), we can obtain a normalized velocity. The information transmission velocity "u", therefore, equivalent to the velocity of sound "c", is a function of the variation of pressure and density; the higher the pressure variation or the lower the density variation, the faster the information transmission velocity. It is a value that can be used as a classification criterion and as a criterion of choice, in the case of needing events with a high transmission velocity. If you work with a small number of sections, it is possible that null values for the density variation appear; therefore, it is necessary to avoid this fact.

Of course, the dynamics of the global economy, whether agile or stagnant, shape and influence the evolution of each event. It is impossible to separate the overall dynamic from the specific dynamics of each event. As we know, everything is interconnected, and these interconnections must be taken into account. Therefore, it is logical to think that most economic events, as part of the structure of each country, are affected by the global economy, and vice versa. It follows logically that all events will exhibit similar variations in their information velocity, understanding that this velocity is closely tied to or dependent on global dynamics. The speed at which information about an economic event spreads is profoundly influenced by the global economic situation or dynamics. This can be explained through various interrelated factors:

Global Market Interconnection

Greater economic interdependence in a highly interconnected global economy means that economic events in one region have direct or indirect implications for other regions. This increases the attention and urgency with which information spreads. For example, a Federal Reserve interest rate decision impacts financial markets worldwide almost instantly.

Speed of Reactions

Interconnection drives economic agents—ranging from governments to investors—to react more swiftly to changes.

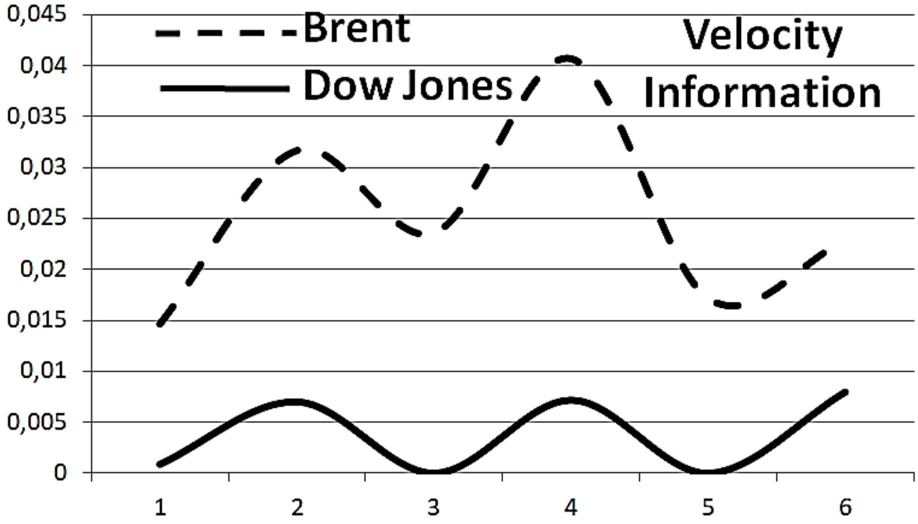
- Information and Communication Technology: Technology has enabled economic information to flow in real time. However, the attention and speed of dissemination depend on the event's importance in the global context. During times of high economic uncertainty, such as global financial crises, information systems prioritize these events, accelerating their dissemination even further.
- Relative Importance of the Event in the Global Context: If an event aligns with critical global economic dynamics, its dissemination will be faster. For instance, a monetary policy change in an emerging economy will have less immediate impact than one in a major economy like the United States or China, due to their weight in the global economic system. During periods of global crisis or uncertainty, even minor events can spread rapidly due to the amplifying effect of uncertainty.
- Global Social and Political Factors: Geopolitical dynamics also influence the speed at which economic information spreads. During times of global political tension or economic warfare, any financial, commercial, or resource-related event gains particular urgency.
- Psychological and Behavioral Factors in Global Markets: In periods of high volatility, such as during recessions or pandemics, financial markets are more attuned to any signal. Both trading algorithms and human investors respond more quickly, further accelerating information dissemination.

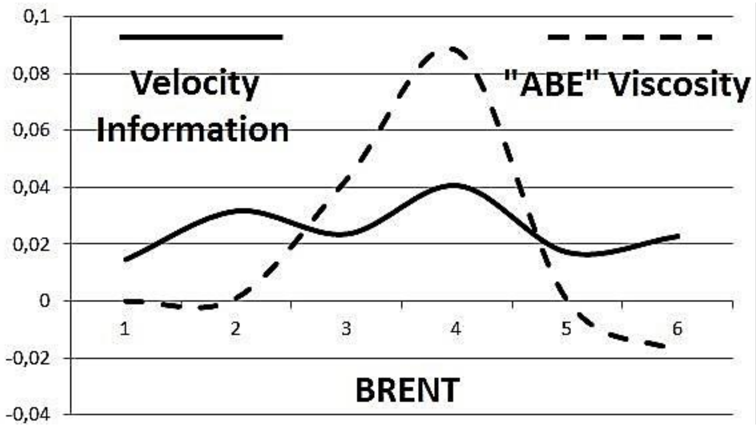
Measuring Influence: Global economic dynamics determine the relevance and impact of an economic event. The more synchronized global markets are and the more sensitive they are to certain trends (inflation, recession, growth), the faster information spreads. Additionally, the development of technologies like social media, global financial media, and algorithmic systems amplifies this velocity.

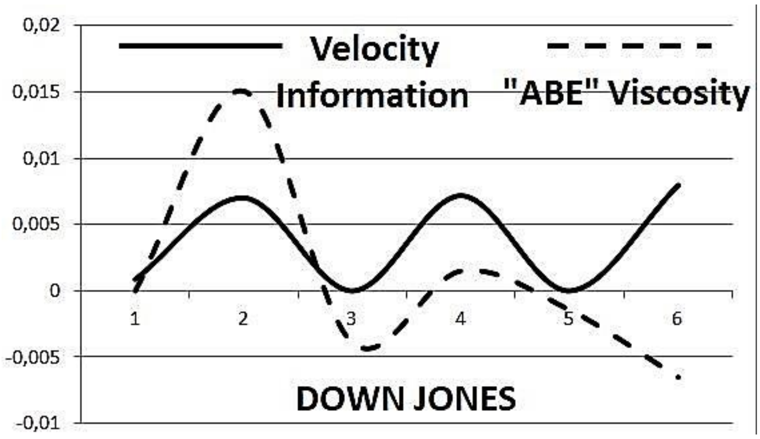
Example

Calculation of Information Transmission Velocity for 3 isolated events: Considerations about

Let the events of the event group "E2" be:







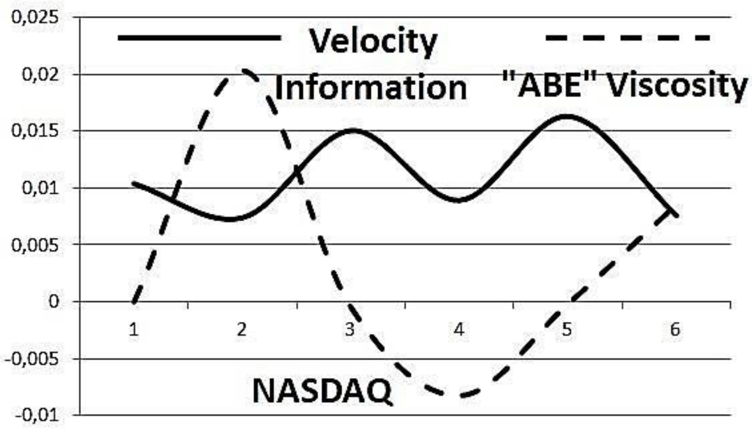


Figure 14

Comparison between transmission velocity curves of the analyzed events, during the last 6 periods of 25 values each, and viscosity "ABE".

Observe that, at least in the first approximation, it seems that the relative velocities are more or less constant; it appears as if they are inherent to the analyzed event itself.

Therefore, and observing the curves generated, the information transmission velocity of the Brent is the highest of all: The speed of information transmission in the price of Brent crude oil is higher than in indices such as the Dow Jones or the Nasdaq due to several characteristics unique to commodity markets compared to stock markets. Key reasons include:

Global Nature of the Oil Market: Brent crude is a globally traded commodity, with its prices reflecting changes in global supply and demand almost immediately. Factors such as geopolitical tensions, OPEC decisions, climatic events, or production disruptions directly impact prices, transmitting information more quickly. Lower Diversification of Factors: While indices like the Dow Jones or the Nasdaq aggregate multiple companies from different industries, goals, and contexts, Brent crude focuses on a single commodity. This means any relevant event has a direct and immediate impact on its price, whereas stock indices take longer to process events as they must account for impacts across multiple sectors.

Liquidity and Continuous Trading: The oil market operates nearly continuously with high liquidity, especially in international markets such as ICE and NYMEX. This almost roundthe-clock trading allows information to be incorporated into prices more quickly, unlike stock indices, which depend on specific trading hours and the exchanges on which they are listed.

Sensitivity to Macroeconomic Factors: Brent crude is a key barometer for the global economy. Changes in monetary policy, fluctuations in the dollar, and economic growth projections quickly impact its price because they directly influence the supply and demand for this commodity. Conversely, stock indices are generally less sensitive to immediate global factors and more influenced by corporate reports or sectorial dynamics.

High Reaction from Speculative Actors: The oil market is heavily influenced by speculative traders and hedgers who react immediately to news. This contrasts with stock indices, where responses may be mediated by deeper analysis of corporate fundamentals or long-term decisions by institutional investors. The faster speed of information transmission in the price of Brent crude reflects its sensitivity to immediate global factors, its unique focus on a single commodity, and the constant operation of its markets, making it more agile than diversified stock indices. The fact that the speed of information transmission in Brent crude oil prices is higher than that of the Nasdaq or Dow Jones indices has significant implications for investors. These implications affect both strategy and risk management. Some of them include:

Competitive Advantage for Agile Investors

- **High-Frequency Traders (HFT):** Traders using algorithms or high-frequency systems can leverage the rapid transmission of information in Brent crude prices to react within milliseconds, giving them an edge in such a sensitive market.
- **Faster Informed Decisions:** Investors with access to real-time data and advanced analytical tools can benefit from this speed to adjust their positions before others.

Increased Volatility

- **Impact on Hedging Strategies:** The market's rapid reaction to global events can heighten price volatility. This forces investors reliant on Brent, such as energy companies or transport operators, to implement more frequent and

precise hedging strategies.
- **Exposure Risk:** Investors must be prepared for sudden fluctuations, as unexpected events (conflicts, OPEC policy changes, natural disasters) can quickly alter prices.

Opportunities and Challenges in Diversification

- **Correlation with Other Assets:** Oil prices affect entire sectors, such as energy, transportation, and manufacturing. Rapid changes can indirectly influence stock indices, impacting the correlation between these assets and commodities.
- **Effective Diversification:** Understanding the relationship between Brent's rapid information transmission and its impact on other markets allows investors to better diversify their portfolios to mitigate risks.

Impact on Macroeconomic Investments:

- **Global Economic Barometer:** Since Brent crude reflects changes in the global economy almost immediately, investors can use its price as a leading indicator to adjust strategies in other markets, such as bonds or equities.
- **Impact on Monetary and Fiscal Policies:** Movements in Brent prices can signal potential decisions by central banks or governments, providing early cues for portfolio adjustments.

Differences in Investment Horizon

Short-Term: Short-term traders may find more opportunities in Brent due to its high sensitivity and immediate reaction to global news.

Long-Term: Long-term investors must consider that Brent's speed can amplify temporary reactions, which may distract from more sustainable fundamental trends.

The higher speed of information transmission in Brent crude oil prices can create both opportunities and risks. Investors must adapt their strategies to respond to this dynamism, using advanced analytical tools and diversification to capitalize on rapid market reactions while managing the risks associated with volatility.

In the Earlier Graphs, Two Things are Evident: Each event has a specific magnitude of information velocity and its information velocity occupies a particular "height." This indicates that each event has a characteristic and inherent information velocity (which can be perfectly considered as a criterion for classification and comparison). This also reflects the degree of dependency of each event on the global situation. An important confirmation of the above: The three analyzed events show identical variations over the same time periods (depending also of one gap originate by viscosity). This demonstrates that the time or moment of analysis entirely and evidently conditions the information velocity of each event.

Transmission Information Between Events (Relative)

Key concepts to explain:

- **Speed of Information (or Information Transmission Speed):** This refers to how quickly information about an economic event becomes known, is processed, and generates a reaction in another event or economic variable.
- **Time Lag Between Events (Lag):** This is the time that passes from when an input (cause) occurs until the output (effect) becomes evident.
- The faster the transmission of information between two events, the shorter the time lag between them. In other words:
- **High information speed:** Low lag.
- **Low information speed:** High lag.

Let's Suppose These Two Events

- **Event A:** The central bank announces an interest rate hike.
- **Event B:** A drop in the stock market is observed.

If economic agents receive and process that information quickly (high speed), the lag between the announcement (input) and the market drop (output) will be short. It can be almost immediate if markets are efficient. Now, if we are analyzing an event with slow information transmission (for example, a change in inflation expectations that takes time to be noticed), the effect may take weeks or even months to be reflected in prices, investments, etc.

Input-output in Economic Chains: What you're describing also applies to value chains, business cycles, and dynamic system models.

- Input: Economic stimulus (fiscal policy, monetary policy, external shock).
- Process: Transmission of information and adjustment of expectations.
- Output: Reaction of variables (output, prices, employment).

The Lag Between Input and Output Is Directly Influenced by:

- The speed of information propagation.
- The flexibility of agents to react (e.g., sticky prices).
- The structure of the economy or the market.

By analyzing the speed of transmission of economic information and its relationship with the time lag between event, using the European Central Bank's (ECB) monetary policy and its impact on the eurozone economy as a case study, we find:

Context: ECB Monetary Policy and Time Lag: The transmission of ECB monetary policy decisions to the real economy involves significant time lags. Research indicates that, on average, it takes between 12 and 18 months for an interest rate adjustment to affect the Gross Domestic Product (GDP) and inflation in the eurozone. This lag is due to various factors, such as price and wage rigidity, the structure of the financial system, and the expectations of economic agents

Speed of Information Transmission: The speed at which information about ECB decisions spreads and is processed by economic agents influences how quickly economic effects materialize. For example, the ECB has used forward guidance to communicate its monetary policy intentions, which helps reduce uncertainty and accelerates decision-making in financial markets.

Relationship between Information Speed and Time Lag: There is, at least initially, an inverse relationship between the speed of information transmission and the time lag between economic events:

- **High information speed:** Economic agents receive and process ECB decisions quickly, allowing for a more immediate response in variables such as interest rates, prices, and economic activity.
- **Low information speed:** Information takes longer to spread and be understood by agents, which delays the economic response and extends the time lag.

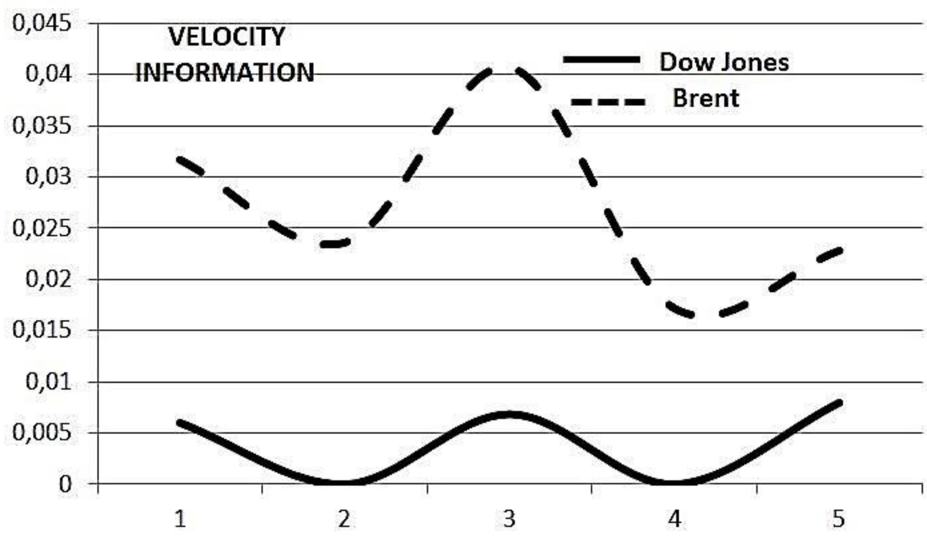
During the 2022–2023 monetary tightening cycle, the ECB implemented significant interest rate hikes to combat inflation. Research indicates that the transmission of these decisions was faster compared to previous cycles, possibly due to clearer communication and increased market anticipation.

We can conclude so:

- **Information Speed:** The speed at which economic information is disseminated and processed directly influences the effectiveness of economic policies.
- **Time Lag:** A higher speed of information transmission can reduce the time lag between policy implementation and its effects on the economy.
- **ECB Strategies:** The use of tools such as forward guidance has allowed the ECB to improve the transmission of its policies, reduce uncertainty, and accelerate the economic response.
- **So:** lag as a viscosity, is essential in order to know the information transmission velocity.

Example: Calculation of Information Transmission Velocity between 2 events (working together): Considerations about

In the present analysis, a clearer correlation is observed between the variations in the information transmission rate of both events. This direct relationship appears more pronounced compared to previous studies. Similarly, a significant correlation is evident between the information transmission rate and absolute viscosity. The observed increase in correlation coefficients can be attributed to the simultaneous and synergistic interaction of the two phenomena under investigation.



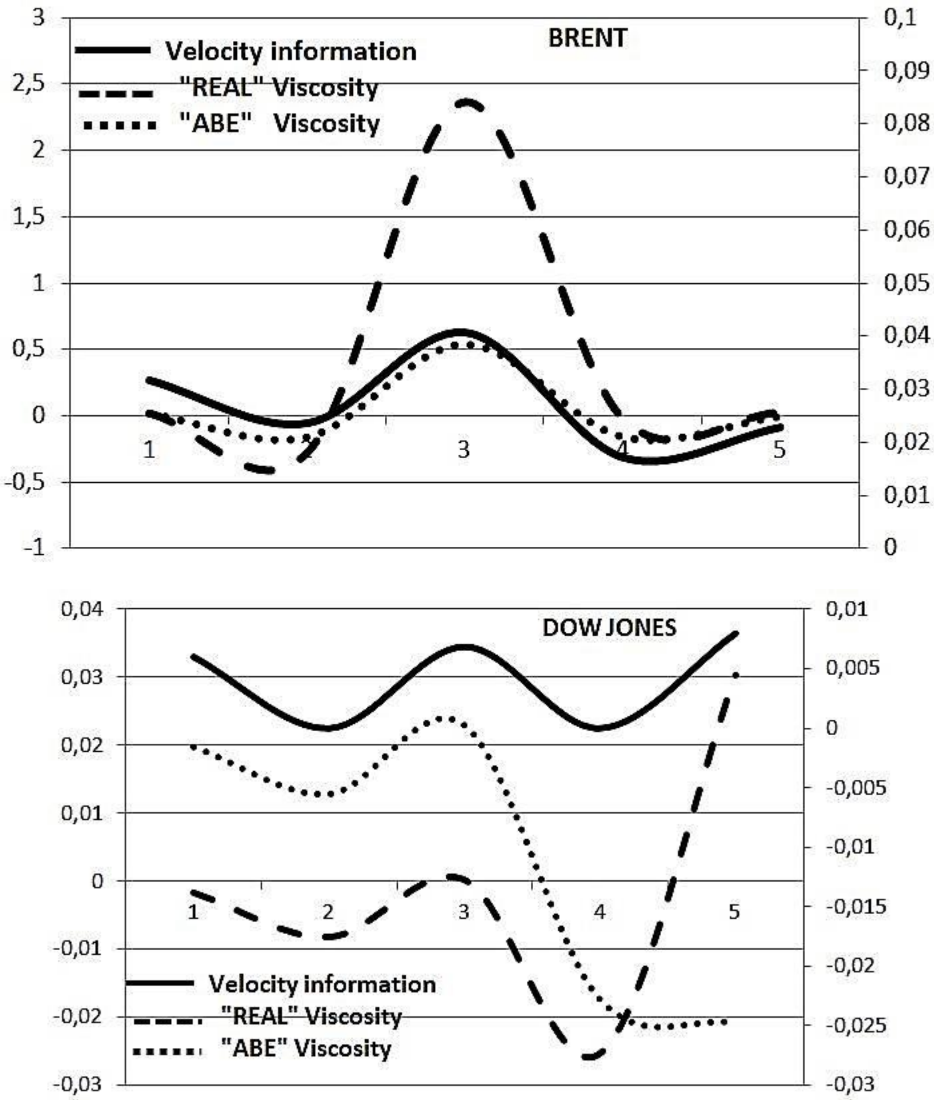


Figure 15

Comparison between transmission velocity curves of the analyzed events, during the last 6 periods of 25 values each, and viscosity "ABE".

Note: The speed at which information is transmitted by an event is intrinsic to the event itself. That is: Each event, regardless of the event it interacts with, will have an information transmission speed that is practically identical or analogous. This speed can be measured using viscosity.

Note: The speed of information, as has been demonstrated, constitutes an intrinsic property of the event itself; however, this condition remains valid only within the temporal interval during which the analysis is conducted. In this context, such speed can be formalized as a parameter or reference criterion for the comparison between events. This same rationale extends, as previously noted, to all definitions, parameters, or comparative criteria employed throughout the present study.

Optimization of the "ABE" Model: Turbulent Viscosity

The so-called viscous term has been included in the model; in it, viscosity is found; if we work with fluids, we work with the Navier Stokes equations, and in them, we also find the same term; but in these equations, we can find big errors if we want to model or work with turbulence; this is so because turbulence itself has a viscosity called turbulent viscosity, which is necessary to incorporated ("C_μ" is an experimental coefficient):

(48) $$\nu_T = C_\mu \frac{TKE^2}{\varepsilon}$$

This viscosity depends on 2 values "TKE" (turbulent kinetic energy) and "ε" (dissipation of "TKE": Represents the rate at which kinetic energy is converted into internal energy (heat) per unit mass of the fluid) which are experimental values. To calculate these 2 important values, one can resort to the use of experimental tools such as CFD techniques. The calculation of "TKE" is direct and immediate, providing its value at any point. Calculating the value of "ε" is perhaps not so direct; however, a mathematical channel can be generated in a CFD simulation to calculate the dissipation at a given point; if "y" is the coordinate through which the dissipation is to be known and "δ" is the length in that coordinate or direction within which "ε" ("ρ" density and "u" velocity) is to be calculated:

$$\varepsilon = \int_0^{\delta} (\rho u) dy \tag{49}$$

In the above equation, the density varies and the velocity "u" is the difference of velocities between the 2 points where you want to calculate the turbulent kinetic energy dissipation. There is another way to calculate the dissipation of "TKE" between 2 points which is to calculate the difference of "TKE" at both points. We can now define another concept related to the quantification of the intensity of a turbulence "It":

$$It = \frac{\sqrt{(2/3)K}}{\bar{u}} \tag{50}$$

The use of the values of "TKE" and "ε" corresponds to the use of the so-called "K-ε" model ("TKE" = "K") used in fluid dynamics [37]. In, we can analyze an example: In a nozzle, the flow is studied by applying the Navier Stokes equations; the values for both sides of the equation at 2 given points are: The first table corresponds to the non-use of turbulent viscosity and the second, the opposite [38]:

Table 6

	$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)$	$-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$
Point 1	-4.45E-02	-9.08E-05
Point 2	-1.21E-02	5.31E-05

Table 7

	$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)$	$-\frac{\partial p}{\partial x}+\mu_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$
Point 1	-4.45E-02	-4.32E-02
Point 2	-1.21E-02	-1.21E-02

With the use of turbulent viscosity, the similarity between the two parts of the equation is clear.

Kolmogorov Scales

Turbulence in a fluid can be as large at the beginning as the scale of the fluid itself; there are other turbulences that are formed from the larger ones and so on; but there is a time when the viscosity is able to eliminate or absorb the small turbulence created, often transforming its energy in the form of temperature (or velocity, in short); this limit is called Kolmogorov scale "Lk" (scale of the smallest turbulence). There is also the so-called Kolmogorov "t-time" and "u velocity":

$$L_k = \left(\frac{\mu^3}{\varepsilon}\right)^{\frac{1}{4}} \leftrightarrow \varepsilon = \frac{h}{t} \qquad t_k = \left(\frac{\mu}{\varepsilon}\right)^{\frac{1}{2}} \qquad u_k = (\mu\varepsilon)^{\frac{1}{4}} \tag{51}$$

It is a measure of the resistance of a fluid to flow. The higher the viscosity, the faster the kinetic energy will dissipate and therefore the shorter the Kolmogorov length. These are very important values when designing a CFD simulation; "h" can be calculated experimentally and is very useful in pedestrian dynamics, for example, but also in information transmission in general.

It has been seen that pressure is a measure of energy; it quantifies therefore what energy a wave has mainly as a function of its velocity. This energy is defined as "TKE" or turbulent kinetic energy of the event "E ("ui" is the velocity of "E" on the "i" axis) at a given point; in other words: The energy of a particle:

$$TKE_E = m\sum_{i=1}^{n} u_i^{\prime 2} \quad (52)$$

It is therefore possible to calculate its dissipation by means of an empirical test; this is very useful in the case, for example, of analyzing the dynamics of groups of pedestrians, automobile traffic, flocks of birds, etc. From the above expressions, it is obtained that the turbulent kinetic energy dissipation depends exclusively on the viscosity (at least in first approximation).

