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Economic Performance as the Enduring Reality: A Closed-Loop Diagnostic Reconstruction of FCFE FCFF and EVA

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Abstract

This article develops a closed-loop method for diagnosing corporate economic performance before interpreting value. The central proposition is that valuation is not the primary object of analysis: it is a confirmatory representation of a deeper economic circulation in which capital is raised, employed in operations, converted into after-tax operating income, reinvested or released as cash, allocated across financing claims, and carried into the next operating state. Using a controlling five-year model for 2005–2009, the study reconstructs free cash flow to the firm from free cash flow to equity for every explicit year and at the continuing-value boundary. Under the workbook sign convention, FCFF equals FCFE plus the decrease in debt plus after-tax interest. The annual reconstruction closes with zero residual in all five years. At the continuing boundary, FCFE continuing value of 108,791.1362, terminal debt movement of 2,121.0000, and a workbook-reported financing and tax-shield component of 9,377.9090 reproduce FCFF continuing value of 120,290.0452 exactly. The reconstructed FCFF stream discounted at the declared WACC produces operating value of 74,268.4586; the asset-and-claim bridge produces common-equity value of 73,718.4586, or 335.0839028 per share, which converges with the direct FCFF and EVA routes. The contribution is not a new cash-flow identity. It is an auditable diagnostic architecture that integrates annual reversibility, capital-state continuity, terminal claim closure, multi-method convergence, and a management intervention protocol. The results support a disciplined distinction between persistent economic performance and valuation outputs that vary with claims, discount rates, timing, and terminal assumptions.

Keywords: Economic Performance, Closed-Loop Analysis, FCFE, FCFF, EVA, ROIC, WACC, Continuing Value, Financial Reconciliation, Management Diagnosis

JEL Classification: G12, G31, G32, G34, M41

Research Question

Can established corporate-finance components be made to operate as one time-linked and reversible diagnostic system that explains managerial economic performance before valuation is interpreted?

Introduction

Corporate finance often begins its public conclusion with an estimated value. That order is useful for transactions, but incomplete for management diagnosis. A value estimate compresses operating performance, reinvestment, financing, risk, timing, and terminal assumptions into one output. The compression is informative only when the underlying economic circulation has first been reconstructed. Otherwise, a mathematically correct present value may coexist with an economically weak explanation of how management created, preserved, or destroyed productive capacity.

The closed-loop approach reverses the order. It begins with the anatomy of the firm: operating invested capital, nonoperating assets, debt, equity, and other claims. It then follows the transformation of capital into NOPLAT, economic profit, reinvestment, FCFF, financing payments, FCFE, and the next operating state. Valuation enters after these movements close. It is therefore an end-point control and a comparative representation, not a substitute for performance analysis.

The empirical proof in this article is intentionally narrow. It asks whether the same cash history can be reconstructed across the operating and equity boundaries without unexplained leakage. The model reports FCFE directly. Adding the declared decrease in debt and after-tax interest reconstructs FCFF for each explicit year. The same principle is extended to continuing value, where the terminal bridge must identify debt movement and any financing or tax component separately. Finally, the reconstructed FCFF route is required to converge with direct FCFF/DCF and EVA.

The article makes three contributions. First, it converts a standard FCFE–FCFF identity into a year-by-year audit trail tied to debt states and operating capital. Second, it treats continuing value as a claim-boundary problem rather than a single residual formula. Third, it translates reconciliation into management questions and intervention tests. The contribution is integrative and procedural; it does not claim priority over the established theories from which the components are drawn.

Main Finding

The five annual bridges close exactly, and the terminal bridge closes arithmetically to four decimal places. When the reconstructed enterprise cash flow is valued under the declared operating discount rate and passed through the asset-and-claim bridge, it converges with the model's FCFF and EVA values. This establishes internal coherence under the model's stated assumptions. It does not, by itself, validate every economic assumption or identify the terminal tax component independently.

Theoretical Location and Literature

The framework stands at the intersection of several mature traditions. Modigliani and Miller separate operating value from financing effects and establish the importance of taxes and capital structure. Free-cash-flow theory distinguishes cash available to all capital providers from cash available to common equity. Residual-income and clean-surplus models connect accounting states to value. EVA makes the capital charge explicit. ROIC decomposition locates performance in margin and capital turnover. Value-driver models connect growth, reinvestment, return, and value. Performance-measurement systems add strategy and nonfinancial causation.

No single component is presented here as new. The standard claim-boundary identity is well known: FCFE equals FCFF less after-tax interest plus net borrowing. Likewise, the equivalence of DCF methods under consistent assumptions is established in the literature. The research opportunity lies in operational integration. Textbook identities are commonly stated once; a diagnostic system must reproduce them annually, connect them to opening and closing capital states, carry them through a terminal boundary, and explain any divergence among value representations.