For the Kolmogorov "limit" values to be reached, it is necessary, as already mentioned, that the viscosity destroys or prevents the formation of the smallest turbulences. For this, the viscous forces must be equal to the inertial or velocity forces; that is: The Reynolds number must be equal to 1; combining this fact with the above-mentioned expressions, we obtain:

$$\varepsilon = \frac{(\rho u)^4}{\mu}; \varepsilon = \frac{L\rho u}{t^2}; \varepsilon = \frac{u^3}{L\rho} \quad (53)$$

Thus, by combining them, a relationship between the 3 minimum Kolmogorov values is obtained; a very interesting relationship to be able to appreciate mutual variations between the fluid values and the Kolmogorov values:

$$\frac{(\rho u)^4}{\mu} = \frac{L\rho u}{t^2} = \frac{u^3}{L\rho} \quad (54)$$

Example: Calculation of Turbulence Intensity

Let us calculate the turbulent kinetic energy, for the Brent and the Dow Jones, belonging to event group "E2", in the last 7 intervals, with "m=1":

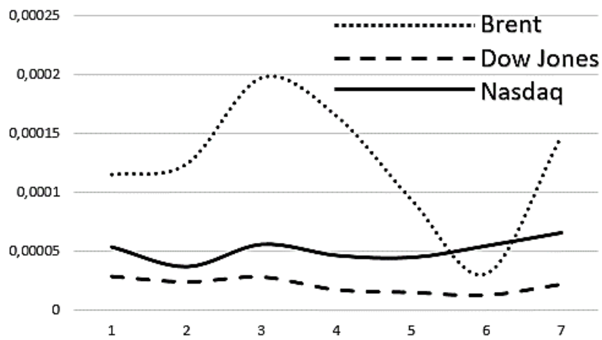


Figure 16: Turbulent Kinetic Energy Curves

Table 8

	Integral	Average
Brent	0,00087494	0,00012499
Dow Jones	0,000148402	2,12003E-05
Nasdaq	0,000357377	5,10538E-05

Kolmogorov Scales in Economics

In the context of economics, this theory has been applied in some economic growth models to analyze the distribution of wealth and the evolution of economic inequalities. In particular, some studies have used the theory of Kolmogorov scales to analyze the distribution of income in an economy and how this distribution changes as the economy grows. Kolmogorov's theory suggests that the income distribution in an economy behaves similarly to a turbulent fluid, and that fluctuations in income at different levels of the distribution follow a stepwise pattern similar to turbulence in fluids. In this approach, the income distribution is analyzed at different scales, from the lowest to the highest income. Each scale is defined by a given fraction of the population, for example, the poorest 10% or the richest 1%. It has been found that income distribution follows a staggered pattern at each of these scales, and those inequalities between different income levels are persistent and persist even as the economy grows.

There is no single numerical model of Kolmogorov scales in economics, since the application of this theory to economics may vary according to the focus and specific objectives of the study. However, some common elements can be identified that are usually present in numerical models of Kolmogorov scales in economics. In general, these models are based on the idea that the distribution of income in an economy follows a staggered pattern, with inequality being greater at the higher scales of the distribution. To simulate the evolution of this distribution over time, numerical models usually include three main components:

- **Initial Income Distribution:** This component specifies how income is distributed in the population at the beginning of the simulation. This distribution can be obtained from empirical data or from a theoretical distribution.
- **Economic Growth Rate:** This component describes how the average income of the population changes over time, which can be related to factors such as investment, technological progress, foreign trade, among others.
- **Income Redistribution Rules:** This component describes how income is redistributed across the population over time, and may include taxes and transfers, changes in the production structure, and other economic policies that affect income distribution.

Once these components are specified, the numerical model is used to simulate the evolution of the income distribution over time. This simulation is performed at different income scales, from the lowest to the highest, and analyzes how the income distribution changes at each scale as the economy grows. Information loss refers to the situation in which the original information is lost or degraded in some process or transformation. On the other hand, Kolmogorov scales refer to a measure of the complexity of a data series, in other words, how much information is needed to describe it completely and accurately.

The relationship between information loss and Kolmogorov scales is in the sense that, in general, when information loss occurs, the complexity of the data decreases and the corresponding Kolmogorov scale becomes smaller. For example, if one has an original highresolution image and reduces its size to fit a smaller size, information is lost and the complexity of the image decreases. As a result, the Kolmogorov scale corresponding to the reduced image will be smaller than that of the original image. In short, the loss of information is usually accompanied by a decrease in data complexity, which is reflected in a smaller Kolmogorov scale. Finally, it is necessary to find analogies between the different Kolmogorov scales and economic structures.

Turbulent Temperature

The term Viscous from the "ABE" model already has the so-called Turbulent Viscosity; in other words: The term viscous combines all viscosities. In order to calculate the Viscosity from the velocity, density and pressure and to gather also the turbulent viscosity, it has to gather also the temperature variation, from which the turbulence becomes temperature; this value is defined as Turbulent Temperature "Tt".

$$Tt = \frac{viscous\_term}{\mu} / \mu = function(P, \rho, V, Tt) \quad (55)$$

In other words: This turbulent temperature is the factor that the viscosity needs to be equal to the value of the viscous term. The ideal is to have a definition of viscosity equal to the viscous term, where "Tt" is null (may be μ = function (V,P,ρ,T) ?).

Random Term, Modelling of Human Decisions in the "ABE" Model and its Relationship with Quantum Mechanics

The basic idea is to consider the fluid variables (velocity, pressure, etc.) as random functions in time and space, and to introduce randomness into each variable. But also, the random term can be introduced as stochastic external force acting on the fluid and modeled as a random function. This stochastic force can represent random perturbations in velocity, density, temperature, viscosity, among others, that affect the behaviour of the fluid. In mathematical terms, a random term is added to the "ABE" model which is written as:

$$Dv/Dt = \nabla \text{Potential} - \mu \nabla^2 v + A + Rw \quad (56)$$

Where "Rw" is the random term and "A" is any other deterministic external force. The random term can be modeled as a stochastic process, such as a white noise process or an Ornstein-Uhlenbeck process. Randomness also needs to be introduced, in some form, into human decisions; a decision needs this degree of randomness; it is not completely deterministic and needs to be contemplated. For this, an example: Let there be a population growth model given by "N" as the number of inhabitants; the simplest model that can be defined ("a" constant, "t" the time) for the instantaneous growth rate:

(57) $$\frac{dN}{dt} = aN$$

If a stochastic or random perturbation (instability) is introduced "a=r+αε" ("r" and "α" constant) and "ε" stochastic term:

(58) $$dN = rNdt + \alpha NdB$$
$$N_t = N_0 \exp\left(\left(r - \frac{\alpha^2}{2}\right)t + \alpha B\right)$$

Another example, in economic terms: Let an asset "S", "σ" its volatility, "ε" the stochastic term and "μ" the return; the simplest model for the dynamics with respect to time of "S" is: With the introduction of the stochastic term (Wiener or Brownian process), we obtain that:

(59) $$\frac{dS}{dt} = \mu S \qquad \frac{dS}{S} = \mu dt + \sigma\left(\varepsilon\sqrt{dt}\right)$$

Any human decision involves a "certain" degree of "non-Brownian" randomness; this is why the introduction of a fully stochastic term as defining an action is not entirely correct. There are many random functions that can be used to model human behaviour, depending on the type of behaviour one wishes to simulate. One random function commonly used in economics to model human behaviour is the stochastic utility function. This function is used to model the preferences of individuals and how these preferences influence their consumption and production decisions. The stochastic utility function can be written as follows:

(60) $$U(X) = X + \varepsilon$$

Where "U" is the utility function, "X" is the level of consumption or production and "ε" is a stochastic error term representing the random impact of external factors that may influence the consumption or production decision. For example, in the case of consumption decision making, the stochastic utility function is used to model how consumers assign value to different goods and services based on their characteristics and prices. The error term "ε" introduces an element of uncertainty into the consumption decision, reflecting the possibility that consumers make sub-optimal decisions due to lack of information or external factors. In the case of production, the stochastic utility function is used to model how producers assign value to different levels of output as a function of costs and market demand. Again, the error term "ε" introduces an element of uncertainty into the production decision, reflecting the possibility that producers make sub-optimal decisions due to market uncertainty. In general, the random functions to be introduced should be generated experimentally. The search for individual profit as a fundamental part of human behaviour is inherent to the economic dynamics itself; in a market economy, individuals and firms seek to maximize their profits and satisfy their needs and desires. This search for individual profit leads to the production and distribution of goods and services that satisfy the demands of consumers; envy is the main engine of society: envy can sometimes be confused with a desire to improve.

The expression that describes in a certain way, the quantum phenomena, is also a powerful equation to introduce a random term and propose this equation, called Schrodinger equation, as an equation to introduce a modeling of human behaviour; this is because it has a term that contains the so-called Wave Function; this function is susceptible to be used as a probability function when modeling human decisions.

For example, the Schrödinger wave function can be used to model the behaviour of the expectations of economic agents in a financial market. In this context, the Schrödinger wave function describes the probability that an economic agent will make a particular decision at a particular time, based on his expectations and the information available at that time. The Schrödinger wave function in this case could be expressed as a mathematical equation that describes the evolution of the probability of an agent making a particular decision over time, in terms of a quantum wave. For example, the Schrödinger wave function for the expectations of an economic agent could be of the form [39]:

(61) Ψ(x, t) = A*exp(-(x - x0)^2 / (4σ^2) + iEt/ħ)

Where "Ψ(x,t)" represents the probability that the agent makes a particular decision at time "t" and position "x", "A" is the amplitude of the wave, "x0" is the mean position of the economic agent, "σ" is the standard deviation of the probability distribution, "E" is the energy of the system and "ħ" is the reduced Planck's constant. Ideally, this equation, or a term analogous to the wave function, should be included in the "ABE" model.

Relationship Between Freedom (Conditional) of Choice and Human Cooperation

This is an essential point for the eminent economist Milton Friedman: The freedom of choice of individuals within a society where they interact and cooperate with others is not absolute, but it still exists. Living in a community inevitably involves being influenced by norms, cultural values, social expectations, and the decisions of others, which partially shape our own choices. However, within those boundaries, individuals still retain personal decisionmaking capacity. Freedom in a social context can be seen as conditional freedom, meaning a freedom that takes into account the consequences of our actions on others and society's responses, which can sometimes reduce the feeling of freedom.

This framework of social interaction does not eliminate freedom but rather nuances it, as many of our decisions are influenced by the desire to maintain harmonious relationships, avoid social sanctions, or fulfill specific roles. Nevertheless, even though society exerts a strong influence, people still retain a degree of freedom to make decisions within these limits. This tension between individual freedom and social influence is a central theme in philosophy and sociology. According to some perspectives, such as existentialism, freedom is an inherent characteristic of the human being, even in the face of social pressure. Other approaches, like social determinism, emphasize how collective structures and dynamics deeply shape our decisions. Therefore, individual freedom is not entirely erased, but it is influenced and, in some cases, restricted by interaction with the social group.

Relationship Between the "ABE" Model and Other Mathematical Models

There are innumerable mathematical models applied to diverse fields of Science sharing the same essence or physical foundation; we can find mathematical transformations to deduce some models from others: Schrodinger – Diffusion equation; Schrodinger – Navier Stokes; Schrodinger – Black-Scholes-Merton; Navier Stokes – Black-Scholes-Merton; Navier Stokes–Euler Lagrange; Schrodinger–Euler Lagrange; Logistics (typical equation of any population variation model) – Navier Stokes [31,33,40-43].

Another method of deduction of the "ABE" model is based on the introduction in the Euler Lagrange equations of a term representing a "Non-Conservative" force: In this case viscosity as a friction force. Let "L" be the Lagrangian, where "V" being the potential, and "x" being the only generalized coordinate of the problem:

$$L = \frac{1}{2} m \overset{*}{x}^{2} - V \qquad V = m\frac{P}{\rho} = P * Volume \tag{62}$$

Perhaps a "constant (non-dimensionless) = K_f" could be incorporated, modifying the units to be consistent:

$$V = K_f \, m \frac{P}{\rho} \tag{63}$$

If we apply the Euler-Lagrange equation with non-conservative forces: We obtain (1 dimension "x", "t" the time and "x" the only generalized coordinate of the analyzed problem) the Euler-Lagrange equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \overset{*}{x}} - \frac{\partial L}{\partial x} = Q \tag{64}$$

On the other hand, ("Q" external forces and "u" the velocity) and substituting:

$$F(friction) = -\frac{\mu}{\rho} m \frac{\partial^2 u}{\partial x^2} = Q \qquad m\overset{**}{x} + \frac{1}{\rho}\frac{\partial P}{\partial x} m = -\frac{\mu}{\rho} m \frac{\partial^2 u}{\partial x^2} \tag{65}$$

Interesting to note that the mass does not matter, generating the following expression for the acceleration:

$$\overset{**}{x} = \frac{\mu}{\rho} m \frac{\partial^2 u}{\partial x^2} - \frac{1}{\rho}\frac{\partial P}{\partial x} \qquad \text{But:} \quad \overset{**}{x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \tag{66}$$

That, by combining them, the equation of the "ABE" model is obtained:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} + \frac{1}{\rho}\frac{\partial P}{\partial x} \tag{67}$$

In Other Words: It is possible to model an isolated economic event using the EulerLagrange equation (this result is extended to "n" events).

There are many mathematical models that, in essence, are very similar to the "ABE" model; in fact, they share practically the same terms and, therefore, the dynamics they model, are similar; among them: Logistic Equation, Langevin, Fokker Planck, Lotka Volterra model, "SIR" model (pandemic dynamics), Heat Equation, Romeo and Juliet model, Lanchester Model, Alan Turing's biological model, Navier Stokes, Euler Lagrange, etc [32,44-49].

Short- and Long-Term Prediction

Introduction

By removing from the model, we obtain the expression to perform the prediction; note that it is possible to choose a discrete derivative model, of high order, to increase the accuracy so:

$$u(t+1) = u(t) - u\frac{\partial u}{\partial x} + \frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right) - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} \quad (68)$$

Viscosity Classification

It is called Viscosity "ABE" or absolute, as the viscosity extracted from the previous expression, assuming that $\frac{\partial u}{\partial t} = 0$ (this does not calculate the equilibrium of the series, but theequilibrium of the velocity of the series: A kind of laminar regime). It is also called natural viscosity (inherent to the event itself) or absolute viscosity.

$$\mu(ABE) = \frac{\left(\frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right) - u\frac{\partial u}{\partial x}\right)\rho}{\frac{\partial^2 u}{\partial x^2}} \quad (69)$$

- If $\frac{\partial u}{\partial t} \neq 0$ it is called "real" viscosity.

$$\mu(real) = \frac{\left(u(t) - u(t+1) - u\frac{\partial u}{\partial x} + \frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right)\right)\rho}{\frac{\partial^2 u}{\partial x^2}} \quad (70)$$

Note: Viscosity is a parameter that quantifies the laziness of the time series to variation; on the other hand: It can also be used to model groupings of human beings, as well as the diffusion or propagation of rumors, diseases or fashions in the economic structure of each country, or in general, the velocity of information transmission.

Henceforth, the use of natural viscosity implies a natural interpretation of the definition where it is applied, i.e.: Inherent to the event itself, whereas, if actual viscosity is used, it implies a comparison between several events when calculated in relation to other events. For example:

- A ball has a volume, but depending on at what height or on what planet its mass is calculated, it has one mass or another.
- An object is made of one material; but depending on the light shining on it: It has one color or another.

Example

Calculation of "ABE" and "REAL" Viscosities

The 3-Time Series of group "E2" are Studied:

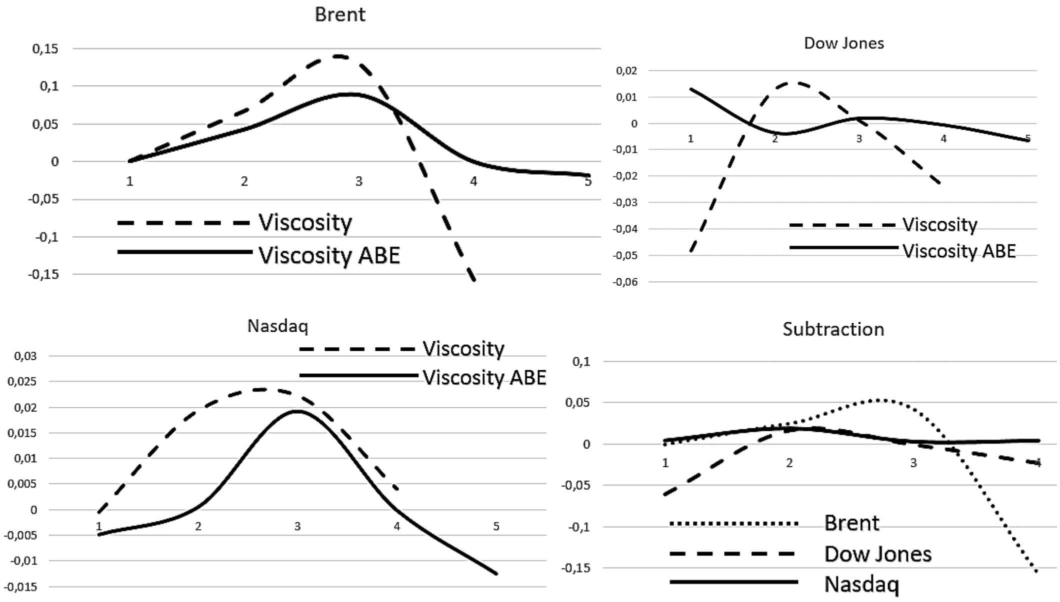


Figure 17

Curves corresponding to the parameterization of the viscosities of the 3 events. The last 5 sections of each series are represented, out of the 8 with 200 values.

Prediction Procedure

In order to predict the value of "u" or the trend of "u" at instant "t+1", it is necessary to know the value of the "real" viscosity at "t"; but this viscosity value is not known:

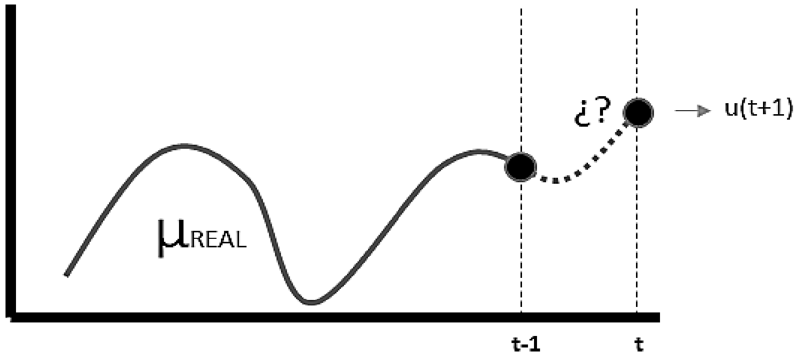


Figure 18: Example Viscosity Curve with Values Points

$$u(t+1) = u(t) - u\frac{\partial u}{\partial x} + \frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right) - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} \tag{71}$$

Protocols for Viscosity Selection in "T"

Let us look at some calculation procedures for this important value:

- Protocol 1

Performing a linear extrapolation of the "real" viscosity curve (based on the last 2 points, i.e.: At "t-2" and "t-1"), or quadratic (based on the last 3 points), etc.

- Protocol 2

Add/subtract to the "t-1" value of the "real" viscosity, the standard deviation of this viscosity. This protocol for selecting viscosity at "t" is ideal because it allows for a very high probability of choosing the value that the analyzed event would select on its own. Therefore, conducting a sensitivity study, as we will see later, of this value provides an excellent determination of the degree of uncertainty in the prediction based on it. That is: Once we have selected the viscosity at "t" by adding and subtracting the standard deviation to the viscosity value at "t-1," we will obtain a range of three possibilities for the viscosity at "t." If this range is entirely above or below the viscosity "V0" for which velocity is zero, it indicates low uncertainty. The greater the immersion (i.e., the greater the difference between this range and "V0"), the lower the uncertainty in the prediction. This is because a greater change in viscosity at "t" will be required to achieve a velocity with a different sign or trend.

- Sure, there are more….

Series Prediction at "t+1"; Short Term Prediction; Minimum Prediction Time the Process of Predicting a Time Series is as Follows:

- Analyze the actual viscosity curve and select the viscosity at "t".
- It is common practice in the world of statistical physics to predict using a set of values that express the uncertainty of the prediction; this is called a sensitivity analysis: To observe whether the prediction is sensitive or not to variations in the initial conditions (in this case the viscosity at "t"); thus, in this case, values higher and lower than the seed viscosity already chosen will be chosen: Normally the standard deviation of the real viscosity is added and subtracted.

When we predict the speed at "t+1", we do so based on the time associated with each segment or interval. Therefore, there is a minimum prediction time, which depends on the number of nodes within each segment.

Notes

- As previously mentioned, selecting the viscosity to predict velocity at "t+1" can be quite challenging; in fact, the greatest difficulty arises when the viscosity curve exhibit very chaotic dynamics. However, we can also find a reasoning approach that provides an indication of the degree of uncertainty in the prediction even before selecting the point.
- It is easier to predict the speed or trend of the event, or even its value (based on viscosity), than to predict the actual value by analyzing its evolution in the original time series.

Long Term Prediction

As far as long-term prediction is concerned, it would be simple; in other words: Incorporate the value of the series (velocity) at "t+1" once the value of "u" at "t+1" has been calculated, as the new value of the velocity curve, repeating the whole process again as many times as desired. The problem is that the velocity is the multiplication of the variation of the series by the density; therefore, the prediction is the prediction of the trend of the series; if you want to calculate the value of the series in "t+1", it is necessary to calculate the density in "t+1", and for that, it is necessary to make a study of predictions based on previous values (backtesting) relating the variations of the predicted velocities and the real values; if it is for example a straight line, we will already have a mathematical relation able to know the value of the prediction in "t+1" knowing its velocity. Knowing the velocity and density, we will know the pressure; the viscosity is immediate as we have a new point to calculate it. The calculated relationship between the density and the variations of the predictions and the values will be specific to the events analyzed and to the model and parameters used. In other words, there is nothing to prevent this relationship from varying. Therefore, if the velocity is positive, the trend of the series is upward, and vice versa.

Viscosity as Absolute Criteria for Fluids Classification

A car placed in the queue produced by a red traffic light, will start its movement when the car in front of it has moved; that is: It will do it with a certain time delay "T_d". This delay also occurs when the price of oil changes due to the variation of the New York Stock Exchange index: It does not do so immediately. Viscosity is defined as: "Viscosity = μ = 1/T_d".

$$\mu = \frac{1}{T_d} \qquad (72)$$

We now calculate the reaction time between 2 particles in a fluid such as water to transmit a sound wave. Let the coordinates of the phase space of this event be: "C" is the velocity of sound in a fluid, "R" is the fluid constant, "x" is the average displacement of the particles (as a Brownian motion), "P" the pressure, "t" is the time, "Nm" the number of particles in 1 linear meter and "NA " is Avogadro's number:

$$T_d = \frac{1/C}{N_m} = \frac{1}{C\sqrt[3]{\frac{P}{RT}N_A}} = \frac{1}{C\sqrt[3]{\rho N_A}} \qquad (73)$$

The Einstein viscosity is ("D" is the diffusivity and "r" the average radius of the particles or molecules) [50]:

$$\mu_E = \frac{RT}{N_A}\frac{1}{6\pi Dr} / x^2 \triangleright D*t \qquad T_d = \sqrt[3]{\frac{\mu_E 6\pi Dr}{PC^3}} \qquad (74)$$

This value of "Td" is now a criterion for classifying fluids. The velocity of sound "C" (propagation of a pressure wave, shock wave or propagation of information) for each fluid depends on the variation of the pressure versus that of the density; in other words, the velocity of sound "C" (propagation of a pressure wave, shock wave or propagation of information) for each fluid depends on the variation of the pressure versus that of the density; in other words:

(75) $$C \triangleright \sqrt{\frac{\partial P}{\partial \rho}}$$

This value of "C", can correspond perfectly to "c" named above as the velocity of light. This expression is therefore equivalent to: The velocity of sound depends on the temperature "T". This is very important:

(76) $$C \triangleright T$$

In other words: The velocity of transmission of information depends on "T"; that is, on velocity, which becomes a vital tool for the economist.

From another point of view, let it be a fluid in which the particles are bound together by a spring with a constant "K" (Hooke's' law), where "x" is the displacement, "u" the velocity", "t" the time and "F" the force with which the spring is elongated:

(77) $$F = Kx = m\frac{u}{t} \quad \rightarrow \quad K = \frac{mC^2}{N_A \mu_G} \triangleright \frac{m}{\mu_G}$$

Viscosity is nothing more than a frictional force between the particles of an event or fluid. But what is Diffusivity "D" as a fluid property? It is the tendency of information to fade or blur. Sometimes, some authors define viscosity as "constant / D"; the larger the "T", the faster the fluid mixing in terms of diffusivity:

(78) $$D \triangleright \frac{T}{\mu r}$$

Comparing this with Einstein's relation for diffusivity ("K_B" is the Boltzmann constant), the great similarity between the two is perfectly clear:

(79) $$D = \frac{K_B}{6\pi}\frac{T}{\mu r}$$

Another excellent analogy to understand the crucial concept of viscosity is the backlash in a gear mechanism. Backlash is defined as the delay between the motion of one gear and another; in essence, it represents a distance and can be interpreted as 1/viscosity. This means that if the distance between the gear teeth is large, the viscosity will be lower. Excessive or significant backlash causes the physical system to operate improperly, making it something that should be avoided.

The viscosity of a fluid always depends on another fluid. For example, discussing the viscosity of water means quantifying its resistance within another medium, such as air. Later on, the viscosity of economic events is analyzed in absolute terms, meaning without considering the specific events in which their main dynamics unfold. This represents an absolute viscosity, assuming the existence of a fluid or a set of events that permeates the entire economy: In ancient astronomy, they assumed the existence of an 'aether'...

Transient Calculation

The so-called transient o transitory term corresponds to the derivative with respect to time "t"; in the 1-dimensional case, the variables with which we work are "t", "x" and "u"; it is therefore possible to substitute the variable "t" for any of the other 2, giving rise to another transient modeling and prediction; an example is given below:

(80) $$a = \frac{Du}{Dx} = \frac{\partial u}{\partial x} + u\frac{\partial u}{\partial t}$$

With this example, $\frac{\partial u}{\partial x}$ it is the new transient term.

Absolute Classification Criteria: Choice of Time Series

Pressure and Viscosity; Graphics and Comparative Values

Another 2 criteria to classify events, are those concerning the amount of "Pressure" and "Viscosity ABE"; these are criteria referring to or based on the capacity of influence that an event possesses; in terms of 1 single time series or economic event, "A" is more important if it possesses greater capacity to suck (ΔP<0) or push (ΔP>0) other events "B" (Pressure) or possesses greater capacity to drag other events (Viscosity); note: capacity is not equivalent to "A" actually doing so, as it depends on the degree of dependence with "B". Thus, it is possible to define the absolute mass of an event as:

$$\begin{matrix} average(P>0) & average(\mu>0) \\ average(P<0) & average(\mu<0) \end{matrix} \quad (81)$$

In addition to these parameters, there are others that quantify in another way, the suction or thrust power and the friction power of an event, such as the value of the standard deviation of pressure and viscosity, the sum (integral of the curve), the mean, the maximum or the minimum, etc. Some examples:

$$\begin{matrix} \sum(P>0) & \sum(\mu>0) & \sum|P| \\ \sum(P<0) & \sum(\mu<0) & \sum|\mu| \end{matrix} \quad (82)$$

Examples: Calculation of Comparison Graphics Between Pressure and Viscosity Capacities

Let be the set "E2" of events, defined above; over the events of this set, the following graphs are calculated:

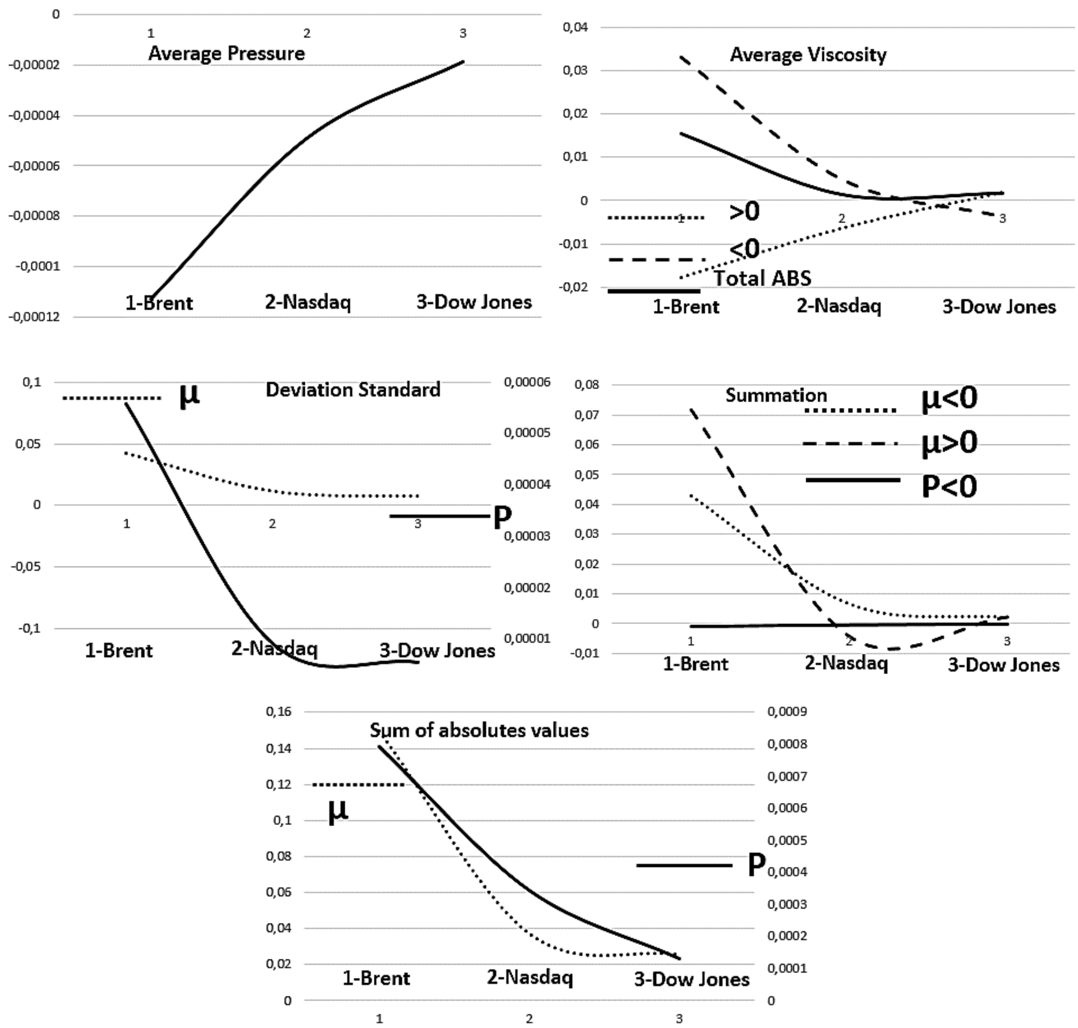


Figure 19: Comparisons of Mean, Sums, Standard Deviation of "P" and "μ"

Uncertainty in Prediction

IC(X)" is defined as the uncertainty index in the prediction of event "X". As we already know, starting from the seed viscosity, a series of values are calculated in order to appreciate the sensitivity of the result.

It is simple posteriori to know an uncertainty index, since it is sufficient to subtract the number of positive values "PP" minus the number of negative values "PN" in the predictions, and the higher it is, the less uncertainty we will have. It is assumed that the range of seed points has the same number of values above and below the central or main seed:

(83) $$IC(X) = |PP(X) - PN(X)|$$

The value of the prediction "PS" for the central seed, is the most important; therefore, if there are only 2 predictions with the same sign, there can be 2 possibilities: That "PS" is included in that pair or not; then the possible values are 3, 2-yc, 2-nc (these last 2 correspond to the inclusion of "PS" or non-inclusion); the lowest uncertainty will correspond to the value "3" and the highest "2-nc". The uncertainty may vary according to the number of sections into which the series has been divided and the number of elements; therefore, it is necessary to calculate the uncertainty by varying both quantities.

Note that this index classifies the series analyzed in the time period analyzed, not another; in other words, depending on the time period of analysis, the nodes perhaps, the seed points, etc, an event may have more or less uncertainty. This is logical, since it is reasonable to think that uncertainty is greater in the long term. The main reasons for working with this uncertainty index are several, since, from a strictly mathematical point of view, low uncertainty is essential, much more than accuracy:

- **Decision Making Under Risk:** In scenarios with important decisions at stake, such as investments or strategic planning, knowing the range of possible future outcomes (uncertainty) is crucial to assess risks and make informed decisions. An accurate forecast, but with high uncertainty leaves decision makers uninformed about the range of possible negative outcomes, which can lead to risky or suboptimal decisions.
- **Robustness to Unexpected Events:** Time series are often subject to unexpected events or changes in trends. A method with low uncertainty provides a wider prediction range that can better absorb these unexpected events, maintaining the reliability of the overall forecast. A highly accurate method, but without considering uncertainty can fail drastically when outlier events occur.
- **Effective Communication:** When communicating forecasts, specifying uncertainty along with the point forecast provides more complete information to users. This allows users to better interpret the forecast, understand potential risks and make sound decisions accordingly.
- **Continuous Model Improvement:** Quantifying uncertainty through prediction intervals or probabilistic error measures facilitates the evaluation of model performance. By analyzing the distribution of errors, biases or weaknesses in the model can be identified, allowing iterative adjustments and improvements to be made to reduce uncertainty in the long term.
- It would be very useful to define this concept as a percentage of accuracy or inaccuracy.
- As we mentioned earlier, the farther the range of possibilities is from "V0," the less uncertainty there will be when predicting velocity using the viscosity values at "t".

Example: Predictions and Uncertainties for an "np"

The predictions of the 2 events of the "E2" group are analyzed. 8 sections of 25 values each have been created: To calculate the velocity values at "t+1," we must select a viscosity at "t".

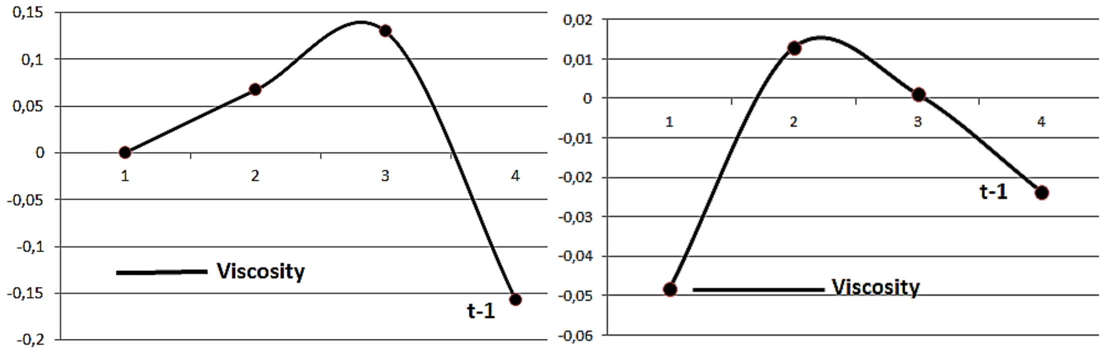


Figure 20

Graphs of viscosity in Brent and Dow Jones; it is necessary to appropriately select the viscosity value for "t".

We can choose, for example, that the viscosity value at "t" and at "t-j" are equal; in this case:

Table 9

	Viscosities in "t"		
	Seed 2	Seed 1	Seed 3
Brent	-0,26399	-0,15712	-0.005026
Dow Jones	-0,0473	-0,02376	-0.000224

	Sign of Velocity in "t+1"			
	Seed 2	Seed 1	Seed 3	I.C.
Brent	>0	>0	>0	3
Dow Jones	>0	>0	>0	3

Table 10

	Viscosity(t)-SD	Viscosity(t)	Viscosity(t)+SD	V0
Brent	-0.263991	-0.157128	-0.050265	0,075865
Dow Jones	-0.0473008	-0.0023762	-0.0002240	0,070757

Brent: Distance Viscosity(t)+SD to V0 = 0.1261
Dow Jones: Distance Viscosity(t)+SD to V0 = 0.071

Values of the viscosity ranges at "t" (initial value ± standard deviation (SD)), along with the difference between the highest value and the value "V0".

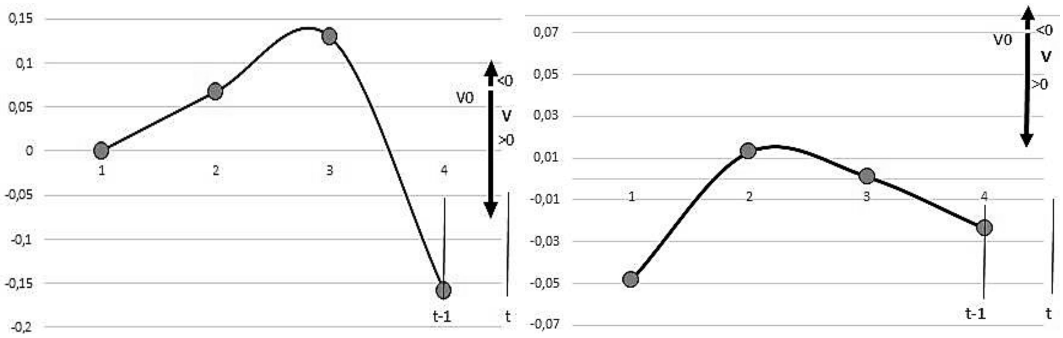


Figure 21 Graphs of Brent and Dow Jones, Indicating the Position of "V0"

The uncertainty of the 2 analyses is very low; this does not mean that the predictions are accurate: It simply means that the variation or sensitivity of the prediction to the viscosity is minimal. For these initial values, the uncertainty is low; however, it is important to emphasize once again that this choice is subject to significant subjectivity, despite the protocols described earlier and others yet to be defined.

Note: It is possible that an increase in uncertainty is caused by an "excessive" number or quantity of points in each interval; this results in a long-term prediction so.

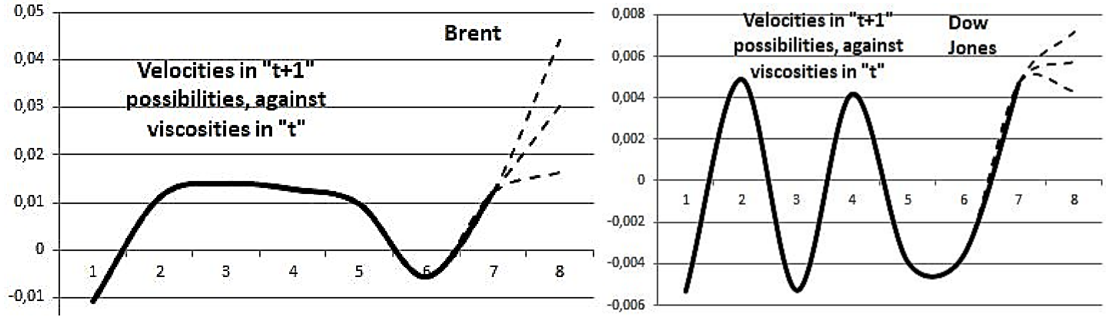


Figure 22: Predictions of the Speed for the 3 Viscosity Values in "t"

In these predictions, the central value is perhaps the most likely, as it results from the viscosity being repeated in "t-" and "t". The upper and lower results correspond to predictions derived from the viscosity in "t" by adding and subtracting the standard deviation. In the case of Brent, if the speed in "t+1" had been the upper value, it would not have followed the speed curve; it would be as if this value had provided a surprise. The same can be said about the central value, though not about the lower value, as it seems that the surprise would have been smaller. In the case of the Dow Jones, the surprise would come from the upper value rather than the other two values. This geometric and trend-based analysis effectively illustrates the validity or logic of the prediction.

Validation of the Trends Predicted in the Previous Example

Each period or interval corresponds to 25 days. We know the values of the series at "t+1" (from November 27, 2023, to January) as well as the values at "t"; therefore, we know the trends of the events. Consequently, we can determine whether the event predictions using the model are accurate or not.

Table 11

	"t"	"t+1"	Trend
Brent	84,396	77.9984	Down
Dow Jones	13520.458	14637.36	Up
Nasdaq	34003.5004	36836.0828	Up

The two results we obtained for velocity at "t+1" are positive; therefore, the prediction is accurate for the Dow Jones but not for Brent. From a geometric perspective, observing the viscosity graphs and the position of "V0," we note that for both events, the viscosity at "t" and its surroundings lie "comfortably" within an area corresponding to a positive velocity prediction; this reduces uncertainty.

Example: Calculation of the Relationship Between Density and Velocity in "t+1": Long

Term Prediction

We worked with the events of group "E2"; we calculated the variation between the predicted velocities and the variations between the values (the analysis was done for the same time interval as for the previous predictions):

Table 12

	Forecasting "u" Δ	Value Δ
Brent	-0.005590113	-6.3976
Dow Jones	-0.004587193	1116.906

The value of the so-called "Value Variation" is the variation of the data of the series. Therefore, a linear relationship is obtained between both columns; it would be necessary to perform more tests on other sections to be predicted, to perhaps obtain another relationship (quadratic, polynomial in general, etc). Therefore, when predicting the velocity, the sign is equivalent to the trend of the value, while its modulus or absolute quantity indicates how much it varies. This procedure must be repeated continuously, using backtesting, in order to increase, adapt or correct the accuracy of the procedure.

Uncertainty Zones or Intervals: Increase in Objectivity

The idea behind this procedure is based on selecting a viscosity at a known time "t"; this will allow the prediction of velocity to be determined directly with the least possible subjectivity. When viscosity is experimentally calculated, its value at time "t" is obtained; therefore, this value is used as the baseline from which the velocity at time "t+1" will be calculated. Additionally, a value for viscosity "V0" is also calculated, at which the velocity is "0". This helps determine whether the velocity increases or decreases above or below that value "V0". The application of this procedure enables the uncertainty of the prediction to be understood, provided we establish a set of rules to assess the chosen viscosity at time "t" as more or less likely.

Example: Modelling and Forecasting a Sample in Trading: SP/500 Index

The following is an example of transcendental importance for understanding the application of this method presented here; minute-by-minute data are available for the SP/500 index on May 13, 2024, from 8:00 a.m.; there are 200 values grouped in 8 sections of 25 values; the prediction is therefore made 25 minutes after the last value.

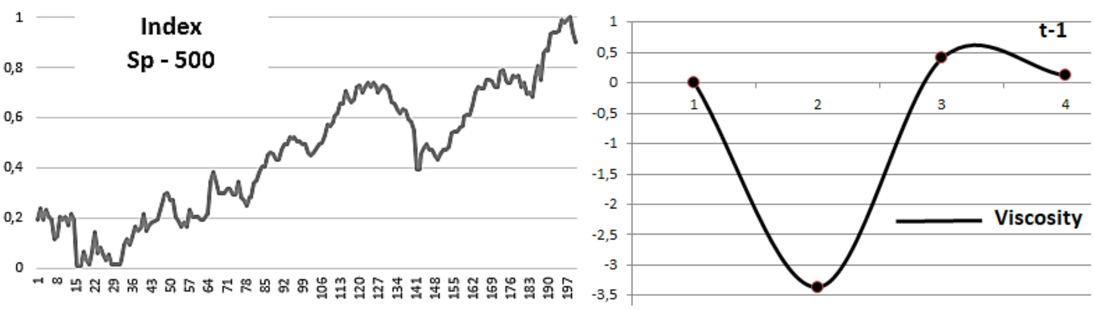


Figure 23: Data and Viscosity Graph

Calculating the values of the last analyzed leg and comparing it with the next one, we obtain that the SP-500 goes down: The value in the last leg is 522.46921 and the value in the next leg is 522.15701:

Seed Viscosities			
	Seed 2	Seed 1	Seed 3
SP-500	-0.054031	0.119519	0.29307

Sign of Velocity				
	Seed 2	Seed 1	Seed 3	I.C.
SP-500	>0	<0	<0	2-yc

Beyond the fact that the prediction is accurate, there is the fact that the method presented here is applicable to this type of time series: Modeling and prediction in real time in the short term. This makes it necessary to create software that acquires and processes data online and in real time, in order to be able to predict quickly. In this case, the election of seed viscosity, is also complicate (analyzing one only event….).

For this purpose, the Python language will be used, using, for example, the ScrapingWeb tool.

"Total" Mass - 2

"m2" is defined as the product of the classification criteria defined above: pressure, natural viscosity. Therefore, this mass corresponds to the influence capacity of an event. Another criterion could be "m1*m2". 7.30. Other Definitions, Such as Classification Criteria

Inertia

Another classification criterion between time series, which in turn becomes a definition, is the concept of inertia; it is a value that indicates the resistance of a time series to changes; therefore, it is closely related to the concept of natural viscosity of an event and to the concept of relative viscosity. Therefore, the inertia of an event "A" is defined as the product between the 1/viscosity and its importance "m":

$$I_A = \frac{m}{|\mu|} \tag{84}$$

Therefore, there are 2 inertias "I_1" and "I_2", using each of the 2 defined masses "m_1" and "m_2". This very important concept is also directly related to the concept of the force needed to modify an event, which will be detailed later: If the economic event has a lot of inertia, the harder it is to stop. As always: If the "ABE" viscosity is used ("m"=1), the inertia is interpreted as the natural inertia of the event; if the 'real' viscosity is used when analyzing the event together with other events, it is interpreted as a comparative inertia (with "m" between them). As a first approximation to the inertia, it is possible to take only the viscosity excluding the mass. This value gives a first and easy idea of the amount of inertia an economic event has.

Ability to Push or Suck

We have already defined above, a parameter that quantified the capacity to influence one event on another (pressure and viscosity); we can think as in the previous case of inertia and define this value as the mentioned capacities, but multiplying them by the mass:

$$CP = m * P \qquad C\mu = m * \mu \tag{85}$$

Force

We have developed many concepts and definitions with the objective of quantifying a state of economic dynamics; being able to do so implies that we are able to model it and know what its future evolution may be. All these concepts are very useful tools for any economic team of any country, since they allow them to vary those elements or events that they want, with the objective of reaching a certain global state. But we need a value that is able to quantify that intervention in the economy, whether it is very difficult or not; that is: We need a value to quantify the effort that must be made to modify an economic event. This value, we denote it as "Force = F": This essential concept is so because it is capable of quantifying the variation of movement from the mass and from the acceleration ("a"). We know that the quantity of motion "Pe" is the product of mass and velocity; if we derive the expression (1 dimension "x"):

$$Pe = mv \rightarrow \frac{\partial Pe}{\partial x} = \frac{\partial m}{\partial x} v + \frac{\partial v}{\partial x} m \qquad F = \frac{\partial Pe}{\partial x} = m * a \tag{86}$$

Therefore, we can know by applying the "ABE" model, what should be the modification of an economic event, to achieve certain equilibrium, for example; but it is absolutely essential, to know what should be the effort to achieve it: This value of "F", allows us to know it. Therefore, the effort depends on the importance "m" of the event (it indicates the inertia to the movement, therefore, the facility or resistance to it), as well as of the variation that is wanted to be applied. It is a concept of maximum importance. It is possible to have an absolute value if you work with a global "m", but it is much better to work with the force as a curve, because in reality, the mass and variations are not constant. Notice something important: The velocity of information transmission and the force "F" necessary to alter an economic event are inversely proportional:

The velocity at which information is transmitted between two economic events and their resistance to change are closely related, and this relationship can have significant implications in market dynamics. How are these concepts related? A higher transmission velocity implies 2 things:

- **Less Resistance to Change:** When information circulates rapidly between two economic events, market reactions are more immediate. This means that prices, interest rates and other economic variables can adjust quickly to new information, which in turn can generate greater volatility.
- **Greater Market Efficiency:** A high velocity of information transmission is usually associated with more efficient markets, where prices more accurately reflect the information available. However, this efficiency may be accompanied by greater instability.

A lower transmission velocity implies 2 things:

- **Greater Resistance to Change:** If information takes longer to spread, markets may take longer to adjust to new developments. This can result in lower volatility in the short term, but can also generate imbalances and bubbles in the long term.
- **Lower Market Efficiency:** A low velocity of information transmission may limit market efficiency, as prices may not fully reflect the available information.

The factors that influence this relationship are varied:

- **Technology:** The development of information technologies has significantly accelerated the transmission of data, reducing resistance to change in the financial markets.
- **Market structure:** More fragmented or less liquid markets may have a lower velocity of information transmission and, therefore, greater resistance to change.
- **Nature of Events:** Unexpected or large events can generate greater volatility, regardless of the velocity of information transmission.
- **Monetary policy:** Central bank decisions can influence the velocity at which the market adjusts to new developments.

Understanding this relationship is essential to:

- **Investors:** Allows them to make more informed investment decisions and better evaluate the risks associated with different assets.
- **Political:** Helps to design more effective economic policies and to anticipate market reactions to changes in economic conditions.
- **Companies:** Enables companies to better manage their financial risk and adapt to changing market conditions.

Note

- There is therefore a relationship between viscosity and velocity variation or acceleration. The strength or force of an event can be understood, from an aerodynamic perspective, as a drag force.
- Acceleration is the variation of velocity. Therefore, if the change in velocity is small, it means that the event is highly susceptible to being modified, since the force has a very small value.

Example: Inertia Calculation for 3 Events

We work on 2 events from the "E2" group: Price of a barrel of Brent Oil and Dow Jones Index; we calculate their force curves. We calculate the inertia for the two events taking "m=1" in the last 5 intervals:

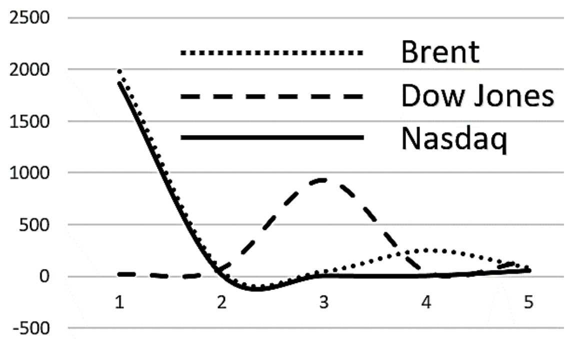


Figure 24: Inertia Absolute Curves of 3 Events

Example: Calculation of the Force for 3 Events Analyzed Isolated

We work on 2 events from the "E2" group: Price of a barrel of Brent Oil and Dow Jones Index; we calculate their force curves absolutes in 6 last sections:

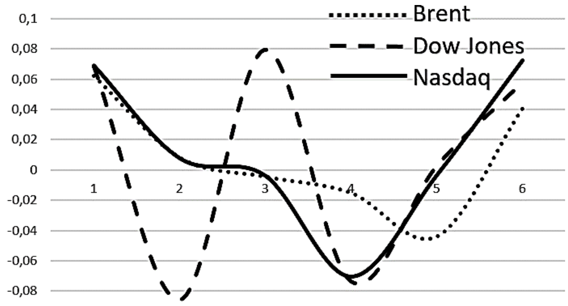


Figure 25: Curves of the Force of 3 Events

Therefore, the force necessary to "move" the price of Brent is sometimes, greater than that necessary to move the Dow Jones. The question of which is more difficult for a government, varying the price of Brent or the Dow Jones index, may seem incorrect. Neither the Brent price nor the Dow Jones index are instruments that a government can directly manipulate in a meaningful and sustained way over time. Meanwhile, Brent is almost always costlier to move than the Nasdaq. The Nasdaq Index and Brent prices have different dynamics, which affect how easily they can be influenced: Nasdaq Index: As a stock market index composed of multiple technology companies, its behaviour is influenced by a wide range of economic, political, and business factors. Significantly altering it requires large-scale events, such as changes in economic policies, major movements in the tech sector, or financial crises. This makes it relatively difficult to manipulate in the short term. Brent Prices: Although also influenced by global factors such as oil supply and demand, OPEC decisions, and geopolitical tensions, Brent can be more sensitive to specific short-term events, such as production disruptions or changes in inventory levels. These sensitivities make it, in certain contexts, easier to influence than a large and diversified stock index like the Nasdaq.

- **Why can't governments directly manipulate these indexes?**
- **Brent:** The Brent price is an indicator of the crude oil futures market. It is determined by global supply and demand, influenced by factors such as OPEC countries' production, industrial demand, geopolitical tensions, environmental regulations and investor expectations. An individual government, even a major oil producer, has limited power to dramatically and sustainably alter this price.
- **Dow Jones:** The Dow Jones index is an average of the stock prices of 30 major US companies. Its value reflects investor confidence in the US economy and in these companies in particular. It is influenced by a multitude of factors, including corporate earnings, interest rates, monetary policies, geopolitical events and general market sentiment. A government, while it can influence the economy through fiscal and monetary policies, cannot directly control the behaviour of individual investors that determine the value of the index.