Literature stream	Established strength	Closed-loop requirement
Capital structure	Separates assets and financing claims	Reconcile annual debt, equity and operating-capital states
FCFF and FCFE	Defines claim-consistent cash flows	Reverse the bridge annually and at continuing value
EVA and residual income	Prices surplus after a capital charge	Link economic profit to capital continuity and cash release
ROIC decomposition	Locates margin and turnover effects	Trace drivers into reinvestment, FCFF and the next state
DCF equivalence	Shows value convergence under consistent assumptions	Use convergence as a controlled error test
Performance systems	Adds strategic and operational causation	Place financial conservation inside a broader diagnostic process

Defensible Distinctiveness

The article therefore advances a claim of integrative and procedural distinctiveness. The distinctive object is the complete proof chain: defined economic anatomy; numerator–denominator conservation; income transmission; state continuity; bidirectional cash and claim closure; multi-path valuation convergence; and diagnosis before value. This wording is deliberately narrower than claiming that the individual equations or concepts are unprecedented.

The relation to prior work is complementary. Fernández demonstrates equivalence among valuation methods when assumptions match. Mielcarz and Mlinarič compare FCFF, FCFE, and EVA in capital budgeting. Stewart and Young and O'Byrne develop the economic-profit logic. The present framework asks an additional operational question: can those representations be made reversible inside one time-linked record that also specifies what management should investigate next?

The Research Gap as a Systems Problem

The research gap is not an absence of valuation equations. It is the absence, in ordinary analytical practice, of a single evidence chain that obliges each equation to preserve the same economic anatomy through time. A DCF model may correctly discount FCFF while leaving the operating-capital roll-forward implicit. An EVA model may calculate a capital charge without proving how the same capital produces cash available to claimants. An FCFE model may value equity without displaying how debt movements reconcile to closing debt. Each calculation can be locally correct while the combined corporate history remains incompletely explained.

This problem becomes material when management decisions alter several layers simultaneously. A price increase may raise margin, NOPLAT, cash, and value without requiring additional capital. Capacity expansion may raise NOPLAT and EVA while reducing current FCFF because investment precedes the operating return. Debt repayment may lower FCFE while strengthening the future financing state. A conventional endpoint comparison can misclassify all three events unless the model preserves the sequence from decision to operating state, cash release, claim allocation, and successor state.

The closed-loop proposition is therefore falsifiable. If opening capital, income, reinvestment, cash flow, financing movement, and closing claims cannot be reconciled, the loop fails regardless of whether the final valuation appears plausible. If the loop closes only after changing signs, rates, or terminal conventions from one representation to another, the apparent convergence is not evidence. The framework earns its conclusion only when definitions remain stable throughout the proof.

Analytical question	Conventional endpoint answer	Closed-loop evidence requirement
Did growth create value?	Value or earnings increased	Incremental return exceeded incremental required return and the successor state remained productive
Did cash generation improve?	FCFF or FCFE increased	NOPLAT and investment explain the change without hidden financing or underinvestment
Did leverage help equity?	ROE or FCFE increased	Debt, interest, tax, closing claims, and resilience reconcile
Is continuing value credible?	Perpetuity formula is finite	Growth, reinvestment, return, debt policy, tax treatment, and discount date form one state
Do valuation methods agree?	Reported values are similar	Independent routes converge under matched boundaries, rates, dates, and opening claims

Closed Loop Diagnostic Architecture

The unit of analysis is not a terminal value or a share price. It is a sequence of economic states. Opening operating invested capital supports activity during the year. Activity generates NOPLAT. Management then reinvests, releases, or reallocates capital. The resulting net investment determines FCFF and creates the closing operating state. Financing claims convert FCFF into FCFE through after-tax interest and net borrowing. Cash retained or distributed affects the equity and liquidity states. The next period begins from the state that these decisions produced.

$$\text{Opening } OIC_t + \text{Net operating investment}_t = \text{Closing } OIC_t = \text{Opening } OIC_{t+1}$$

$$FCFF_t = NOPLAT_t - \text{Net operating investment}_t$$

$$EVA_t = NOPLAT_t - WACC_t \times \text{Opening } OIC_t$$

$$FCFE_t = FCFF_t - \text{After-tax interest}_t + \text{Net borrowing}_t$$

These identities perform different diagnostic jobs. The capital roll-forward tests state continuity. FCFF tests whether operating income was converted into cash after reinvestment. EVA tests whether the operating return exceeded the required return on capital. FCFE identifies the residual cash available to equity after financing. None is sufficient alone. Their combined closure constrains interpretation: a claimed improvement must appear in the relevant income, capital, cash, and claimant states.