Governments cannot directly manipulate these indexes; they can implement policies that influence the factors underlying them:

Brent Price

- **Energy Policy:** regulate domestic oil production, tax production or consumption, invest in renewable energy.
- **Foreign Policy:** Influence international relations with other oil producers, participate in international agreements to stabilize prices.

Dow Jones Index

- **Fiscal Policy:** Adjust corporate taxes, implement fiscal stimulus to boost the economy. - Monetary policy: Influence interest rates through the central bank to encourage economic growth.
- **Regulation:** Implement regulations that affect the companies included in the index.

The price of Brent and the Dow Jones index are the result of complex, global market forces beyond the direct control of any single government. Governments can influence these indexes through broader policies, but cannot manipulate them directly.

We can describe other methods to quantify the modification of the dynamics of an economic event:

- **Sensitivity Analysis:** This allows us to assess how the outcome of an economic model varies in the face of changes in different variables.
- **Resilience Indicators:** Measure the capacity of an economic system to absorb shocks and recover.
- **Predictable vs. Unpredictable Events:** Events with greater uncertainty are more difficult to modify.
- **And of Course:** Applying the "ABE" model to observe and analyses changes.

Instabilities

We can generate series in which the prediction announces an extremely large variation; in other words: We can develop a method or conditions that a series must fulfill, so that the prediction results in a large instability; this would mean that the series itself predicts an instability. Therefore, this generation of series that provide a highly unstable prediction is of vital importance since, in this way, it is possible to "learn" from them to know what conditions a series must have (what information it must provide) for a crack to occur; in addition, knowing this possibility, we anticipate the crack, being able to minimize its consequences or modify them. It has been said from the beginning that this model does not take into account "unexpected" instabilities such as a meteorite; but there are others, such as human interventions, which are not taken into account either, but which can perhaps be predicted; that is: If, for example, Banco de Santander makes a brutal capital increase or sells part of its structure or anything that involves a big change, obviously, such acts will affect its dynamics; but those actions may be due to a series of "prior" conditioning factors to the action; therefore, it is possible to analyze such conditions and learn from them.

Problems to Be Solve; Choice of Events – 1

The problem we will try to solve now corresponds to the modeling and prediction of a group of "n" economic events, expressed by means of a group of "n" time series. The optimal choice of the series is essential for a good and simple modeling and, of course, for a prediction with the least possible uncertainty; for this purpose, and as we have already seen, it is absolutely essential to choose series under a series of premises:

- Use series with as much information or absolute mass as possible.
- They should be as dependent as possible or, in other words: With the highest mass and relative viscosity between them (defined and analyzed below); the maintenance of series that are not very dependent (useless) reduces the information used and, therefore, increases the uncertainty in the prediction.

Relative Mass

Definition Information by Sections and Global

The absolute and relative importance curve of an event, in terms of the amount of information and the capacity to affect or influence other events, is extraordinary and decisive. But how do we calculate the relative importance or, in other words, the influence of one event on another?

Given 2 dependent events "X" and "Y", there is a way to quantify the dependence between them in both directions; this method is the calculation of the so-called information "I"; the information of "X" on "Y", that is, how "X" influences "Y" ("I(X,Y)"), is calculated by means of the expression ("H" is the entropy):

$$I(X,Y) = -\sum_X \sum_Y p(x,y) \log_2\left(\frac{p_{X,Y}(x,y)}{p_X(x)\, p_Y(y)}\right) = H(X) + H(Y) - H(X,Y) \quad (87)$$

$$I(X,X) = ENTROPY(X) \quad (88)$$

This expression is denoted as the mass of "X" in "Y"; in other words: the importance of "X" in "Y" (also called relative mass or cross-mass between "X" and "Y"); the more information, the more dependence:

$$I(X,Y) = m_{X,Y} \quad (89)$$

In this way we obtain a general value and a curve (section by section) for the comparison between both series. The idea of being able to obtain an influence or dependence curve lies in the fact that this influence is not constant; that is: there are variations in "X" which, depending on their intensity, affect the "Y" series more or less. On the other hand, we will define the global information or global information dependence index "IGI", as the sum of the information by sections divided by their number of nodes.

$$IGI = \frac{\sum INFORMATIONS}{INTERVALS\_QUANTITY} \quad (90)$$

When calculating mutual information, we must do it with:

- Different numbers of sections (and therefore nodes), to minimize the possible errors at the ends of the sections, calculating the average of the results.
- Dividing the result by the number of nodes or elements of the section.

The main objective is to establish a quantification of the relationships between events to classify them and obtain the relative weights; this can also be done using the so-called distance coefficient. Mutual information and the distance correlation coefficient (Distance Correlation, or dCor) are statistical measures that evaluate relationships between two variables, but they approach this task from different perspectives. Let's see how they are related:

- Mutual Information: Mutual information (I(X;Y)) is an information theory measure that quantifies the amount of information shared between two variables X and Y.
- It is nonlinear: it captures both linear and nonlinear relationships between variables.
- It is based on joint and marginal distributions: it measures how much the uncertainty about X is reduced by knowing Y (and vice versa). o It is measured in bits (or nats, depending on the logarithmic base).

For example:

- If I(X;Y)=0, X and Y are independent.
- High values indicate stronger dependence.
- Distance Correlation Coefficient: The distance correlation coefficient (dCord) measures the dependence between two variables, accounting for all types of relationships (linear and nonlinear).

It is based on distances between pairs of observations: it uses distance matrices to compute relationships between the geometric configurations of X and Y. o It ranges from 0 to 1:

- dCor=0: independence.
- dCor>0: existence of some dependence.

It is particularly useful in relationships where Pearson correlation is zero, but the variables are still related (e.g., circular or quadratic patterns).

Relationship Between Mutual Information and dCord

Both measures are conceptually related as they evaluate dependence, but their approaches and computations differ:

General Nature

Both can detect nonlinear relationships, but mutual information focuses on probability distributions, while dCord operates in a distance space. -Statistical Independence:

- When I(X;Y)=0, dCor(X,Y) will also be 0, as both indicate independence.
- If I(X;Y)>0, dCor(X,Y) will also be greater than 0, but there is no direct quantitative correspondence between their values.

Key Differences

- Mutual information is more robust when relationships depend on the shape of the distributions (e.g., highly asymmetric relationships).
- dCord is better suited for detecting complex geometric relationships in the data.

- Practical Connection in Economic Events: In economic events, these measures can be useful for:
- Detecting linear and nonlinear correlations between variables such as inflation, GDP, or interest rates.

Being Complementary

- If I(X;Y) indicates strong dependence but dCord is low, it could suggest a relationship that depends more on distributions than on geometric configurations.
- If both are high, they indicate a robust relationship in both probabilistic and geometric terms.

Therefore, both measures (information and distance) can capture dependence in complementary ways, and their combination could provide richer insights into the interactions between economic events. It is even possible to combine them to derive relative weights, "assisting or helping" each other in the process. Another correlation coefficient that works similarly to the distance-based ones is the so-called Xi coefficient.

Examples: Calculation Correlation Coefficient Xi

We work with the events from the "E2" group, calculating their relative correlation coefficients with normalization "N2".

Table 14

CORRELATION COEFFICIENT Xi – BY INTERVALS		
	Brent / Dow Jones	Dow Jones / Brent
T1	0.3481	0.4874
T2	0.0636	-0.1379
T3	0.2281	0.1945
T4	0.3482	0.3337

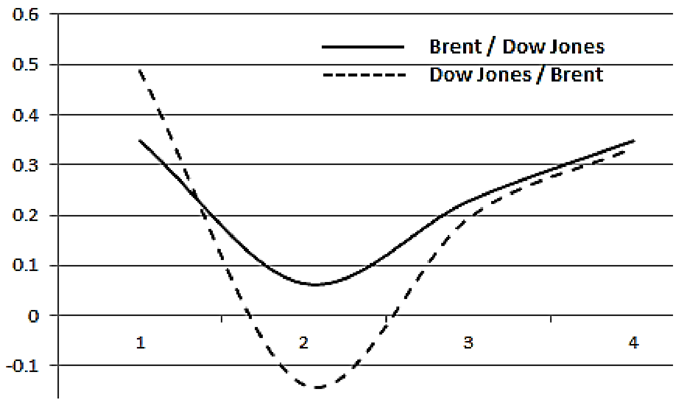


Figure 26

Curve of Correlation Coefficient "Xi" by intervals (4 intervals). As always, it´s possible create more intervals to create the curve, for example. This value and/or curve, is a classification criterion for series; also the average, integral, addition, etc…. The correlation coefficient "Xi" for full series are: Brent / Dow Jones = 0.027, Dow Jones / Brent = 0.0185. These values work as a global index or dependence. The Xi correlation coefficient becomes a complementary value to the mutual information coefficient we have just discussed, with the goal of providing a classification of all events based on their dependencies.

Examples

Calculation of Relative Masses

Let there be 3 events corresponding to group "E2": Price of a barrel of Brent Oil, Nasdaq Index and Dow Jones Index; we calculate by sections, the mutual information between Brent and each of the other 2 events and the so-called "global dependency index" (Python), in order to observe and classify the importance of Brent with respect to the 2 indexes; in this case we use 4 sections of 50 elements and 3 sections of 67 elements. With the 2 curves generated, their average is calculated to obtain the final curve; for this, it is necessary to scale the abscissa axis, being the values "1st" and "3st" of the second curve, the values "1st" and "4st" of the first one (therefore, the value "2st" of the second one, is equivalent the value "2.5st" of the first one; a polynomial of degree 2 is used to approximate both curves (parabolas)). The normalization

"N2" has been used for each series:

Table 15

BRENT / NASDAQ			
T1	T2	T3	T4
3.0028	2.7350	2.4013	2.7910
T1	T2		T3
2.7858	2.6152		2.6359
AVERAGE 2 CURVES			
T1	T2	T3	T4
2.8943	2.6978	2.5009	2.7135
GLOBAL INDEX DEPENDENCE			
2.7016			

BRENT / DOW JONES			
T1	T2	T3	T4
3.0028	2.7350	2.4013	2.7910
T1	T2		T3
2.7858	2.6152		2.6359
AVERAGE 2 CURVES			
T1	T2	T3	T4
3.0422	2.6091	2.6257	2.8096
GLOBAL INDEX DEPENDENCE → 2.7717			

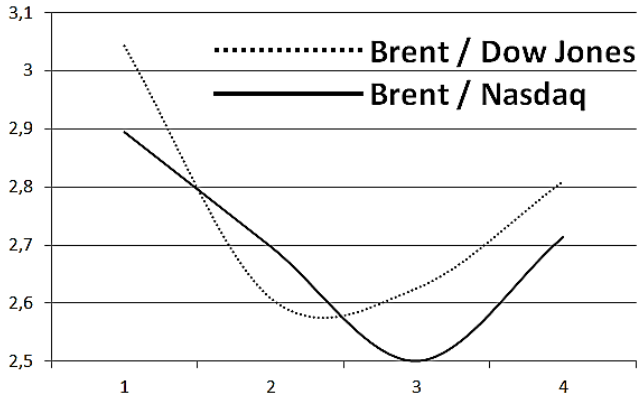


Figure 27

Final (average) curves of mutual information; comparison of the influence of Brent on the Nasdaq and on the Dow Jones. Important: Without introduce the "Xi" coefficient.

Reflections

The influence of Brent on the Dow Jones is greater than on the Nasdaq (only 2.6% more). It must be taken into account that this "general" value implies that the importance is greater in the time interval analyzed.

• Despite the above reflection, it should be noted that there are time intervals in which the importance or influence of Brent on the other two events alternates: sometimes it is greater and sometimes it is less. It is in interval "2" where the influence of Brent on the Nasdaq is greater than on the Dow Jones.

Definition 2 – Transfer Function as a Dependency Criterion

As we already know, the importance of a series "X" on a series "Y", can be defined as the influence of "X" on "Y" to some variation of "X"; in other words: How "Y" (output) responds (when and how much) to an instability of "X" (input); therefore, we can use the socalled Transfer Function as follows: Firstly, calculate the "TF" between "X" and "Y"; secondly, modify the input and apply the calculated function on "X" to observe the variation of "Y"; in other words: we measure how much and when "Y" varies before a variation of "X"; to quantify the variation of "Y", it is sufficient to calculate the subtraction between "Y" and the "Y" modified by the variation of "X" [12-16].

This alternative method to the previous one (definition 1) is necessary, since the calculation of the mutual information by means of entropies does not ensure causality; the complementarity of both methods increases the efficiency of the calculation of the mutual influence. Let "VI" be the average variation of the percentage change of the input and "VO" be the average variation of the percentage change of the output. The quotient of both parameters will represent a "TFI" classification criterion in terms of influence or dependence.

$$TFI = \frac{VI}{VO} \quad (91)$$

It is also possible to apply this procedure to sections, but, on the other hand, it is always convenient to apply it to small sections (to ensure the imposed linearity), but this has the disadvantage of not being able to capture in detail the relationship sought. It will therefore be applied by sections, if the number of elements in each section is representative and adequate. On the other hand, it is necessary to analyze the possibility of using the "TF" with "n" inputs on a single output.

Relative Viscosity

Initial Outline

To quantify the dependence between 2 series "X" and "Y", we calculate the relative viscosity between two events. This viscosity is called relative viscosity between "X" and "Y": "µXY". Viscosity between "X" and "Y" is defined as "1/delay", as we have already seen.

As we already know, it is convenient to calculate the viscosity in each section of the analyzed time series; but it is complicated because the end or beginning of a section may coincide with a repetition with delay, of both series; it is something necessary to automate and it is not simple. If 2 series are chosen for example in which no delay is appreciated, perhaps .... is a problem of the periodicity in the taking of measurements; the other reason is that perhaps, one of them is a linear combination of the other or simply that the series are poorly chosen; or simply that an event is not the cause of the other. For example, with data from 1900 to 2010 on cigarette sales and mortality due to cancer, we obtain a maximum for each time series located in 1960 and 1985, respectively; in other words: A delay of 25 years; this means that, working on these 2 series, we can only detect this delay in long periods of analysis. The delay between 2 series is also related to mutual information, in the sense that it is possible that, as mentioned above, depending on the intensity of a variation of "X", "Y" is affected or not; even if it is in a state of unstable equilibrium (also called in Fluid Mechanics and Aerodynamics, Stagnation Point51), it is possible that with a small variation of "X", "Y" varies intensely and can change the previous tendency to instability. It is necessary to calculate the mutual viscosity over various numbers of sections (and therefore nodes) to eliminate possible problems at the ends of the sections, as was done with the calculation of the relative masses.

Reflections on Viscosity in the Economy and Analogies

Viscosity in economics refers to the extent to which the prices of goods and services change in response to changes in demand or supply. An economy with high viscosity has slowly changing prices, while an economy with low viscosity has rapidly changing prices. This is sometimes identified with inertia. Viscosity in economics refers to the degree of resistance of a market to external changes. Like viscosity in physics, viscosity in economics measures the ability of a market to resist or adapt to changes in external economic conditions, such as changes in prices, supply, demand, government regulation, among others. In general, viscosity is used to describe the rigidity or flexibility of prices in an economy. An economy with high viscosity may have more stable and predictable prices, but it may also be slower to adapt to changes in supply and demand. A market with high viscosity is one that is highly resistant to external changes and is slower to adapt to new economic conditions.

In contrast, a market with low viscosity is characterized by being more dynamic and adaptable to external changes. Viscosity, therefore, is an ideal tool for any economist. Viscosity can have important implications for investors and economic agents seeking to make decisions based on market conditions. For example, in a highly viscous market, it may be more difficult for investors to take advantage of investment opportunities and make profits quickly, while in a less viscous market it may be easier and faster to make successful investment decisions. The reason why, in economics in particular, there are events that have non-constant viscosities among themselves is that one event may "react," or even fail to react, to the movement of another event; the velocity of reaction therefore depends on the magnitude of the change and perhaps other factors. Therefore, we will define viscosity in Economics, between 2 series, as the "delay" curve between them. The economy behaves as a non-Newtonian fluid; that is: Viscosity depends on velocity; just this is the reason for the necessity of the calculation of viscosity as a curve. Viscosity is one of the things that can be changed to change the dynamics; it is possible to facilitate the grouping of people to allow certain changes in the economy (we can observe this fact in groupings of seeds, which, depending on their friction, form some figures or others).

Like all the definitions made in this work, viscosity is a great tool to "control" the dynamics of the economy; for example, making it easier for people to come together in groups (hobbies, work, interests, etc...) is a tool that controls viscosity. To foster groups, associations, and organizations within a country, a government can implement several measures that promote cooperation among individuals and sectors of society. These measures can not only improve the economic dynamic but also strengthen the social fabric, increase citizen participation, and enhance resilience in the face of economic crises. Here are some key measures a government could take:

Fiscal and Financial Incentives

- **Grants and support funds:** The government can offer grants or funding to groups, associations, and cooperatives to help them start or expand. These funds may be intended to cover initial costs, training, or innovation.
- **Tax exemptions:** Offering tax exemptions or reductions to nonprofit organizations or associations that have a positive impact on the economy or society (e.g., associations that create jobs or local development projects).

Facilitation of Formalization and Registration

- **Administrative simplification:** Reducing bureaucracy to make it easier to create, register, and operate an organization, cooperative, or association. This includes streamlining the process of registration, licensing, and legal procedures.
- **Support in the creation of statutes and regulations:** Advising new groups to help them establish their structure and internal rules in a way that aligns with the country's legal framework.

Education and Training

- **Business Training Programs:** The government can fund or promote educational programs for leaders of organizations, focused on business skills, resource management, accounting, marketing, and leadership.
- **Promoting Education on Teamwork and Cooperation:** Developing educational campaigns that highlight the benefits of working in organized and cooperative groups, both personally and in business.

Infrastructure and Collaborative Spaces

- **Innovation centers and coworking spaces:** Creating physical or virtual spaces where associations, entrepreneurs, and small businesses can collaborate, share resources, and ideas. This fosters creativity and knowledge exchange.
- **Access to technology:** Facilitating access to advanced technologies for groups to improve their productivity and competitiveness in the global market.

Support Networks and Alliances

- **Creating collaborative networks between sectors:** Promoting partnerships between associations, private companies, NGOs, and the public sector so they can work together on joint projects that generate social and economic benefits.
- **Encouraging volunteering and citizen participation:** Incentivizing people to actively engage in community associations, which can, in turn, strengthen social capital and create an environment conducive to economic development.

Regulation that Favors Cooperation

Legislation on Cooperatives and Associations: Developing laws that favor the operation of cooperatives and associations over competitive companies, such as a legal framework that facilitates the fair distribution of profits and responsibilities.

- **Legal and Labor Protection:** Ensuring that the members of groups have adequate labor protection, with social security measures, which benefits both workers and organizations.

How can this improve the economic dynamic? Groups and associations can have a positive impact on the economy in several ways:

- **Job Creation:** Cooperatives and associations can generate jobs in key sectors, especially in areas where the labor market is weak. By working cooperatively, it is easier to create quality, inclusive, and sustainable jobs.
- **Better Resource Utilization:** Collaborative organizations allow for resource sharing (such as knowledge, technology, or infrastructure), which can reduce costs and increase efficiency.

This is especially important for small businesses or entrepreneurs.

- **Development of Local Markets:** Associations can promote the local economy by helping create more sustainable and resilient markets. Through networks of cooperatives and associations, small businesses can access markets that would otherwise be inaccessible.

Innovation and Competitiveness: Collaboration between different organizations fosters innovation as it allows the exchange of ideas, technologies, and best practices. More innovative economies tend to be more competitive in the global market.

- **Strengthening social capital:** When people come together in associations or cooperatives, social capital (trust, a sense of community, and cooperation) is strengthened, which can have positive effects on long-term economic development.

Important: What can an economic team do to alter viscosity, beyond what was explained earlier?

The grouping of various elements, such as seeds or leaves in the field, is primarily caused by the complexity of the leaf geometry (increased friction between them) but also by the environment. This means that they group together when one stops, or when they encounter a tree or a discontinuity in the terrain, etc. Therefore, an appropriate environment and conditions not only facilitate the groupings but also their structure, speed, etc.

Threshold Effect in Economics

Just as a wind turbine requires a minimum wind speed to start rotating, certain economic events or indicators must reach a specific magnitude to influence others. For this reason, two events may appear to have no viscosity, but this is only seemingly so because one of them requires a minimum magnitude to alter the dynamics of the other.

The viscosity also, can be zero: The analogy with a wind turbine can be applied to certain types of economic relationships, and it makes sense to think in terms of thresholds or minimum magnitudes for one economic event to influence another. In economics, this phenomenon is often described in terms of nonlinear elasticities or threshold effects.

Two pendulums of the same length oscillating in a room, even if they are not in the same phase, will tend to oscillate in the same phase over a long enough period of time. What is the reason? The reason is the presence of air; the air transmits pressure waves and causes the pendulums to vary their speed until they swing in unison. If this same experiment were conducted on the Moon, they would never oscillate in the same phase. In economics, as we've already mentioned, events tend to reach equilibrium naturally (just like relative viscosities); this equilibrium is caused by the presence of air; that is, by the environment in which the economy generally operates.

Examples

- **Interest rate and consumption:** Small increases in the interest rate may have no visible impact on consumption, but if the rate surpasses a certain threshold, consumers may begin to significantly reduce their spending.
- **Investment and economic growth:** A small amount of investment in infrastructure might not have an immediate effect, but exceeding a critical level could trigger substantial improvements in productivity and growth.

Nonlinear Relationships:

Many economic relationships are not linear; that is, the impact of a change is not proportional to the magnitude of the change. This can be explained by:

- **Cumulative effects:** The impact of an event becomes evident only after it reach a certain scale.
- **Structural constraints:** Some systems have internal limits that must be overcome to respond.
- **Example:** In the labor market, a small increase in job demand may not affect wages if there is sufficient unemployed labor, but when unemployment falls below a certain level, wages can start rising rapidly.

Practical Applications:

- **Economic Policy:** Policymakers must identify these thresholds to design effective measures. For instance, determining a minimum level of public investment necessary to stimulate the economy.
- **Economic Models:** Incorporating these thresholds into econometric models enhances their predictive capacity and provides better insights into phenomena such as recessions, economic bubbles, or tipping points.

Limitations of the Analogy

- **Complexity of Causality:** In economics, relationships between variables are often multidimensional and influenced by numerous external factors, whereas a wind turbine primarily responds to a single parameter: wind speed.
- **Time Delays:** In a wind turbine, the effect (rotation) is immediate, but in economics, the impact of surpassing a threshold can take significant time to manifest.

The idea of a minimum magnitude required for one economic event to affect another is valid and supported by economic theories. This approach helps identify tipping points in complex systems, improving our understanding of how and when significant changes occur in economic dynamics. However, the complexity and multicausality of economics demand additional analytical tools to accurately model these relationships. Determining the magnitude of an event required to influence another's dynamics is a necessary component to model and integrate into the "ABE" framework. For this, we can define a function of the viscosity such that "k" is a value that controls the slope of the transition between the active and inactive states, and "μcrit" is the critical transition viscosity:

$$\eta(\mu) = \frac{1}{1 + e^{-k(\mu - \mu_{\text{crit}})}} \quad (92)$$

Viscosity Calculation Methods

Introduction

There are several methods that can be used; they will be described below, by means of some practical examples. To do so, we will work on 2 series corresponding to the price of a barrel of Brent oil and the Dow Jones index of the "E2" event group:

Determining whether the influence of the Brent price on the Dow Jones is greater than the influence of the Dow Jones on Brent depends on the specific context and the factors driving the movements of each. Both directions of influence are explored below:

Influence of Brent on the Dow Jones

- **Energy Costs:** The price of Brent directly affects many Dow Jones companies, especially those that are large consumers of energy. An increase in Brent prices can increase operating costs and reduce profits, negatively affecting the value of these companies and, therefore, the Dow Jones.
- **Inflation and Monetary Policies:** An increase in oil prices may lead to higher inflation, which in turn could influence the Federal Reserve's policies (such as raising interest rates), negatively affecting equity markets, including the Dow Jones.

Influence of the Dow Jones on Brent

- **Global Economic Indicator:** The Dow Jones is often seen as a barometer of global economic health, especially the U.S. economy. If the Dow Jones rises, it could indicate a growing economy, which in turn could increase demand for oil, driving Brent prices higher.
- **Market Sentiment:** The performance of the Dow Jones can influence overall market sentiment. If investors are confident that the economy is strong (reflected in a bullish Dow Jones), they may anticipate an increase in oil demand, which could push Brent prices higher.

Comparison of Influence

- **Brent on the Dow Jones:** Brent's influence on the Dow Jones tends to be more direct and specific. Brent's impact on business costs, inflation and monetary policy has a clear connection to the performance of individual stocks within the Dow Jones.
- **Dow Jones on Brent:** The Dow Jones influence on Brent is more indirect and reflects a broader view of the global economy and energy demand. The Dow Jones can influence expectations about future oil demand, but has no immediate impact on crude oil supply and demand, which are the main drivers of the Brent price.

Conclusion

In general, the influence of Brent on the Dow Jones is more immediate and direct due to how oil prices affect operating costs, inflation and economic policies, which in turn affect stock prices. On the other hand, while the Dow Jones can influence oil demand expectations, its impact on Brent is more diffuse and depends on a variety of additional factors. Therefore, the influence of Brent on Dow Jones is arguably greater than the influence of Dow Jones on Brent.

In summary, therefore, the answer to the question of who has more influence logically depends on the specific context and the factors that are dominating the market at any given time:

- **Long-Term:** The price of oil tends to have a greater influence on the Dow Jones, as energy costs are a fundamental factor for the global economy.
- **Short-Term:** The relationship can be more volatile and depend on a combination of factors, including investor expectations and geopolitical events.

The calculation of the viscosity between the two events attempts to resolve this question.

Method of Cross Correlation

It is possible to calculate delays by calculating "cross-correlations"; i.e.: How much a series has to be shifted for the correlation between the two to be maximum.

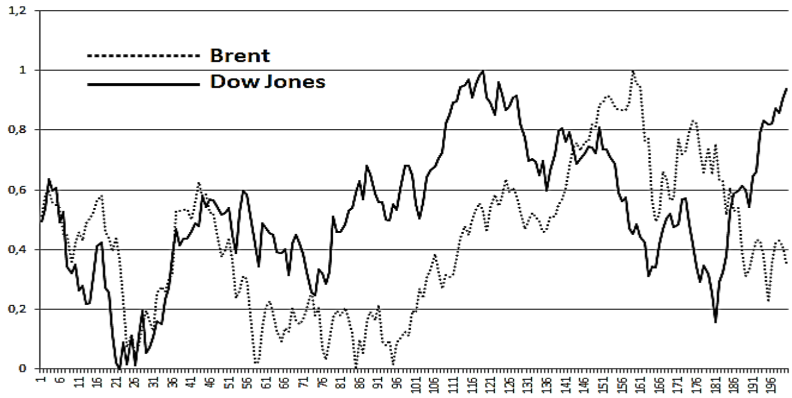


Figure 28: Curves of the 2 Series Analyzed

The normalized cross-correlations are analyzed in the last 5 sections of 25 elements each; for this purpose, an average has been made between the values corresponding to sections containing 25 elements and sections containing 20 elements; in this way, it is obtained:

Each of the generated curves is observed and the "delay" or "gap" (local maximum or minimum) closest to "0" is chosen; in this way, the following curve for viscosity is obtained:

Important: To avoid an infinite value (1/delay) in the case of 1/0, a section with positive viscosity followed by a section with positive viscosity (or vice versa), implies that the curve has to pass through "0". Another way to avoid this infinity is to calculate viscosity curves with sections of different quantities of nodes.

This correlation method attempts to "match" the curves depending on the displacement, but there could be peaks in both the input and the output without the need for both graphs to be similar. In other words, the peaks would repeat but not the dynamics; this makes the method complicated and confusing to apply.

Method Applying the "ABE" Model

It is even possible, as we will see when creating the 2D "ABE" model, to calculate the relative viscosities between 2 events, by clearing them from the model applied to both events.

Geometric Method

There are series with constant viscosity and others with variable viscosity (Non-Newtonian fluid); the casuistry is tremendously high. We can measure the relative delay, in each zone or section.

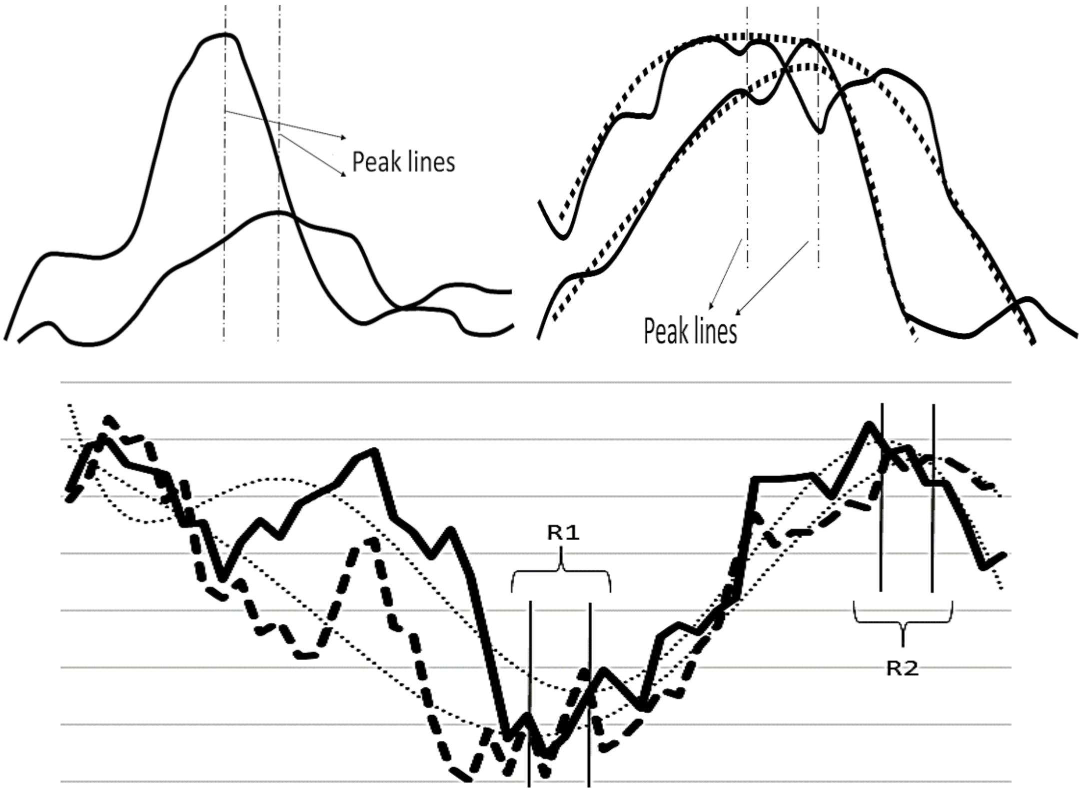


Figure 29

First Image: Delay measurement in 2 'obvious' zones and easy to detect (2 central peaks).
Second Image: It is possible to interpolate a curve (with dots) to each of the black curves to obtain peaks and to be able to measure their distances more easily.
Third Image: Interpolation curves and measuring differences: It can be positive and change to negative.

For the other delay values, it is necessary to automate the process and therefore use a well-defined action protocol.

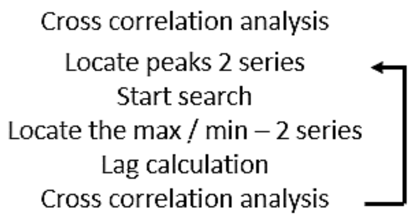


Figure 30: Protocol Generated for the Automated Calculation of the Delay Curve.

Important: The calculation of viscosity is mainly based on the delay or advance of peaks between the 2 series analyzed; therefore, it is essential that the existence of peaks is really due to the interaction between both signals; this is the reason why it is necessary to treat the signals to clean them of contaminating signals; that is: To be able to work with the purest possible signals; remember the use of the "FFT", for example.

Note: The peaks should be peaks of the smoothed curves. This makes the method more accurate, along with an analysis by segments of varying lengths in detail.
Note: o detect and quantify peaks and differences between peaks, we can work with the following two series:
• The initial series (without nodes).
• The series composed of the nodes.
A well-balanced combination of both series will lead to a more accurate analysis and calculation of viscosity.

Viscosity by FFT

Another possibility exists: it is a variation of the previously explained geometric method. This is applied by obtaining the "FFT" (Fast Fourier Transform) to "smooth" the initial time series, extracting its most important harmonics and replacing the original series with the sum of these harmonics. In this way, a "new" time series is generated, preserving its key peaks, troughs, and general variations. These variations are useful for the geometric calculation of viscosity between two series.

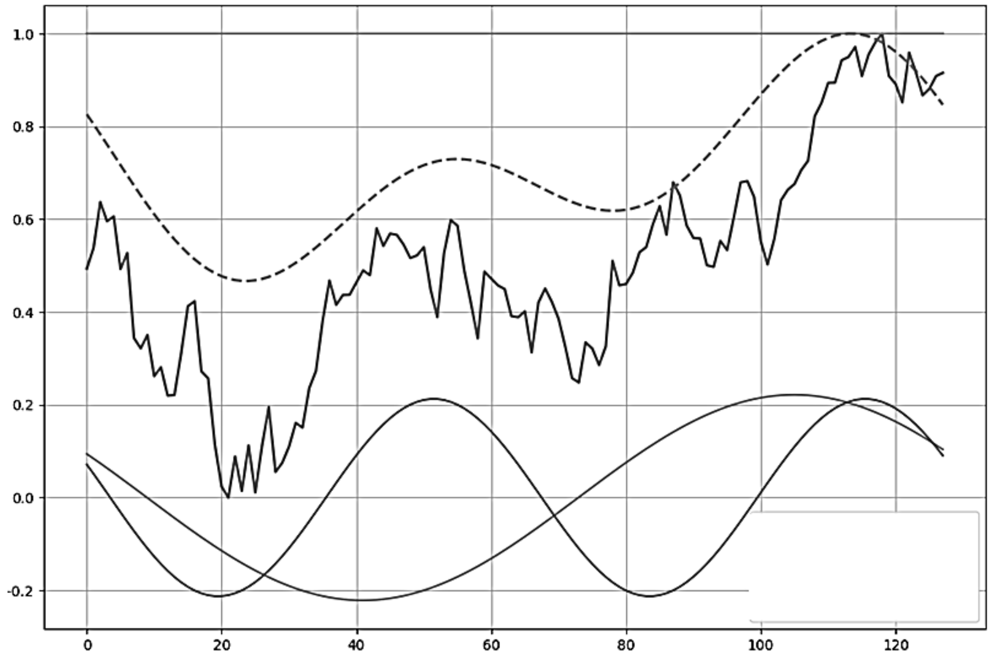


Figure 31

Comparison between a time series, its two main harmonics, and the sum of these harmonics. The dashed line is the sum of the two harmonics represented below.
It is a very good method in principle, and it is necessary to seriously analyze its usefulness.

Example: Calculation of Viscosity Between 2 Events, Using the Geometric Method, Without Smoothing and Smoothing

Let there be 2 series corresponding to the price of a barrel of Brent oil and the Dow Jones index of the "E2" group of events; a software has been generated that works on the complete series, detecting the peaks of both curves on which it has to calculate the lags or leads, as well as the choice of the points to subtract; the peaks are marked with stars and diamonds, respectively. In this way, we obtain the peak curves and the viscosity curves in 3 cases: Unsmoothed series and series smoothed by the exponential method with factor 0.8 and 0.4, respectively [51]:

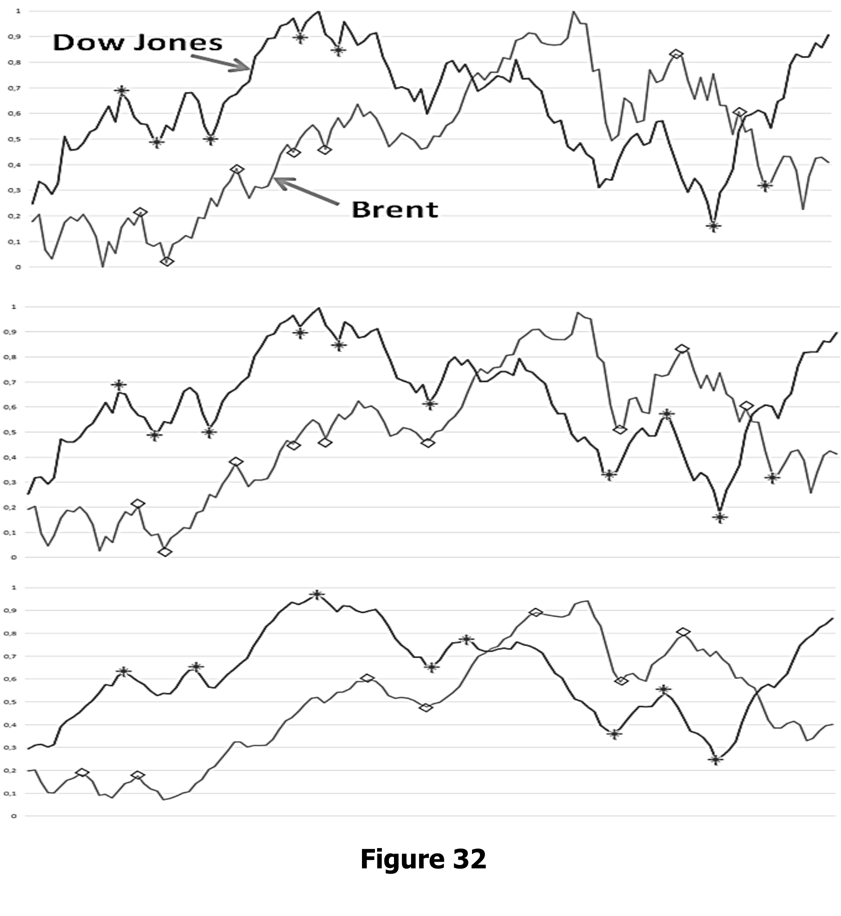


Figure 32

Comparative curves between the 2 events and viscosity curves; unsmoothed series, and smoothed series with the exponential parameter 0.8 and 0.4, respectively. Through this procedure of simplifying or smoothing the curves by calculating the distances between peaks, it is possible to generate the viscosity curve between both events in a simple and relatively accurate way; there will be an ideal smoothing that generates the ideal viscosity.

Some Notes

The viscosity curves generated by various methods are curves to be used as a basis for the viscosity that should be substituted into the "ABE" model. The range of values indicated by these curves must be adjusted to fit the range of values in the "ABE" model. The "real" range of values can be determined from the viscosity calculated by applying the "ABE" model.

The geometric method is one of the best procedures for calculating viscosity; however, the calculation of distances between peaks also has its challenges (some of which have already been mentioned): There may be peaks, especially in the series considered as output, that are caused by other series or events not analyzed or included in the joint study. This is another reason why analyzing a large number of dependent events together is the best option.

Transfer Function Method

This method, is mainly used in dynamic systems and allows to characterize the relationship between the two series (Input and Output) in terms of a mathematical function [12-17]. The delay is associated with the parameters of the transfer function. To do this, if "X" and "Y" are the 2 series where we want to calculate their relative viscosity in sections, we calculate the "TF" in each section, generating a step-type instability at each input, and calculating when "Y" responds to this instability; the time space required corresponds to the 1/viscosity:

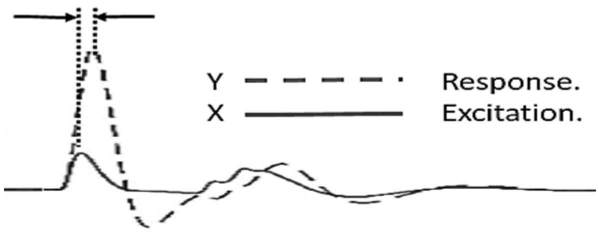


Figure 33: Excitation of "X" and Response of "Y"

In the case of sufficient values, this method is very efficient and of very valid theoretical conception.

Chat GPT

It is simply a matter of saying that this artificial intelligence prompt can calculate the advances or delays between curves, either from the graphs or from the numerical series. The tests conducted indicate good accuracy in the calculations.

Recommendations and Conclusion

More than methods in themselves, there are 3 recommendations that we believe are very efficient:

- Normalize the series using the "N2" process; this provides a greater accentuation of the variations and with it, greater ease in finding them.
- Smooth the series; thus, despite reducing the number of points and peaks to measure, the viscosity curve is smoothed and it is easier to calculate, by avoiding possible errors of contaminating peaks.
- Clean the signal from contaminating peaks, coming from interactions with other events, perhaps even not participating in the analysis.

The viscosity curves generated by all methods help to calculate the "real" viscosity curve; that is: That to be replaced in the "ABE" model. The cross-viscosity or relative viscosity is calculated by geometrically joining (average, etc) the 3 calculation methods seen. The range of values of the generated curve corresponds to the range of values of the curves when calculating them from the "ABE" model in 2 dimensions. As for the reliability of the 3 methods: the geometric method and the cross-correlation method are the most reliable, while the method of applying the "ABE" model is the least reliable because of its high uncertainty (in fact, we only work with its range of values, not with its geometric shape). Important: The viscosity from "ABE" model, must to be the same than viscosity "experimental".

Choice of Events-2

We are now in a position to make a better choice of events, since we already know:
- Viscosities and "natural" pressures as important parameters.
- The absolute entropy of each event.
- The relative masses and viscosities between all the events involved in the analysis.

From these values and curves, the events participating in the global analysis are "conveniently" selected, eliminated or substituted.

Therefore, in the case of two events, they should be selected in such a way that the information provided by both is maximized. This includes both individual information and dependency information, or intersection information. Ideally, this would occur when both series change only once; in this way, both would provide the maximum amount of information, and the dependency between them would be clearly identified. If it is possible to detect dependencies in different segments or intervals, the series should vary in those segments to understand and quantify the dependency. Important: It is therefore possible to simplify or smooth out the areas where no dependencies are detected in order to increase both individual and mutual information.

Depending on the detection of dependencies, there will be more or fewer intervals or segments in each time series. Additionally, each segment where dependencies are detected may have a different number of elements or nodes. Therefore, the possibility of dividing each series into segments of varying lengths is something that needs to be analyzed and studied. Thus, one of the possible methods to apply to each of the series that are subject to joint analysis is: To smooth out the "random" or highly volatile segments and separate the series into segments (of any length) where dependencies are detected: Very important series pre-treatment.

2D Model "ABE"

Model Generation

The objective is to model 2 time series and, therefore, to predict them jointly; this is the essence of this article: Joint modeling. For this, we generate a mathematical model that is an extension to 2 dimensions of the 1-dimensional model previously exposed; the objective is to increase the information with which we work, since the information of 2 series analyzed jointly, is the sum of the information of both plus the information of their dependence; this reduces the uncertainty and the precision in any prediction (the relative masses and the relative viscosities, are information): 2

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} + \frac{1}{\rho}\left(\frac{\partial P}{\partial x} - \frac{P}{\rho}\frac{\partial \rho}{\partial x}\right) - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} \qquad (93)$$

The idea of Extending this Model to 2-D (2 series "X" and "Y") is as Follows:

The acceleration of one of the events "X" is equal to the sum of the accelerations produced on "X", minus the sum of the decelerations (brake) produced by the frictions on "X".

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + PX - FX \qquad PX = \frac{1}{\rho_x}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right) + \frac{1}{\rho_x}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + PY - FY \qquad PY = \frac{1}{\rho_y}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right) + \frac{1}{\rho_y}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right)$$

$$FX = \frac{1}{\rho_X}\left((\mu)_{XX}\frac{\partial^2 u}{\partial x^2} + (\mu)_{Y,X}\frac{\partial^2 u}{\partial y^2}\right)$$

$$FY = \frac{1}{\rho_Y}\left((\mu)_{YY}\frac{\partial^2 v}{\partial y^2} + (\mu)_{X,Y}\frac{\partial^2 v}{\partial x^2}\right) \qquad (94)$$

Nomenclature: "x" are the values of the "X" series and "y" are the values of the "Y" series.

Cross Viscosities

- μ_{XX} , μ_{YY} Are the "natural" viscosities of "X" and "Y".
- μ_{XY} , μ_{YX} Are calculated, as we have already seen, in two main ways: By the process called "geometric" and by the process of cross-correlations; but there is another method:

When the "ABE" model is applied to 2 events, 2 equations are obtained in which, in each of them, the only unknown is precisely the cross viscosity sought; therefore, it is sufficient to clear the respective cross viscosity from each equation. It is necessary to consider:

- The cross-viscosities cleared from the 2D model are approximations to the real ones and, ultimately, the ones to be replaced in the model; they must be used to calculate the final viscosity together with the other 2 viscosities (calculated by the 2 methods described above).
- These viscosities are approximations, since they are removed from the "ABE" model applied to only 2 events; therefore, the uncertainty is high due to the lack of information.

• In fact, something tremendously important: If both viscosities are cleared from the model and at the same time calculated geometrically, it is possible to compare them; this comparison is of vital importance, since it will certainly serve to adjust (by incorporating coefficients) the model as a whole. It is extremely essential that this possibility exists. This comparison can certainly lead to a better management of the infinity resulting from a viscosity with a zero delay.

Example: Calculation of Viscosity Between 2 Events, Using the "ABE" Model

The series used are the price of a barrel of Brent oil and the Dow Jones index, both belonging to the "E2" event group. The real viscosity curves of the Brent with against to the Dow Jones and of the Dow Jones with against to the Brent are calculated:

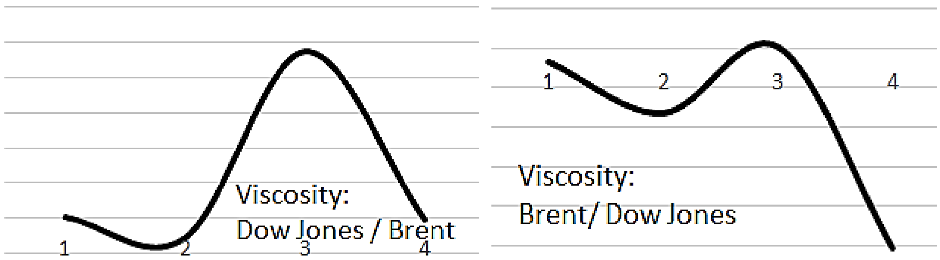


Figure 34

Relative viscosity curves between Brent and the Dow Jones, in the last 4 tranches of 25 values each (not in "t"). Ideally, these curves and the "real or experimental" curves would be identical. To achieve this, it is necessary to introduce a series of experimental coefficients, using techniques such as neural networks or artificial intelligence in general. To experimentally calculate the viscosity between both events, it is helpful to know a bit of theory regarding these two events. The relationship of lag or lead between the economic events of the Brent price and the Dow Jones index is not fixed, as it depends on a variety of factors such as the type of economic event, the nature of market shocks, and investor expectations. However, there are certain patterns and observations based on previous studies and the historical behaviour of these two indicators.

- **Lag of the Dow Jones relative to the price of Brent: In general, the Dow Jones tends to react with a lag of days to weeks to changes in the price of oil (Brent). This la**g is due to the fact that investors in the stock markets often adjust their expectations slowly as they understand the implications of changes in crude oil prices on inflation, interest rates, and economic growth expectations.
- **Short-Term Lag (days):** Immediate movements in the price of Brent (such as quick reactions to geopolitical news, natural disasters, or supply disruptions) may trigger a reaction in the Dow Jones within 1 to 3 days, as investors start to assess the impacts of events, but there is not yet a full adjustment in corporate profit expectations.
- **Medium-Term Lag (weeks):** If the change in the price of oil is sustained over a longer period
- (for example, a prolonged increase due to OPEC production cuts or a supply crisis), the Dow Jones could begin to more fully reflect the economic impact of this change, with a lag of 2 to 4 weeks. This is particularly true if the price of crude affects expectations regarding economic growth and inflation.
- **Factors Affecting This Lag:** Speculation in the oil market: Often, oil prices are influenced by short-term speculative factors, while the Dow Jones reacts to these changes with a lag because investors consider not only the immediate effects but also the long-term consequences.
- **Monetary and Fiscal Policy:** Decisions made by central banks and fiscal policies have an impact on financial markets, and the Dow Jones may reflect the effects of changes in oil prices with an additional lag of 1 to 3 weeks due to expectations regarding interest rates and inflation. - Nature of the economic shock: In the case of events such as military conflicts, international sanctions, or global economic crises affecting both oil and stock markets, the lag may be shorter. In these cases, both the price of oil and the Dow Jones tend to react quickly, within 1 to 3 days.
- **Empirical Evidence:** Historical studies suggest that the effects of oil prices on stock markets, such as the Dow Jones, are often felt more slowly, with a lag of between 1 to 4 weeks. This is because changes in crude prices affect investor expectations regarding business costs, inflation, and monetary policy, which takes time to fully reflect in stock indices.
- **Short-Term Effects:** In some cases, the Dow Jones may show more immediate reactions if the price of oil changes unexpectedly due to geopolitical events or market disruptions. However, the full market reaction to changes in Brent price concerning inflation or economic growth expectations takes between 2 to 3 weeks.

Information from the Transfer Function

Both masses and cross viscosities are methods of quantifying the dependencies between events; the essential thing, and this is the main objective, is to have as much information as possible of the analyzed series. Another method that can be applied to obtain more information from the series is the transfer function "TF" that we already know. It is used in time series analysis to model the causal relationship between two or more series (cause of several series (inputs) on a single series (output)). In other words, it allows us to understand how one time series (called output) is affected by another time series (called input). Therefore, given 2 series, the application of the "TF", considering one of them as input and the other as output, implies:

- **Identify the Causes of Instability:** By modeling the relationship between the input and output of the system using the transfer function, the factors contributing to the instability can be identified. This may include:
- **System Parameters:** The parameter values in the transfer function can reveal system characteristics that lead to instability, such as gain, time constant or the presence of poles in the unstable region of the complex plane.
- **External Influences:** The transfer function can help identify whether the instability comes from disturbances external to the system or whether it is an intrinsic property of the system.
- **Designing Control Strategies:** Once the causes of instability have been identified, the transfer function can be used to design controllers to mitigate the instability. There are several control methods, such as PID control, state feedback control or robust control, which rely on the transfer function to stabilize the system.
- **Predicting Future Behavior:** Although the transfer function cannot predict with certainty the occurrence of future instabilities, it can be useful in assessing the sensitivity of the system to changes in operating conditions or system parameters; this is the essence and true utility. By analyzing the system response to different hypothetical scenarios, the probability of future instabilities can be identified.
- **Simulate the System:** The transfer function allows simulating the behaviour of the system under different inputs and disturbances. This can be useful for evaluating system performance under conditions that might be difficult or dangerous to test in practice.

Regarding the Prediction of Other Instabilities: The transfer function alone cannot predict with certainty the occurrence of other instabilities, since it does not take into account random factors or unpredictable events that could affect the system. However, the information obtained by analyzing the transfer function, such as the sensitivity of the system to changes or the presence of unstable poles, can be useful to identify situations or operating conditions that increase the risk of future instabilities. Therefore, the objectives of its application to 2 series "X" and "Y" and in general, to all series, are:

- To be able to predict the output in the face of input variations; considering several restrictions, mainly that the relationship is linear.
- In addition to the time delay / lead time of the output, to know the time lags (one series can have a maximum at "t" and the other series a minimum at "t"; this means that they have a time lag of 180º for example). This is done by calculating the so-called "Bode Plot".
- With the application of the "TF" it is possible, therefore, to know their joint dynamics (with many restrictions inherent to the "TF"); and perhaps, such dynamics are intrinsically inherent to "X" and "Y" in such a way that they are the same regardless of when they are measured.
- Analyze instabilities; even predict them. Just as "TF" is applied to model the suspension of a vehicle to predict its behaviour on a bumpy road (instabilities), it can also be used to predict the effects of a crisis.
- Analyze the joint dynamics among all events; that is: If "n" is the total number of events analyzed, apply the "TF" for "s" events as input (s <= n-1) and a single output.

Another extremely important application of the "TF" is the mitigation of the effects of shocks: In the case for example of a car suspension system (the typical initial model of a quarter vehicle), we can excite it from a step or bump; this will produce an alteration of the spring and damper lengths; under given values in the spring and damper, the system can even go into resonance; in other words: One can know the dynamics by knowing the "TF" of the system; in the case of the economy the same thing can happen: given "n" events, one can calculate its "TF" with respect to another event; by modifying especially the viscosity, it is possible to control the dynamics "after" the instability.

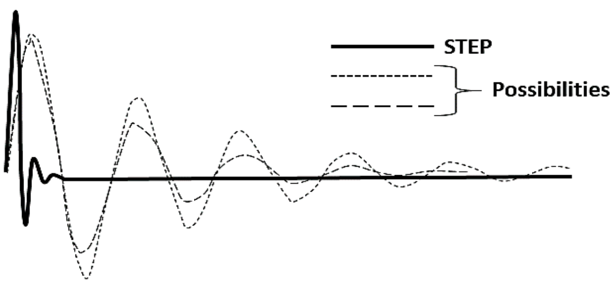


Figure 35

Different evolutions of the suspension, before a step. Example analogous to the economy. For all these reasons, the "TF" is an excellent method for predicting a time series as a first approximation, in other words: to have information on the evolution, although it does not work with the transport and diffusion phenomena; in spite of this, it is able to recognize the mutual information between 2 series in order to recognize their special and particular joint dynamics.

Example: Application of the "TF" to Predict the Output and Know the Dependency

Let the event be "IBEX 35 - Index of Spain" from June 26, 2017 to October 26, 2023 (input); the other period (output) corresponds to the "Price per barrel of Brent oil" (output) in the same time period. The price of Brent oil is now taken as input and the IBEX-35 index of Spain as output; the temporary period is from November 1, 2018 to October 21, 2022:

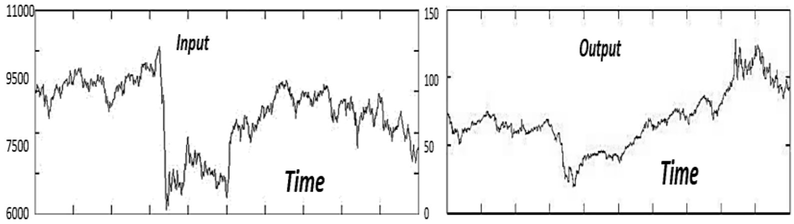


Figure 36: Graphs with Respect to Time of the 2 Series Analyzed

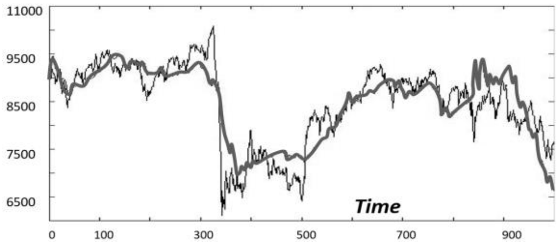


Figure 37

"TF" applied on the input; the "approximate" output is obtained (in black line); the "TF" is a dynamic model between input and output.
The "TF" is:

$$\frac{9.074\, s^{3} + 0.4085\, s^{2} - 0.0005001 s + 2.847\, 10^{-6}}{s^{5} + 0.5502\, s^{4} + 0.346\, s^{3} + 0.004387\, s^{2} + 1.537\, 10^{-5} + 3.449\, 10^{-8}} \quad (95)$$

Calculated with 5 poles and 3 zeros. Once this "TF" is obtained, it is possible to know how the output will vary according to the variations of the input; it is necessary to take into account the linearity imposed on the system, with the errors that this may entail; for this reason, it is recommended to use this procedure with caution and as already hinted, as an information input or a first approximation.

Prediction for "t+1"

It is sufficient to clear the transient term at "t+1" from each equation; each equation is selfsufficient to evaluate the velocity at "t+1"; let's see it for 2D:

$$u(t+1) = u(t) - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + PX - FX$$
$$v(t+1) = v(t) - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + PY - FY \quad (96)$$

Pressure and Viscosity Weighting as a Neural Network

Try to weight the contributions of each event to each "ABE" equation; i.e.: Multiply the pressure term and the viscosity term by a weighting factor. Example for 2 events:

$$PX = \frac{m_{XX}}{\rho_x}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right) + \frac{m_{YX}}{\rho_x}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right) \qquad FX = \frac{1}{\rho_X}\left(m_{XX}(\mu)_{XX}\frac{\partial^2 u}{\partial x^2} + m_{YX}(\mu)_{Y,X}\frac{\partial^2 u}{\partial y^2}\right)$$
$$PY = \frac{m_{YY}}{\rho_y}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right) + \frac{m_{XY}}{\rho_y}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right) \qquad FY = \frac{1}{\rho_Y}\left(m_{YY}(\mu)_{YY}\frac{\partial^2 v}{\partial y^2} + m_{XY}(\mu)_{X,Y}\frac{\partial^2 v}{\partial x^2}\right) \quad (97)$$

The sum of the masses in each equation must be "1" (for pressure and for viscosity).
The learning process of a neural network is based on the proper calculation of the weights of the terms in the function "f", where "x" represents the input value, "w" the weights, and "y" the desired output:

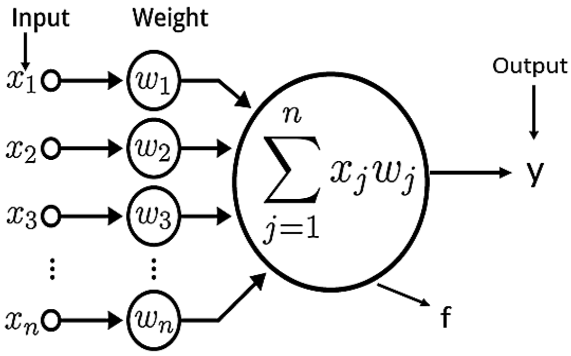


Figure 38: Function as a Linear Combination of Input and Output

Is it possible to adjust “f” to account for dependencies between the variables? Yes, in fact, we can replace it with the “ABE” model. The mass values effectively act as weights that can be calculated. These masses can be considered as initial approximation values, and by applying the neural network concept, they can be refined through model training. This training is carried out through back testing.

Example: Modeling and Prediction of 2 Economic Events

In this example, we rely on 2 of the events of the “E2” group: Brent and Dow Jones. For this, the 2D “ABE” model is generated. We need weighting factors in the mathematical model consisting of relative entropies.

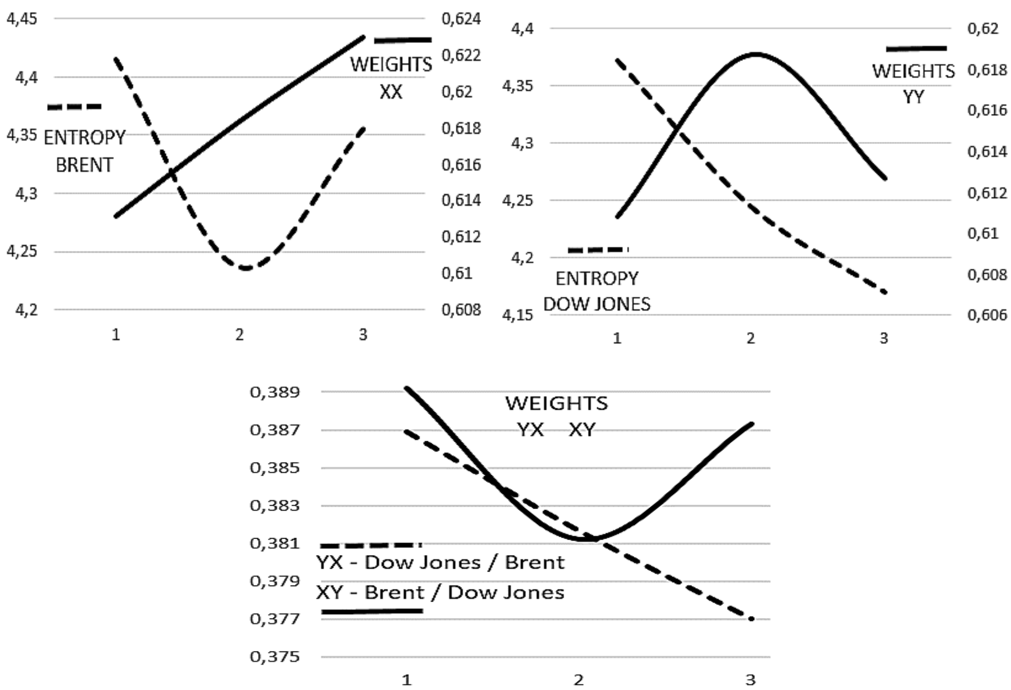


Figure 39: Curves of the Relative Weights or Masses in the 2D Model

If the viscosity is calculated experimentally, a curve composed of points is obtained, including the viscosity at “t”. In contrast, the calculation of the viscosity curve using the “ABE” model does not include this latter term. This is the critical reason for applying experimental methods to determine the viscosity at “t”. We have the viscosity curve from the “ABE” model; it is necessary to select a viscosity value for “t”.

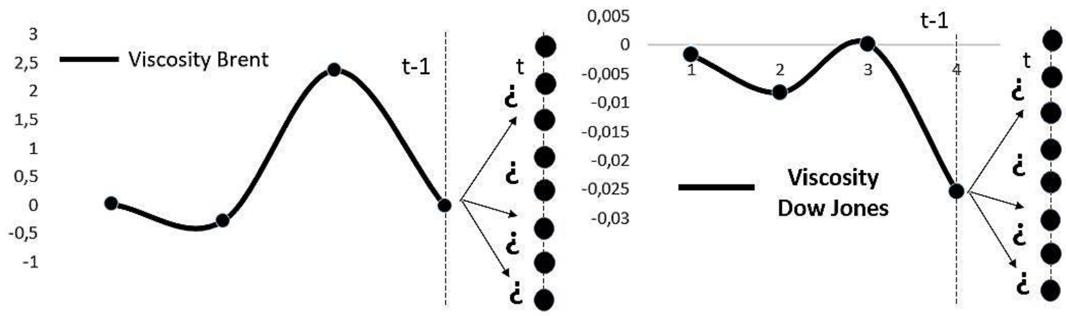


Figure 40: The Brent and Dow Jones Viscosity at “t” can have Many Values

Depending on this value, the velocity prediction will be positive or negative; that is, the value of the time series at "t+1" will either increase or decrease. Therefore, it is very useful to know the value of viscosity at "t" from which the sign of the velocity changes from positive to negative, or vice versa, above or below that point; let this point be referred to as "V0".

Table 16

Viscosities in "t"

	Seed 2	Seed 1	Seed 3
Brent	-0.151668	-0.019205	0.113258
Dow Jones	-0.035419	-0.0248	-0.015281

Sign of Velocity in "t+1"

	Seed 2	Seed 1	Seed 3	I.C.
Brent	>0	>0	<0	2-yc
Dow Jones	<0	<0	<0	3

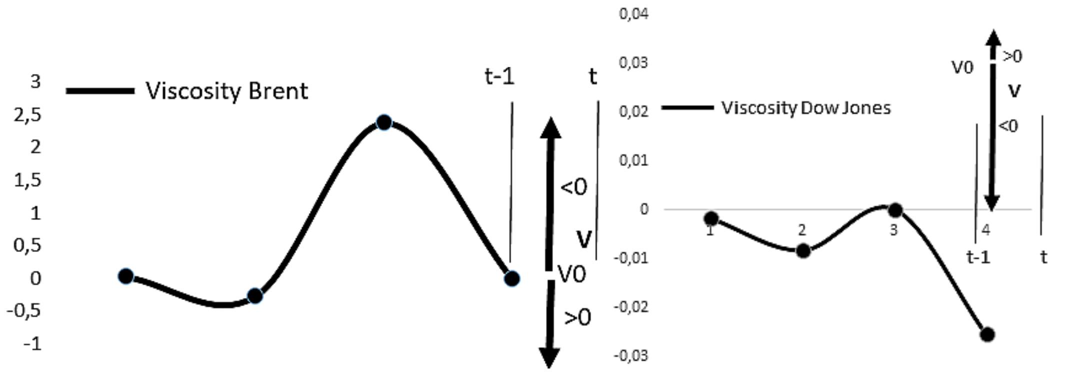


Figure 41

Position of "V0" against the viscosity curve, especially in "t".
High uncertainty for Brent and low uncertainty for the Dow Jones.

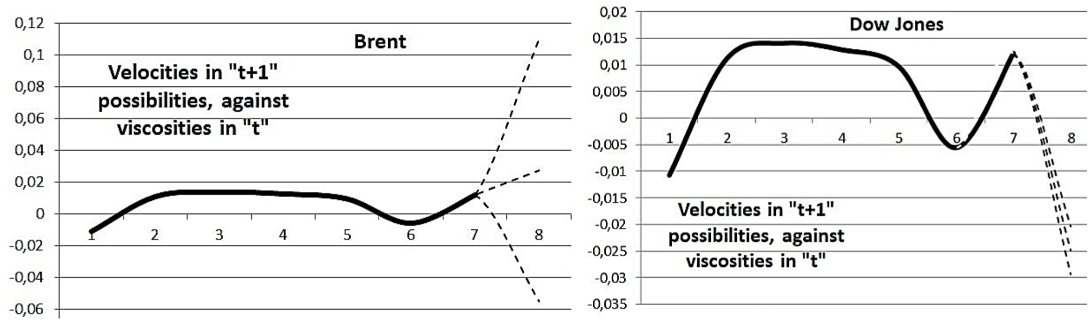


Figure 42: Predictions of the Speed for the 3 Viscosity Values in "t"

Starting with the Dow Jones, any of the three predictions would result in a significant surprise. Regarding Brent, the central value would undoubtedly result in the least surprising outcome, as geometrically speaking, it appears the most logical. It is also important to note that the three predictions for Brent create a range of possibilities, most of which are logical and lack surprises.

Conclusion

The fact of modeling and predicting a group of time series, implies more than the smoothing of the joint graph between the viscosities as already mentioned, the facility to "choose" a real viscosity to calculate the velocity at "t+1"; this is the essence; if such a similar evolution or joint smoothness is not given, it is an unequivocal sign of a bad choice of the analyzed series; that is: A new choice will be necessary, taking perhaps, other dependent series that act as "bridges" between the series that need to be modeled.

Model In "N" Dimensions

This is the extension or generalization of the 2 previous models, to "n" dimensions or economic assets. In this way and with the nomenclature already used and described, we obtain:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - .... + PX - FX$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - .... + PY - FY$$

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$$PX = \frac{m_{XX}}{\rho_x}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right) + \frac{m_{YX}}{\rho_x}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right) + \frac{m_{ZX}}{\rho_x}\left(\frac{\partial P_z}{\partial z} - \frac{P_z}{\rho_z}\frac{\partial \rho_z}{\partial z}\right) + ...$$

$$PY = \frac{m_{YY}}{\rho_y}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right) + \frac{m_{XY}}{\rho_y}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right) + \frac{m_{ZY}}{\rho_y}\left(\frac{\partial P_z}{\partial z} - \frac{P_z}{\rho_z}\frac{\partial \rho_z}{\partial z}\right) + ...$$

$$PZ = \frac{m_{ZZ}}{\rho_z}\left(\frac{\partial P_z}{\partial z} - \frac{P_z}{\rho_z}\frac{\partial \rho_z}{\partial z}\right) + \frac{m_{XZ}}{\rho_z}\left(\frac{\partial P_x}{\partial x} - \frac{P_x}{\rho_x}\frac{\partial \rho_x}{\partial x}\right) + \frac{m_{YZ}}{\rho_z}\left(\frac{\partial P_y}{\partial y} - \frac{P_y}{\rho_y}\frac{\partial \rho_y}{\partial y}\right) + ...$$

****

***

$$FX = \frac{1}{\rho_X}\left(m_{XX}(\mu)_{XX}\frac{\partial^2 u}{\partial x^2} + m_{YX}(\mu)_{Y,X}\frac{\partial^2 u}{\partial y^2} + m_{ZX}(\mu)_{Z,X}\frac{\partial^2 u}{\partial z^2} + ...\right)$$

$$FY = \frac{1}{\rho_Y}\left(m_{YY}(\mu)_{YY}\frac{\partial^2 v}{\partial y^2} + m_{XY}(\mu)_{X,Y}\frac{\partial^2 v}{\partial x^2} + m_{ZY}(\mu)_{Z,Y}\frac{\partial^2 v}{\partial z^2} + ...\right)$$

$$FZ = \frac{1}{\rho_Z}\left(m_{ZZ}(\mu)_{ZZ}\frac{\partial^2 w}{\partial z^2} + m_{XZ}(\mu)_{X,Z}\frac{\partial^2 w}{\partial x^2} + m_{YZ}(\mu)_{Y,Z}\frac{\partial^2 W}{\partial y^2} + ...\right)$$

****

(98) ***

The representation of the data corresponding to the "n" events constitutes the so-called phase space. How can a phase space be represented in "n" dimensions? By assigning, for example, to each point a color, corresponding to a "unique" color combination of the "RGB" palette for example (256 colors / 256 events). An important thing to note is the extraordinary increase in information when analyzing "n" series together; in other words: If you analyze "n" series in isolation, you are working with "n" information, but if you analyze them together, the information increases to "2n2": "m" and "μ" are information.

In tensorial notation:

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{P} - \mathbf{F} \quad (99)$$

$$P_i = \frac{1}{\rho}\sum_j m_{ij}\left(\frac{\partial P}{\partial x_j} + P\frac{\partial \rho}{\partial x_j}\right), \quad i,j \in \{x,y,z\}$$

$$\mathbf{P} = \frac{1}{\rho}\mathbf{M}\cdot(\nabla P + P\nabla\rho)$$

$$F_i = \frac{1}{\rho}\sum_j m_{ij}(\mu)\frac{\partial^2 u}{\partial x_j^2}, \quad i,j \in \{x,y,z\}$$

$$\mathbf{F} = \frac{1}{\rho}\mathbf{M}(\mu)\cdot\nabla^2\mathbf{u}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho}\mathbf{M} \cdot (\nabla P + P\nabla \rho) - \frac{1}{\rho}\mathbf{M}(\mu) \cdot \nabla^2 \mathbf{u}$$

"ABE" GENERAL METHOD

Definition of the Problem to be Solved

- Initial choice of series.
- Normalization of the series.
- Entropies.
- FFT (Fourier) to each series.
- Absolute masses.
- Choice of series.
- Possible identification of harmonics.
- Possible simplification of each series.
- Possible noise reduction.
- Calculation of nodes, density, velocity and pressure, of each series.
- Calculation of relative masses.
- Calculation of relative viscosities.
- Final choice of the series.
- Generation of the "n" dimensional model.
- "t+1" prediction calculation of each event.

As part of the protocol, it is possible to learn from the past; that is: When working with a phase space corresponding to "n" events, it is possible to change some past value and observe the behaviour involved; this provides decisive information about the conditions for achieving a given dynamic.

"GT" MODEL

Definition: Minimum Action

A particle in motion within a dynamic system will tend to go where it needs the least energy to move. This is called the principle of minimum energy and is a universal principle; in economics, it could be used to capture the efficiency of an economic path, in other words, the ability to reach a desired state with the least "cost" in terms of resources or time. We can analyze the following analogy: Be a person, pushing a wall composed of 2 parts, and each arm pushing one of them; one of the 2, it is not known which one, is made of paper while the other is made of brick; basic question: Which one will give way first and make the person put his arm through it? The paper area will yield sooner, since it offers less resistance (it is weaker); that is: It has less Energy to oppose the hand. An economic event will not move as long as it does not feel pressure; we have already seen that pressure is responsible for the economic dynamics and is energy in itself. This principle of minimum energy is called principle of minimum action "S"; that is: Lagrangian "L" is defined as the subtraction of the kinetic energy "Ec" minus the potential energy "Ep" (the potential energy is a store of energy available to the particle); in our case (in 1 coordinate "x"):

$$Ec = \frac{1}{2} m v^2 \qquad Ep = \frac{m}{\rho}\frac{\partial P}{\partial x} \tag{100}$$

The action "S" between 2 points "1" and "2" (initial and final) is defined as the integral of "L" between the 2 times "t" (initial and final) of the path traveled:

$$S = \int_{t1}^{t2} L dt = \int_{t1}^{t2} \left(\frac{1}{2} m v^2 - \frac{m}{\rho}\frac{\partial P}{\partial x} \right) dt \tag{101}$$

Therefore, an economic event will tend to go where the action is minimal (it depends on "m" and "v").

Definitions: Space and Metric

The field of events in "n" dimensions or economic events is defined as the space in which each dimension is an event. The phase space of the "n" measured events will be a line or trajectory in the "n" dimensional space. This trajectory must comply with the principle of minimum energy or minimum action. Geodesy is defined as the line that joins 2 points or particles of space with the smallest distance; in the beginning, the geodesies and the trajectories or phase space of the set of events analyzed are not equal; but, if we define the distance (metric) between 2 points as the action "S", the "n" dimensional phase space or trajectory is a geodesy.

That Is: The "ABE" model is capable of calculating dynamic pressures. The application of Takens' theorem can also be useful for determining high- and low-pressure zones as time progresses. These dynamic pressures can be modeled using Gaussian potentials or other tools; in this way, a pressure field model is obtained. This pressure field is dynamic:

It is like a sea with dynamic waves in "n" dimensions. The trajectory or phase space of the problem evolves through all these waves. The n-dimensional surface contains all the geodesics or dynamic trajectories of all the events used to generate it. We can test variations of certain events to understand how the pressure field changes within a temporal neighborhood of an event. This will allow us to determine whether the potential field can be considered more or less constant, enabling the analysis of a new seed point's dynamics over a larger or smaller interval; that is: A new value for an event. Otherwise, the variations in the field can be examined with the same goal, perhaps to identify a pattern or rule that enables its prediction based on event variations.

Tools for Generation Mathematical Model Pressure Field in First Approximation

Using Takens' theorem and other procedures, it is possible to determine, in a first approximation, the attractor and repulsor zones (negative pressure and positive pressure) of the phase space of the problem solved with "ABE". These zones can be simulated using the socalled Gaussian Potentials ("σ" is the width, "V_0" is the depth or intensity, and "x" and "y" are the center of the potential in two dimensions).

$$P(x,y) = \pm V_0 e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad (102)$$

With the "+" sign, a peak or high-pressure zone is obtained, and with the "−" sign, a low-pressure zone or well is created. The pressure field of a problem with "n" economic events is not constant; that is, it varies over time. Therefore, the values of the Gaussian potential center, the depth value, and the width value must depend on time.

$$P(x,y,t) = \pm V_0 f(t) e^{-\frac{(xg(t))^2+(yh(t))^2}{2(\sigma i(t))^2}} \qquad (103)$$

There are other types of potentials to simulate the pressure field: vector fields simulating rotation, damping, acceleration, etc. However, using Gaussian potentials is an excellent tool due to its simplicity. That said, the use of vector fields provides greater accuracy and reliability, despite being more complex to generate and fine-tune. With this modeling approach, a method is obtained to experiment with different seed points.

Geodesies from Seed Points

Given a set of events to be analyzed, we can draw its phase space inside a full pressure field. This phase space will have an initial point and an end point; on this pressure field "SP", The dynamics, trajectories, or geodesics correspond to the dynamics of all the events analyzed in the model. If we want to study the dynamics of an event but with different (seed) values starting from a time "t0"; that is: To understand how an event evolves when initial values change, the geodesics on the initial surface will be valid in those neighborhoods of the initial seed point where we consider the pressure field to remain unchanged.

Important: Therefore, we can analyze the model of "n" events to understand the pressure field and its variations against changes in each event. The goal is to gain a deep understanding of it, as this would allow us to predict the longterm evolution of any given seed point. Therefore, it is an ideal tool to test economic policies and, in short, to acquire knowledge that may be useful at some point in time, such as the detection and analysis of instabilities, trends, objectives, measures to be implemented, etc. With this tool, it is possible to analyze and learn from different dynamics in the face of possible changes from the initial seed point or intermediate points.

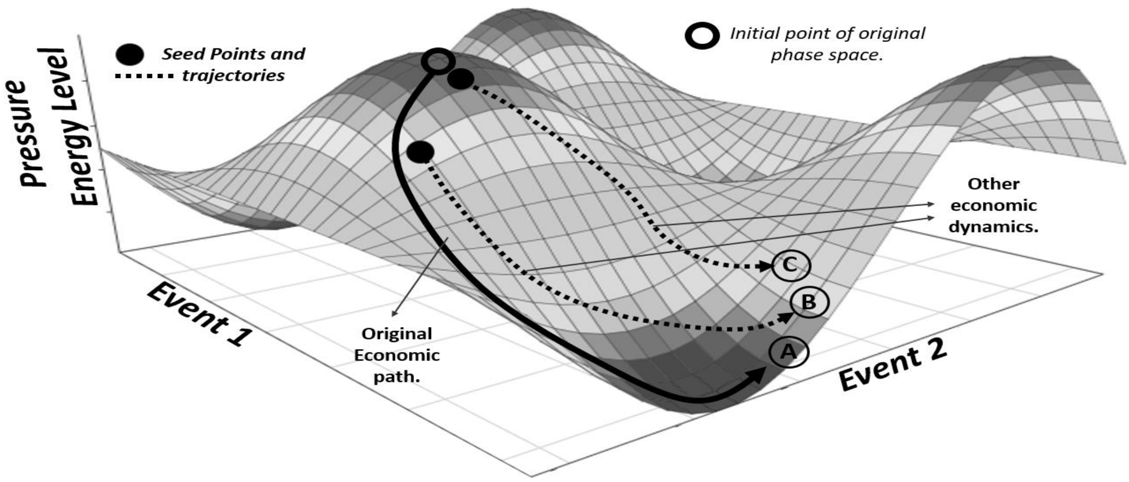

Figure 43

Example in 2D: The black line represents the measured phase space formed by the "2" events studied ("n" in general); the other 2 lines are other phase spaces generated by 2 different seed points, originating 2 different final destinations "B" and "C": It´s possible to know the new dynamic and destination, if we change any value.

Geodesies in Economy; Sample in 2 Dimensions

$$L = \frac{1}{2}mv^2 - \frac{m}{\rho}\frac{\partial P}{\partial x} \tag{104}$$

$$\frac{d}{dt}(mv_x) = m\frac{dv_x}{dt}, \quad \frac{d}{dt}(mv_y) = m\frac{dv_y}{dt}$$

$$\frac{\partial L}{\partial x} = -\frac{m}{\rho}\frac{\partial^2 P}{\partial x^2}, \quad \frac{\partial L}{\partial y} = -\frac{m}{\rho}\frac{\partial^2 P}{\partial x \partial y}$$

$$\frac{dv_x}{dt} = -\frac{1}{\rho}\frac{\partial^2 P}{\partial x^2}$$

$$\frac{dv_y}{dt} = -\frac{1}{\rho}\frac{\partial^2 P}{\partial x \partial y}$$

With Initial Velocity:

$$\dot{x}(0) = u_x, \quad \dot{y}(0) = u_y$$

$$\dot{x} = u_x - \int_0^t \frac{1}{\rho}\frac{\partial^2 P}{\partial x^2}dt'$$

$$\dot{y} = u_y - \int_0^t \frac{1}{\rho}\frac{\partial^2 P}{\partial y^2}dt'$$

$$x(t) = x_0 + u_x t - \int_0^t \int_0^{t'} \frac{1}{\rho}\frac{\partial^2 P}{\partial x^2}dt''dt'$$

$$y(t) = y_0 + u_y t - \int_0^t \int_0^{t'} \frac{1}{\rho}\frac{\partial^2 P}{\partial y^2}dt''dt'$$

$$(x(t), y(t)) = \left(x_0 + u_x t - \int_0^t \int_0^{t'} \frac{1}{\rho}\frac{\partial^2 P}{\partial x^2}dt''dt', \quad y_0 + u_y t - \int_0^t \int_0^{t'} \frac{1}{\rho}\frac{\partial^2 P}{\partial y^2}dt''dt'\right)$$

Particular Case: Pressure "P" as a Potential Gaussian: (105)

$$P(x,y) = -V_0 e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial P}{\partial x} = \left(-V_0 e^{-\frac{x^2+y^2}{2\sigma^2}}\right)\left(-\frac{x}{\sigma^2}\right) = \frac{V_0 x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 P}{\partial x^2} = \left(\frac{V_0}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right)\left(1 - \frac{x^2}{\sigma^2}\right)$$

$$\frac{\partial^2 P}{\partial x \partial y} = \frac{V_0 x y}{\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$x(0) = x_0, \quad y(0) = y_0, \quad \dot{x}(0) = u_x, \quad \dot{y}(0) = u_y$$

$$\frac{d^2 x}{dt^2} = -\frac{1}{\rho}\left(\frac{V_0}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\left(1 - \frac{x^2}{\sigma^2}\right)\right)$$

$$\frac{d^2 y}{dt^2} = -\frac{1}{\rho}\left(\frac{V_0 x y}{\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}\right)$$

For 2 Gaussian Potentials "P":

$$P(x,y) = -V_0 e^{-\frac{x^2+y^2}{2\sigma^2}} - V_1 e^{-\frac{(x-x_1)^2+(y-y_1)^2}{2\sigma_1^2}}$$

$$\frac{\partial P}{\partial x} = \frac{V_0 x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{V_1(x-x_1)}{\sigma_1^2} e^{-\frac{(x-x_1)^2+(y-y_1)^2}{2\sigma_1^2}}$$

$$\frac{\partial P}{\partial y} = \frac{V_0 y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{V_1(y-y_1)}{\sigma_1^2} e^{-\frac{(x-x_1)^2+(y-y_1)^2}{2\sigma_1^2}}$$

Geodesic Sample with 2 Gaussian Potentials (Pressure Field Constant)

Initial Velocity: (0.5,0.5). Seed point: (-2,0).

1 Potential: "V0=1", "σ=1", center: (0,0). 2 Potential: "V0=0.8", "σ=0.8", center: (2,2).

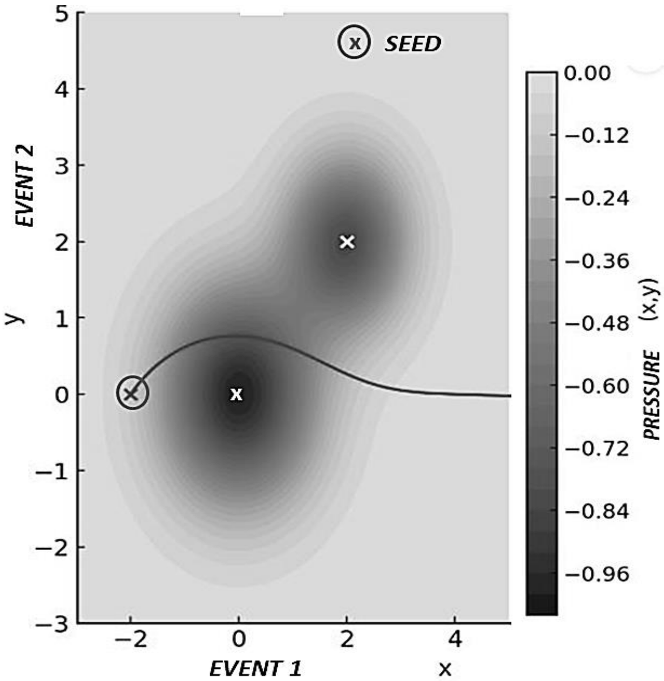

Figure 44: Geodesic Sample (2 Dimensions) for 2 Gaussian Potentials of "P"

Geodesic sample (2 Dimensions) for 2 Gaussian Potentials of "P" (Pressure Field Constant)

- Potential: "V0"=10, "Sigma"=1.5, Center: (0,0).
- Potential: "V1"=8, "Sigma"=1, Center: (2,2).

Seed: (0,-15), Initial velocity: "u"=0, Angle: Pi/4; velocity components:(1.41,1.41).

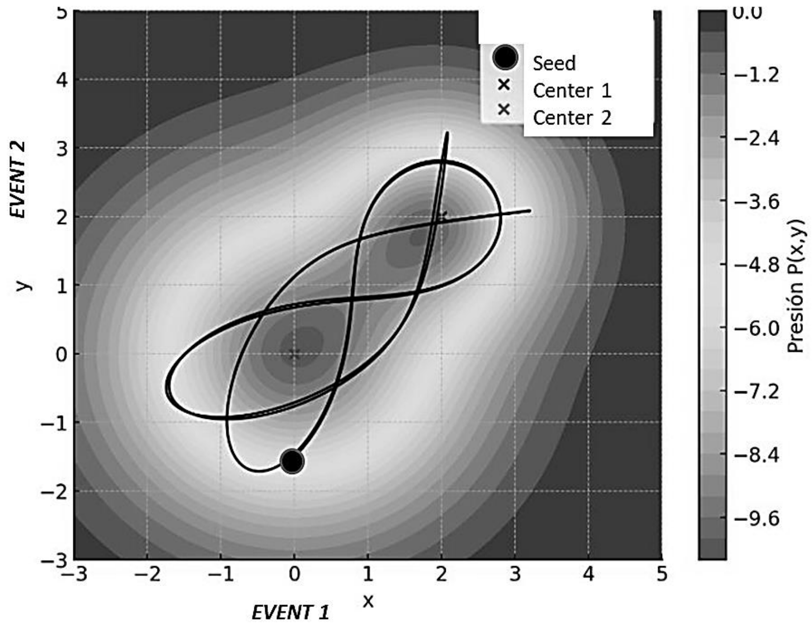

Figure 45: Geodesic Sample (2 Dimensions) for 2 Gaussian Potentials of "P"

Important: It is possible to model and quantify the attractive and repulsive pressure zones using Gaussian pressure wells and peaks. This is a simple and efficient way to create, in a first approximation, the geodesics that simulate the dynamics of events in "n" dimensions.

Geodesic Sample (2 Dimensions) for 1 Gaussian Pic Potential and 1 Gaussian Well Potential (Pressure Field Constant)

$$V_0 = 5, V_1 = 3$$

$$\sigma = 1, \sigma_1 = 0.8$$

$(x_1, y_1) = (2, 2)$, other center: (0,0).

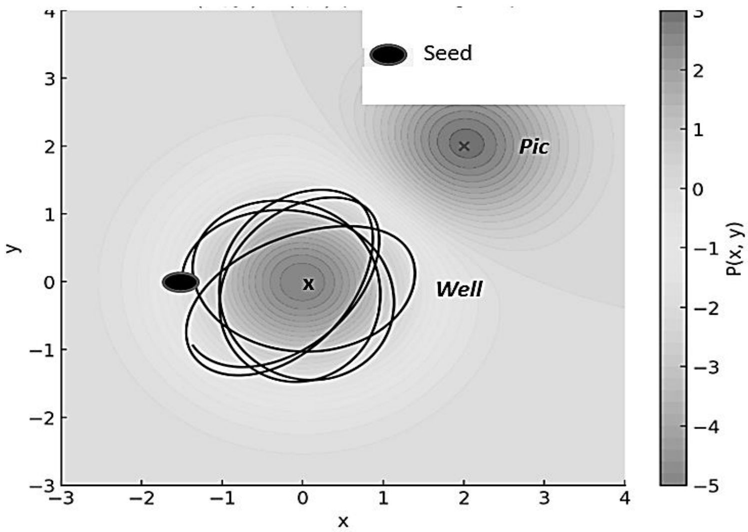

Figure 46: Geodesic Sample (2 Dimensions) for 2 Gaussian Potentials of "P" (Pic and Well).

Geodesic sample around 2 Gaussian potentials in movement (Pressure Field not Constant)

Black lines continues: Movement circular of potentials:

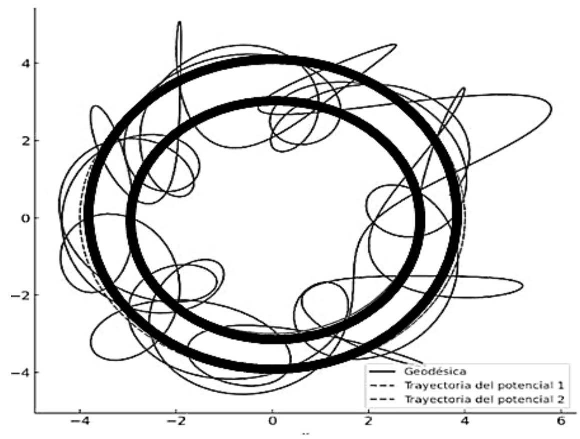

Figure 47: Geodesic Around 2 Potentials Gaussian in Movement

Relationship Between the 3 Concepts

The Lotka-Volterra equations, as prey (P) - predator (D) population evolution equations, are a mathematical model that describes the dynamics of a population. However, a variation of these equations can be used to simulate supply-demand equilibrium in a market. In this case, two populations are considered: The producer population and the consumer population. The equations describing the dynamics of these populations can be modified as follows (the simplest model):

dP/dt = aP – bPC

(107) dC/dt = -cC + dPC

Where: "P" is the population of producers; "C" is the population of consumers; "a" is the production rate, which describes the ability of producers to generate supply; "b" is the demand rate, which describes the probability that a consumer will purchase a product; "c" is the abandonment rate, which describes the probability that a consumer will stop consuming a product; "d" is the consumption rate, which describes the amount of products each consumer purchases in a given time. These equations describe how the population of producers increases as they produce, while the population of consumers decreases as they abandon consumption, but increases as they purchase products. Equilibrium between supply and demand is reached when the rate of production equals the rate of consumption multiplied by the rate of demand. The computable general equilibrium model "CGEEM" is a complex mathematical model used to analyze the economy at the macroeconomic level. This model is based on general equilibrium theory and uses equations to describe the supply and demand of different goods and services in an economy. The equations of the "MEGC" model for supply and demand depend on the type of good or service being analyzed and may vary from one model to another. However, the following are some of the basic equations of the model: Supply:

(108) Qs = f(P, Pf, Ps, Y, T)

Where: "Qs" is the quantity of the good or service offered; "P" is the price of the good or service; "Pf" is the price of the factors of production used in the production of the good or service; "Ps" is the price of the goods and services related to the good or service being analyzed; "Y" is the income or production of the company; "T" is the taxes applied to the production of the good or service. Demand:

(109) Qd = g(P, Y, Pr, Pg, Pe)

Where: "Qd" is the quantity demanded of the good or service; "Pr" is the price of goods related to the good or service being analyzed; "Pg" is the price of substitute goods for the good or service being analyzed; "Pe" is the expected price of the good or service in the future. These are just some of the equations that can be used in the "MEGC" model to describe the supply and demand of different goods and services in an economy. The complexity of the model depends on the number of goods and services being analyzed and the variables being considered. The heat equation is a partial differential equation that describes the diffusion of heat in a continuous medium. To simulate supply and demand, this equation can be used as a mathematical model in the propagation of prices in a market. For this, a technique known as a price diffusion model can be used; the price diffusion model assumes that the rate of change of the price of a good at a given time is proportional to the difference between the current price and the average price in a local environment. To modify the heat equation to simulate supply and demand, the temperature variable can be replaced with the price of the good or service. In addition, the terms in the equation should be adjusted to reflect market supply and demand. One possible way to do this is to use a heat source in the equation representing supply, and a cooling source representing demand. The term representing the heat source could increase the price when the supply of the good is high, while the term representing the cooling source could decrease the price when demand is low. The modified heat equation to simulate supply and demand could be written as:

(110) $\partial u/\partial t = k\ {}^2u + Qo - Qd$

Where "u" is the price of the good or service, "k" is the thermal diffusivity, "Qo" is the heat source representing supply, and "Qd" is the cooling source representing demand. To modify the heat equation to simulate supply and demand, one can replace the temperature variable with the price of the good or service and adjust the terms in the equation to reflect market supply and demand. This allows the use of price diffusion modeling techniques to predict price propagation in the market.

Thus, by modeling supply and demand behaviour, it is possible to calculate the equilibrium, starting from a seed point. If the time series of fox population "A" and rabbit population "B" in a given territory are analyzed separately or in isolation, it is not possible to determine whether the fox or rabbit population will disappear: It is possible that if the number of rabbits is low, the foxes will die due to lack of food (and much more possibilities), and also, if a pattern or trend is detected, both will last forever. The interaction or dependence is something that cannot be predicted by analyzing the series in isolation: It is impossible because that information "I" is not included in the individual series. However, it is included in the information from the intersection: $I(A,B) = I(A) + I(B) + I(A \cap B)$; this is the reason why a global or joint analysis of several events (in this case, supply and demand together) is essential.

Entropy is used in economics to analyze complex economic systems and to help understand how factors such as competition, innovation, regulation and public policy can influence economic efficiency and wealth distribution. The concepts of Entropy, Action and Equilibrium are intimately linked; in fact, previously, the following question has been suggested: what does it mean that an event tends to the most probable state? Analyzing the existing relationship, the answer is evident and emerges by itself.

- When a particle or system of particles moves, it moves towards a state of minimum Energy; that is: Minimum Action; at that point, the particle is "calm", with a minimum sum of "tensions"; in other words: The particle tends to have more and more Entropy (in isolated systems, without intervention of external forces, or with conservative forces).
- Equilibrium is a point/zone of maximum Entropy.
- Is it better to "do something/little" or "do nothing" for a change or to get the economy on track?

As we already know, the dynamics of the economy runs by itself, "naturally"; a human intervention, for example, is not capable of modifying it if it is not intense or of great importance. When studying to become an ultra-light pilot, one of the things they teach us when we start flying is that in case the plane does "strange" things, rolls over or seems to go crazy, the best thing to do and the technique to use is to "let go of the controls". All airplanes tend to equilibrium; they are self-stable by nature and any action on them affects such stability; another case is that of the car in a curve: If at that moment we let go of the steering wheel, the wheels tend to straighten out; it tends to the directional equilibrium generated by the special design of the suspension and the steering.

The expressions "the economy regulates itself" or "the economy regulates itself" are very commonly used; in a certain way, as we have already seen, this is really the case if the economic structure meets a series of conditions; the problem is that most economies in the world are not perfect at all; this means that there are inherent instabilities and unstable equilibriums, which require almost constant control.

Important: The fact that a dynamic system is in equilibrium does not mean at all that all the particles are in equilibrium; there may exist "links" or relations between some particles that are in "non-normal" tension; the only condition that equilibrium must fulfill is that the sum of forces - tensions be the minimum possible; this is what is really essential and defining of the problem. In any dynamic system, there is no single point or zone of equilibrium; there may be zones in which, in each of them, there is an equilibrium. An economic event evolves and flows through these zones, depending on the field of pressures and their effects. The Economy is composed of Events; for example, an event can be the sale of melons; its existence implies that the economic dynamics is a determined one and not another with its interactions and so on; if that event did not exist or others were created, the dynamics would be a different one with its particularities. Every time a "need", a "fashion" or an event in general is created, the economy responds with whatever viscosity it may be, to adapt its "new" dynamics to these "new" events, always seeking "equilibrium". In turn, these events can become elements with more information and therefore ideal to be included in a joint analysis (a magnificent tool for an economist). On the other hand, new events are continuously being created and measured, so perhaps a certain part of the stochastic signal that practically always appears in the data may come from this adaptation of the global dynamics to the new conditions. New events, fashions and economic events are constantly being generated in the world. Today's society is constantly evolving, driven by technological advances, cultural changes and economic developments. These factors contribute to the emergence of new events and trends in different areas. In economic terms, events and trends can arise as a result of changes in economic policy, advances in technology, and fluctuations in financial markets or changes in people's consumption habits. For example, the emergence of new technologies such as artificial intelligence, e-commerce and the collaborative economy has led to the emergence of new business models and economic opportunities. In terms of fashions, the fashion and lifestyle industry is also constantly changing. New fashion trends, lifestyles and consumer preferences are generated as people seek to express themselves and adapt to new cultural currents. Today's world is dynamic and in constant motion, leading to the constant generation of new events, fashions and economic events. These changes reflect the evolution of society and offer new opportunities and challenges in different sectors. Obviously, some events are more important than others and therefore have a much greater impact on global dynamics; for example, until cars did not exist, the event of car sales did not exist; when they began to be sold, it had an extraordinary impact, both because of the event itself and because of all the events that go along with the sale of cars. It is important to note that market equilibrium does not automatically guarantee equality in the distribution of economic benefits. In fact, in many cases, markets can generate inequalities in the distribution of income and wealth. Factors such as the concentration of economic power, differences in skills and education, and barriers to entry can influence economic inequality. In economics, an example that illustrates the principle of minimum power is the process of perfect competition in a market. Let there be a market for a specific product in which there are numerous sellers and buyers. In this scenario, each seller tries to maximize his profit and each buyer tries to maximize his satisfaction. Both actors make individual decisions, without centralized coordination.

The principle of minimum energy states that, in a competitive market, prices and quantities will be adjusted so as to reach equilibrium in which energy, understood as the effort or cost required to maintain that equilibrium, is minimized. In this case, the effort or cost translates into excess production or shortage of the product. It is assumed that initially the market price of a product is too high, resulting in oversupply. Sellers face competition and seek to attract buyers by reducing the price. As the price falls, demand increases and, at the same time, sellers reduce production due to lower profitability. This process continues until price and quantity adjust to equilibrium where supply and demand equalize. Similarly, if the initial price of the product is too low, shortages are created and buyers compete to purchase it. In response to this situation, sellers increase the price to obtain higher profits and, in turn, increase production. Again, this process is repeated until equilibrium is reached where supply and demand are equal. In both cases, the process of price and quantity adjustment in a competitive market follows the principle of minimum energy, as supply and demand forces interact in a way that minimizes the effort required to maintain market equilibrium. As equilibrium is reached, the energy required to maintain it decreases, since there are no significant excesses or shortages.

Equilibrium Solution or Trajectory

We can find an analogy between the "GT" method and neural networks: Neural networks are trained by minimizing an energy function, commonly referred to as a cost or loss function. Simply put, when a neural network is deciding on a model or adjusting its weights, it does so by attempting to find the state that minimizes this loss function. In some contexts, this can be associated with a principle of minimal energy: The function that neural networks aim to minimize corresponds to the difference between reality and simulation. However, minimizing this function can present several challenges and problems: The main issue is that the network may not always choose the path of minimal energy or the correct path. This is due to (as can also happen with the "GT" model):

- **Local Minima:** The cost function may have multiple local minima. The network can get stuck in one of these, preventing it from finding the global optimal solution.
- **Regularization:** Regularization techniques are often used to prevent overfitting and improve the network's generalization. These techniques can influence parameter choices and may prevent the network from always

converging to the global minimum of the cost function.

- Stochastic Nature of Training: Many training algorithms use stochastic techniques, which introduce randomness into the process. This can lead the network to explore different regions of the parameter space and find solutions that are not necessarily global minima.

Equilibrium and Stagnation Zones: Stagnation Value "St"

We know that there are primarily two types of equilibrium: Stable and unstable equilibrium zones. Unstable equilibrium is defined as a state in which any small variation leads to a significant change, triggering complex and intense dynamics. These unstable zones can also be interpreted as stagnation zones.

In aerodynamics, a stagnation zone is an area where the kinetic energy of the flow is fully converted into static pressure; in other words, velocity is transformed into pressure. In fact, speed becomes pressure, the expression of pressure.

$$P = \rho v^2 \rightarrow P = \rho \quad (111)$$

The velocity must be minimal and the density maximal:

$$VT_j = \rho T_j * sign\left(\sum_{i=1}^{n}\left(x_i T_j - x_{i-1} T_j\right)\right) * \frac{\sum_{i=1}^{n}\left|x_i T_j - x_{i-1} T_j\right|}{n} \quad (112)$$

At equilibrium points and stagnation points, the velocity is zero.
We define now, the "STAGNATION Value" or "St" as:

$$Sv = \frac{V}{\rho} \quad (113)$$

Therefore, the areas likely to be stagnation zones will correspond to small "Sv" values; let's look at an example:

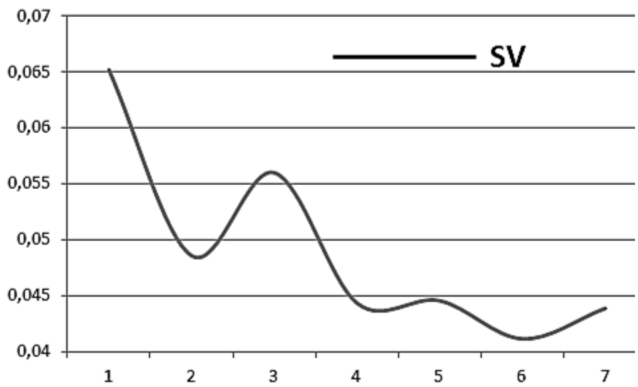

Figure 48

Curve "St" for the event Dow Jones of "E2" group. With respect to this graph, we can state that the optimal time to intervene in the market is right near the end. By identifying local and absolute minima, we can assess whether it is optimal or advisable to intervene at the specific moment of analysis. In fact, in areas with low "Sv", as in the case of the previous graph, the analyzed event is more susceptible to being altered in any direction; this is why, precisely at these moments, it is more convenient to attempt to influence the event, since it is when it can shift in any direction. This is, precisely, the definition of unstable equilibrium.

We can observe that, depending on the interval in which we find ourselves, there is a greater or lesser probability of it being a stagnation zone. Identifying these zones in two dimensions is important, as it is possible to apply Takens' theorem to locate them in "n" dimensions.

A necessary condition for the existence of equilibrium is that the change in velocity, i.e., acceleration, must be zero. More precisely, this condition implies that acceleration should reach a minimum. Under such circumstances, an equilibrium zone may emerge. This phenomenon is observed in many of the points or values discussed throughout this work: rather than isolated points of equilibrium, what actually exist are equilibrium zones. Within these zones, equilibrium can be either stable or unstable, allowing the system's values to oscillate to varying degrees, potentially exhibiting chaotic dynamics.

Note: The identification of a potential stagnation zone corresponds to the detection of an area with low acceleration. As a result, such regions are highly susceptible to variation, given that the force required to alter the system's state is minimal.

We can think in an analogous way about economic dynamics in general, with some examples or analogies:
After the stock market crash of October 1929, the U.S. economy entered a state of paralysis. Consumption and private investment plummeted, and unemployment skyrocketed. The financial system was frozen. There was no momentum from either demand or credit. In other words: zero velocity, like the stagnation point in aerodynamics. But the most interesting part is that in this state, small variations could trigger disproportionately large economic movements. Some examples:

- The collapse of a regional bank could cause a nationwide bank run.
- A slight drop in consumer confidence could lead to a massive decline in spending.
- A minor misstep in monetary policy (like raising interest rates in 1931) could trigger another severe contraction.

All of this created a chaotic dynamic: No one knew where the economy was headed. It was an unstable equilibrium, where the system could either collapse further... or recover, if the right policy was applied.

Another example

By late 2008, after the collapse of Lehman Brothers, the global economy went into a state of shock. What followed was a kind of "stagnation vortex" where:

- Investment came to a halt, even by solvent companies.
- Interbank credit froze, because no one trusted anyone.
- Consumption collapsed, due to fear and job losses.
- Central banks cut interest rates to zero, and still nothing restarted.

It was an economy with almost zero velocity. And any event, no matter how small, could trigger a chain of unpredictable reactions, like in a chaotic system. Chaotic dynamics in response to small disturbances:

- A rumor about a bank's exposure to subprime mortgages could spark a bank panic.
- A simple default by a mid-sized institution could lead to massive sell-offs in global markets. - A delayed decision by the government or central bank could worsen the situation due to a loss of confidence.

All of this was happening in an environment of extreme sensitivity to context. Historical correlations between economic variables stopped making sense: The stock market, gold, government debt, and currencies moved erratically. Pure chaos. What broke the stagnation? Just like in aerodynamics, where you need a new force vector to break the stagnation point, in economics, what helped was:

- Massive interventions by central banks (especially the Fed) through liquidity injections (QE).
- Selective nationalizations and bailouts of banks and insurers.
- Fiscal stimulus plans in the United States, China, and Europe.

These decisions weren't large in volume at first, but they had a signaling effect that helped restore some confidence. Again, small actions, taken at just the right moment in an ultrasensitive system, triggered a phase transition: from stagnation to the beginning of recovery.

Another Example

The Euro crisis (2010–2012) is another clear example of an economic system at an unstable stagnation point, with high sensitivity to seemingly small disturbances. Here, the situation was more political and institutional, but the economic effects were equally chaotic and volatile. After the outbreak of the global crisis in 2008, several southern European countries (Greece, Portugal, Spain, Italy) entered recession, with rising fiscal deficits and accumulated debts. But the euro had a problematic design:

- There was no common treasury in the Eurozone.
- Countries couldn't issue their own currency.
- The ECB had a very limited role (it didn't buy sovereign debt yet).

So, the system became trapped in a kind of structural blockade:

- The economies were in recession, markets demanded more austerity, and governments couldn't devalue or stimulate their economies. In other words, there was no obvious escape route, and that is exactly a stagnation point.
- During those years, the situation was so fragile that any news, rumor, or minor economic data could have enormous consequences. Examples:
- A statement from a Greek minister could cause Italy's country risk to rise.
- A credit rating downgrade by an agency could close access to financial markets.
- A meeting without an agreement in Brussels could cause stock markets across Europe to fall and trigger capital flight.

It was an economy in a "liquid state," with extreme volatility, where confidence was lost and regained at an illogical speed. The market reacted in an amplified way. In July 2012, when the crisis seemed on the brink of collapse, the president of the European Central Bank, Mario Draghi, made a short but decisive statement:

"The ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough." That statement, literally a microvariation in the flow of monetary policy, reversed the chaos.
The markets reacted immediately:
• Risk premiums fell.
• Stock markets rose.
• Capital returned.

And Most Importantly: No one had to intervene yet, but credibility changed the system's dynamics. That was the equivalent of a redirection of the aerodynamic flow: Extreme turbulence was avoided with a single credible gesture at the exact point of instability. Alternatively, a stagnation zone can be defined as a region where even a minor disturbance can produce unpredictable dynamics, potentially branching into multiple trajectories. Therefore, for a zone to be considered a stagnation zone, it must also represent an unstable equilibrium. From a geometric perspective, identifying stagnation zones is relatively straightforward, as they can be detected clearly through the representation of geodesic flows.

Example: Calculation of Action

We calculate the action curve corresponding to the Brent and Dow Jones events of the event group "E2". The phase space curve between these 2 events is a geodesic space:

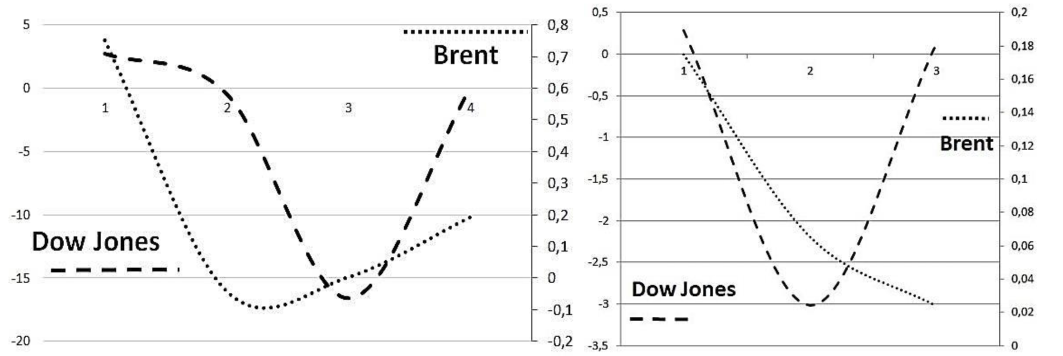

Figure 49

Hamiltonian Stock Curve for the 2 events analyzed jointly and in isolation (last 4 and 3 periods – no in "t"). The action quantified in the graphs corresponds to the actions from which the events move. Important: if the viscosity is low, the event moves easily, which means that the action can be of large magnitude.

Initials and Boundary Conditions; Pressure Field: Attractors and Repellers: Takens Theorem

Introduction

Obviously, as initial conditions we will use the data provided by the phase space (line) of the events analyzed, i.e. density, velocity, viscosity and pressure. Regarding the boundary conditions, Takens theorem states that we can study a complex chaotic system from a single observable variable, instead of needing to know all its internal variables; it is a matter of representing a variable in 2 coordinates, for example, and in each axis the variable with a "delay"; this representation shows the possible attractors and repellers of the global (full) phase space [52-56]. Any multivariable application of the "ABE" model or even Euler Lagrange implies the need to generate initial and boundary conditions, which define the dynamics of the variables. These conditions correspond to pressures, fundamentally. The pressure can be calculated:

$$P = \rho u^2 \rightarrow \partial P \triangleright u \partial u \qquad (114)$$

Which is nothing more than the transport equation; this means that the pressure variation depends mainly on the transport; that is: When other terms such as viscosity are introduced, the pressure variation will no longer depend exclusively on the transport; in any case, it is an excellent value as a "seed" value for the iteration that will solve the problem. On the other hand, if the pressure variation is zero, there will be no variation of the dynamics; that is: If the pressure variation is zero, the velocity variation will also be zero.

Full Pressure Field Generation

To generate the complete pressure field in "n" dimensions, we can proceed in two ways:
• Applying Takens' theorem: This theorem allows us to identify attractive and repulsive regions. However, calculating their intensities or exact values is complex. Still, we can use the identified regions as a first approximation of the real and full field.
• Repeatedly applying the "ABE" model: By analysing the initial data series, we obtain a line corresponding to the phase

space of the initial data. If we alter a value from one of the series, we will obtain another line corresponding to the new phase space. By continuously varying values and generating more lines, these will collectively form the pressure field, with the added advantage that it becomes possible to determine the pressures at each point on each line.

Some Applications of the Method

When we have located and quantified the pressure field in all space, we can do innumerable tests: To know the dynamics of a certain seed point (this point can even be a variation of a known point of the phase space, to find out how the economy would move in its case), to know the conditions before and after an instability, to create a crack or abrupt variation of one or several values, etc.

Knowing when instability is going to occur and what will be its consequences, which are susceptible to change, is something tremendously important; for this, we have 3 methods: "ABE" methods, "GT" method and Reynolds number.

Example: Location Attractors and Repellers in a 2 Dimensional Economic Space Using Takens and More

We are now working on 2 series of values, corresponding to 2 events: "CPI" of Spain between the years 2000 and 2021 and "Unemployment Rate" in Spain between the years 2000 and 2021.

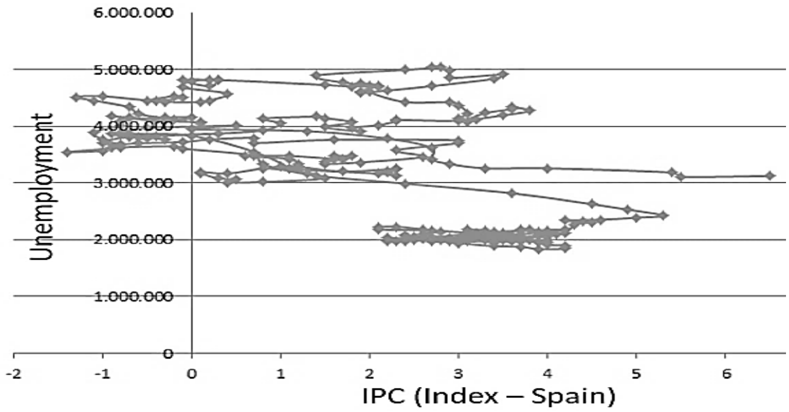

Figure 50: Phase space of the 2 events

We represent below both events (applying Takens) with a delay of 10 measurements; in both representations, we can find 2 accumulations, corresponding to the attractor zones:

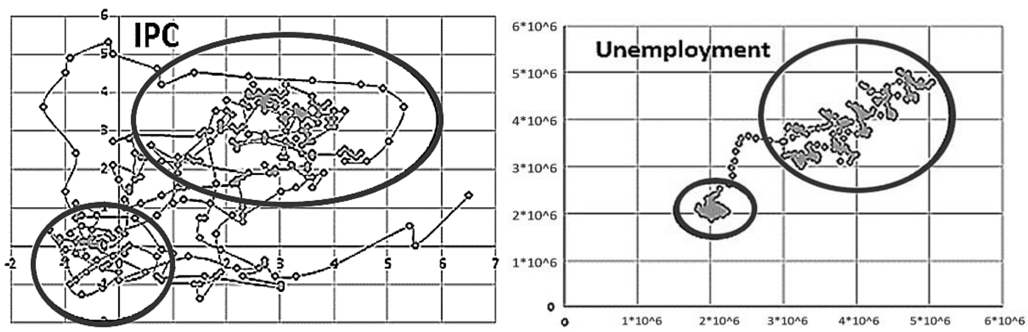

Figure 51

Attractor zones for each of the analyzed events, marked with circles.
As can be seen, the 2 attractor zones of the initial phase space are detected, "working" with each coordinate; note that the coordinates of the attractors in 1 coordinate coincide with the coordinates of the attractors in the joint phase space:

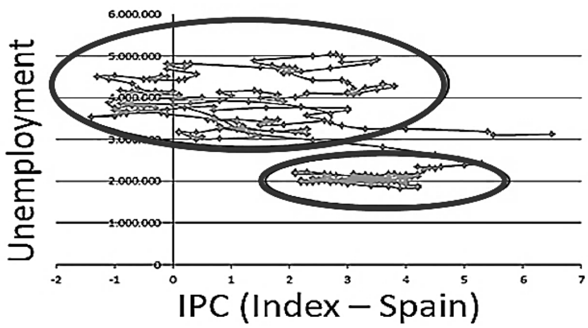

Figure 52: Location of the attractor zones in the joint phase space.

A complementary method to the one we have just discussed to help locate and define the high and low pressure zones is to smooth the phase space; important: We are not saying to smooth the data, but to smooth the line that defines the phase space. Another method we can use to know the pressure field is to draw the phase spaces by pairs, trios, etc. of the analyzed events; this will provide essential information to locate the attractors, repellers and their sizes or intensities. One way to think about the pressure field in "all space" is thinking about the distribution of "dark matter" in the universe; this distribution marks the dynamics of galaxies and, ultimately, the dynamics of the universe.

General Protocol – "Gt" Method

Assuming that the "n" events to be analyzed together have already been conveniently chosen:

- Pressure field detection and quantification.
- Choice of the seed point, depending on what is to be analyzed.
- Generation of the corresponding geodesy.
- Conclusions based on the results.

Examples: Calculation Pressure in Line as Phase Space

1 Event

Let's take the "WIG" index; we choose this event having read the article where it is described; in it, a value proper to the "Wig" is taken which is the "Volume"; therefore, the phase space between "Wig", "Volume" and time, can be considered three-dimensional, but also twodimensional; the structure that reflects the graph, is very peculiar [22]:

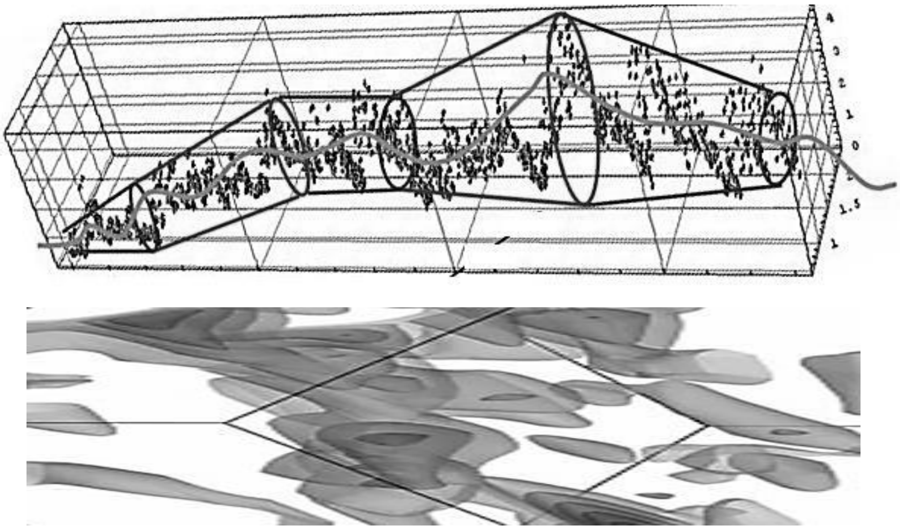

Figure 53

The dynamics or trajectory of the phase space shows a kind of "spiral" around a core. The image below represents an example distribution of the pressure field.
Inside this kind of deformed cylinder, around which the phase space "rotates", there is the distribution of low- and high-pressure zones; this distribution will mark the dynamics of any point located inside the space.

2 Events

We work with the events Brent Oil barrel price and Dow Jones index, belonging to the "E2" group of events. First, let us calculate the importance curves of the 2 events and the weighted curves of "P" and "Viscosity" (in last 4 sections, not in "t"); they will be used to represent the phase space:

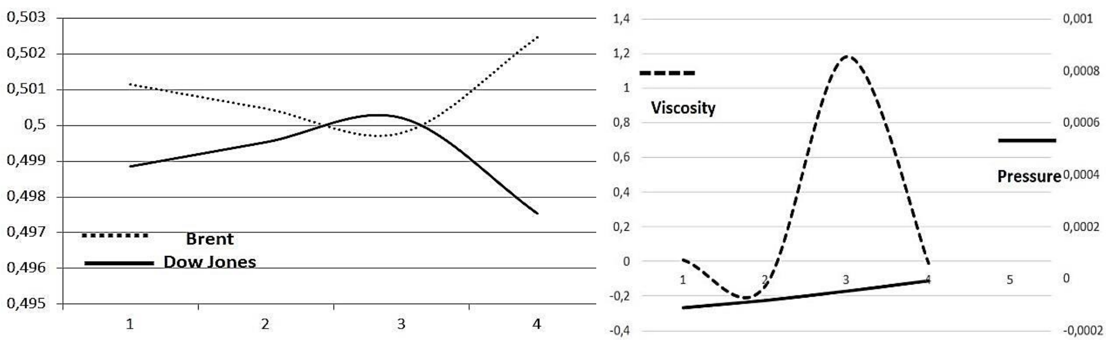

Figure 54

Curves of absolute importance between the 2 events and weighted curves.
The phase space is:

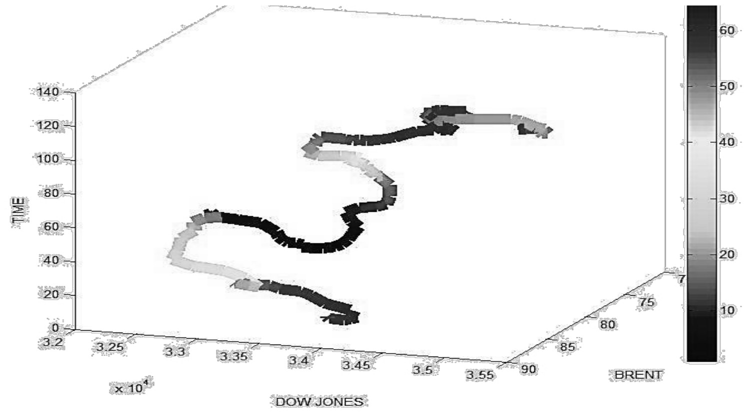

Figure 55

Phase space of the 2 events (last 5 sections) representing the weighted pressure in gray scale.

With this, we have calculated the pressures on a line corresponding to the phase space of the analyzed measures; from now on, it is necessary to create other dynamics from a distribution of seed points until we know the whole field of pressures; at least, to have it as well defined as possible. Only then will we be able to use it to test different economic policies, punctual actions, crises, instabilities, etc.

Government Intervention in the Control of the Economy

"Some" Mathematical Tools

There are instabilities inherent to the dynamics of the economy itself, produced by the interaction between the different events that make up its imperfect structure. These instabilities, whether slight or not, of greater or lesser magnitude, can cause serious problems to the economy; mainly, they can originate unstable equilibriums that are very dangerous, as well as sudden and "unexpected" alterations of the natural dynamics. It must be an essential task of any government to control such instabilities by modifying the structure and/or human decision making. It is not about politics, it is about efficiency and, therefore, making the economic process more efficient and making natural dynamics prevails over anything else. But it must also control other instabilities, even the "surprise" ones, in the sense that, knowing their harmful effects, make them less so and mitigate their consequences. Moreover: It is possible, by taking advantage of periods of instabilities, to create effects that would otherwise be difficult to achieve; in other words: Instability as a fulcrum for creating or repairing economic structure. The word "intervention" is always the subject of political debate in a multitude of media; but it should not be so; it should be understood as a necessary control, nothing more. With this work, we have tried to generate a series of tools and methods of action by the government in the Economy:

• To be able to create or remove pieces of the economy, knowing what the consequences may be.
• Act on the velocity and nodes of the events; both are key to the dynamics.
• By means of the transfer function: A- Calculate when and how much an event will respond to changes in another or others. B- Model and predict crises or instabilities in general. - By acting on viscosity: A- Promote and encourage groupings of people or markets in general. B- Mitigate or change the effects of crises, also working with "TF". C- Modify the velocity of an event. D- Modify the response of one economic event to another.
• By means of the test bench "GT" described in this work, it is possible to test different economic policies, knowing the evolution, detecting possible instabilities or crises, etc.
• Change the velocity in order to change the turbulence and other´s variables.
• If exist turbulence, is better to have a big velocity (the same occur with aircraft). - "Playing" with all definitions and concepts defined in this work, it´s possible to change all concepts. Etc.
• In short, in our opinion, very useful tools have been created for any government or Company, which greatly facilitate the modeling and prediction of any event or economic measure.

Conclusions

A model has been generated that is applicable to any group of time series, without any restriction, with which it is possible to predict the velocity "u" in the time section or interval "t+1", based on the intervals into which the initial series has been divided for analysis. As it has been verified, the generated model is a regression function that interpolates the points of the analyzed series; for it, a series of values (curves) are calculated so that the regression adjusts to the data; but: This regression function works with the transport and diffusion phenomena. The imprecision or uncertainty of this method can be great; this method, like many others, is based on the information provided by the series; the series has the information it has and its variations cannot be explained by itself: They are the result of interactions with other economic events that are not analyzed jointly; therefore, it is extremely important to be able to work with several dependent events jointly, in order to add to the information of each series the information that the dependence undoubtedly provides. This increases accuracy and reduces uncertainty in forecasting. In addition:

• It has been able to somehow predict instability, with all that this entails, mainly in terms of anticipating it, mitigating pernicious consequences or creating other desired effects.
• Controlling the dynamics of the economy, from a scientific point of view.

Economists, not without reason, complain that economics, to be a proper Science, should have a method of experimentation more or less similar to physics; the experiments carried out in economics (and received the Nobel Prize a few years ago) are of a Natural nature; that is: Based on measurements taken in countries, in such a way that it is possible to experiment with them, analyzing their consequences. On this occasion, a complete method of experimentation has been exposed, suitable for asking questions of all kinds and learning; the acquisition of knowledge is essential in Science. From this test bench, it is possible to test what are called various "Economic Policies"; that is: To test every kinds of economic actions:

- Predict crises or instabilities in general; therefore, reduce its possible consequences.
- Observe the impact that a given tax could have.
- To observe the re-distribution of the "equilibrium" in the case of applying shocks or instabilities to the system in any time series.
- What to do or not to do to achieve a certain objective.
- Choose some time series to observe their suitability.

Important
- Testing new economic "structures" (markets, etc) to observe the resulting new unstable equilibria.
- Detect and analyze unstable equilibria that may occur.
- To know what happen tomorrow, if today we change something (test).
- Etc.

In summary: The dynamics of the economy obviously depend on the structure and human decisions, as we have seen; but control by the government is essential; as an example, we can appreciate the evolution of the "GDP" of the Euro zone and EEUU, both before the crisis at the beginning of the century and in the Covid pandemic; EEUU, under certain measures, managed to remain after Covid, at just 1% of the previous "GDP", while the Euro zone, with other policies, remained at 5%. It must be taken into account that the intervention of the government represents a possible change in the structure of the economy and in human decision-making and for this, knowing in advance of possible instability is absolutely essential.

Important: In fact, the recent (2024) Nobel laureates in Economics, Daron Acemoglu, Simon Johnson, and James A. Robinson, were awarded for their studies on how institutions influence prosperity. Their findings highlight the point: The proper selection and creation of institutions, as part of the economic structure, directly affect economic dynamics, societal wellbeing, fair and efficient distribution of wealth and sustainable growth.

On the other hand, and very important, to point out that the magnitude of the government measures to be implemented must be proportional to the moment of their implementation; that is: If the arrival of turbulence is close, logically, the measures and changes must be deeper and more far-reaching. Important:

Knowledge is Power: Being able to apply science in a decision, and, therefore, having a greater probability than doing so by chance, implies guaranteed long-term gain. Freedom is freedom with knowledge to choose. Therefore: Having tools to model and predict the economy from a mathematical perspective with low uncertainty would be very interesting for several fundamental reasons:

- **Improvement in Economic Decision-Making:** A tool that allows predicting the economy with low uncertainty would help governments, businesses, and individuals to make informed decisions based on concrete data. Monetary policy, fiscal policy, and large-scale investment decisions could rely on much more reliable projections, reducing risks and optimizing outcomes.
- **Governments:** They could plan economic policies more precisely, such as public spending, tax collection, or inflation control, anticipating the impact of their actions.
- **Businesses:** They could anticipate economic cycles and adjust their production, investment, and expansion strategies, increasing their competitiveness.
- **Individuals:** Citizens could make better decisions regarding savings, consumption, or investment, minimizing financial risks.
- **Reduction of Risks and Vulnerabilities:** The economy is full of uncertainties and risks. A reliable mathematical economic model would allow forecasting economic crises, financial bubbles, or recessions before they occur, providing time to implement corrective measures. This would reduce the likelihood of falling into major financial crises like that of 2008, which was partly due to a lack of adequate foresight.
- **Optimization of Resources:** A precise tool would allow optimal resource allocation at both the microeconomic and macroeconomic levels. Governments could better direct public funds towards sectors that maximize economic growth and social equity. Businesses could optimize their capital resources and identify the most productive areas for investment. In general, the waste of economic, labor and technological resources would be reduced.
- **Improvement in Long-Term Projections:** The economy is often influenced by unforeseen events and long-term structural changes, such as demographic evolution or technological advancements. If mathematical tools can better model these factors, long-term projections would be more reliable, allowing for more solid future planning. This is particularly relevant for global challenges like climate change, which directly affects economies in the long run.

- **Deeper Understanding of Complex Economic Phenomena:** The economy is a highly complex system, influenced by a large number of interconnected variables. Advanced mathematical tools could help break down and better understand these relationships. For example, understanding the specific impact of monetary policy on inflation or how wealth distribution affects economic growth. This more detailed understanding would help identify effective levers for intervention and improve general welfare.
- **Development of More Just and Efficient Economic Policies:** Precise modelling would allow the creation of more balanced and equitable policies, as it would better predict how different social classes or sectors of the economy are affected by certain decisions. This could reduce inequalities and ensure inclusive economic growth.
- **Strengthening of Economic Stability:** Uncertainty generates volatility, which in turn creates instability in financial markets and the real economy. If uncertainty could be significantly reduced through precise mathematical models, economies could experience greater stability, which is crucial for sustainable long-term growth.

So: Objectives Achieved:
Optimal Selection of Economic Events
- Calculation of the "wave" model.
- Calculation of the "abe" model.
- Calculation of the pressure field.
- Calculation of the action.
- Calculation of geodesics. "gt" model.
- Calculation of Equilibrium points.
- Selection of economic measures.

Future Developments
General
In this section, we outline some lines of research that are considered relevant to broaden and deepen the results presented in this paper. These lines of research could give rise to future work that will make it possible:
- **Improve the Theory:** It is proposed to deepen the theoretical analysis of the problem, with the objective of obtaining a more complete and precise model than the one presented in this work.
- **Extend the Methodology:** It is proposed to study the possibility of applying the methodology developed in this work to other case studies or different problems.
- **Validate the Results:** Further empirical studies are proposed to validate the results obtained in this work and evaluate their applicability in the real world.
- **Explore New Applications:** We plan to explore new potential applications of the methodology and results obtained in this work.

In short, it is considered that this work opens up a line of research with great potential for future development, both in the theoretical and applied fields. Future work in this line will allow us to broaden our knowledge of the problem studied and to develop new tools and solutions for its approach. In addition to the lines of research mentioned above, it is also suggested:
- **Collect More Data:** It is proposed to collect more data to improve the robustness of the results and to be able to perform more accurate analyses.
- **Consider Other Factors:** Consider other factors that could influence the results, such as, for example, the social, economic or cultural context.
- **Conduct Comparative Studies:** It is proposed to conduct comparative studies with other existing methods or approaches to evaluate the advantages and disadvantages of the methodology developed in this work.

Surely, as always happens and should happen in Science, this theory can be improved:
Through backtesting it may be necessary to adjust some experimental coefficients in "ABE" (as a neural network) or even to add some term or concept to the model; but the theoretical basis is already done and we believe that this is the right way to go. It is expected that this work will be a starting point for future research that will advance the knowledge and understanding of the problem studied.

Software Used and to be Used to Create the Final Code
Of course, one of the fundamental objectives is to create a computational code that is capable of applying all this work on a group of events; the code in question will have several modules:
- **Module 1:** Calculation of classification criteria for all series; optimal choice of events; simplification or smoothing of time series. This first module is responsible for creating the series ready to be analyzed in module 2.
- **Module 2: "**ABE" model to the event group.
- **Module 3:** Back testing analysis: The objective is to improve the global procedure of the method, as well as the calculation of the density necessary for a long-term prediction.
- **Module 4:** "GT" Method.
- **Module 5:** Equilibrium points.
- **Module 6:** Selection of economic measures.

In this article, we have worked with 2 software's: Excel, Matlab and Python; in the future, Matlab and Python will be used to generate the code that will solve the "ABE" and "GT" models with "n" economic events and also as a neural network [57-61].

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