Diagnosis Before Value

The sequence of inquiry is: observed condition, evidence carried by the loop, management question, candidate intervention, and required confirmation. For example, rising FCFF can result from higher NOPLAT, lower investment, or both. Lower investment may reflect efficiency, completion of a capacity cycle, deferred maintenance, or underinvestment. The current cash release is therefore not classified as improvement until the subsequent operating state confirms that productive capacity and economic spread were preserved.

Performance and Valuation

Economic performance is the enduring object because it describes the productivity and continuity of the business process. Valuation is variable because it changes with discount rates, risk perceptions, claim boundaries, timing conventions, terminal growth, and market conditions. This does not make valuation unimportant. It defines its proper role: valuation translates a reconstructed economic history and a controlled expectation set into a present claim value. The quality of the translation cannot exceed the quality of the economic reconstruction.

Method Evidence and Conventions

The evidence is a controlling linked spreadsheet model covering 2005–2009 and its continuing-value state. The article treats the workbook as an auditable computational object rather than as an authority. Reported outputs are accepted only when they can be reconstructed from declared components and tested through independent identities. The analysis therefore distinguishes exact arithmetic closure from economic identification.

Element	Declared convention	Control
Operating boundary	NOPLAT and operating invested capital	FCFF and EVA must use the same boundary
Debt movement	Positive figure denotes decrease in debt	Convert explicitly when using net-borrowing notation
Interest	Workbook reports after-tax interest	Do not add tax a second time
Working capital	Included upstream in net operating investment	Do not add working capital again to ready FCFE
Discount rates	WACC for FCFF; required return on equity for standalone FCFE	Do not require unlike claim values to match
Terminal component	Workbook labels 9,377.9090 as financing/tax-shield component	Report closure; qualify independent identification

Sign Convention

The standard identity uses net borrowing as a positive inflow to equity. The workbook instead displays debt movement as a decrease in debt. Therefore, workbook debt movement equals minus net borrowing. The reversal used in the annual proof is:

$$FCFF_t = FCFE_t + \text{Decrease in debt}_t + \text{After-tax interest}_t$$

For 2005, the workbook records a debt increase of 57, displayed as a decrease in debt of –57. For 2006–2008, positive debt-movement figures represent repayments. The distinction is essential: adding a repayment to FCFE reverses the financing deduction and returns to cash available to all capital providers.

Proof Standard

A local proof closes one identity in one year. A boundary proof reconstructs cash across operating and financing claims. A state proof connects investment to the next capital state. A global proof requires consistent discounting and convergence among FCFF/DCF, EVA, and the transformed FCFE route. Zero residual is necessary, but not sufficient, for an economic conclusion: source classification, timing, and non-double-counting must also be reviewed.

Unit of Analysis and Evidence Perimeter

The unit of analysis is a dated corporate state and its transition to the next state. A state contains the operating-capital base, the financing claims, the income produced during the interval, the investment or release of capital, and the cash assigned to each claimant. This definition prevents a ratio from becoming detached from the numerator and denominator that created it. It also makes the model suitable for controlled comparison: a scenario is not merely a changed value, but a changed sequence of states.

The evidence perimeter separates source data, derived variables, and interpretive claims. Source data include the workbook's opening balances, annual cash flows, debt movements, rates, and declared continuing values. Derived variables include reconstructed FCFF, residuals, spreads, and present values. Interpretive claims include whether cash

release reflects efficiency, whether deleveraging improves resilience, and whether a terminal differential is economically a tax shield. The first two categories can be tested inside the model; the third may require evidence beyond it.

This distinction protects the article from circular validation. A value copied from an output cell does not verify the formula that generated it. A residual defined as the difference between two values does not independently identify its economic nature. Reproduction requires rebuilding the claimed result from upstream inputs whose relationships do not depend on the result being tested.

Data Lineage and Audit Trail

For each material claim, the audit trail records the source amount, sign convention, equation, reconstructed result, comparison value, residual, tolerance, and interpretation. Full precision is retained for calculation even when the article displays rounded values. A zero residual based on displayed figures is reported only where the rounded arithmetic also closes; otherwise, the result must be described as closing within the declared rounding tolerance.

Evidence object	Primary control	Failure signal
Opening and closing OIC	Capital roll-forward	Investment does not create the recorded successor state
NOPLAT and net investment	FCFF reconstruction	Operating cash lacks an income or investment explanation
FCFF and FCFE	Interest and debt bridge	Financing cash or sign convention is missing
Opening and closing debt	Debt roll-forward	Cash repayment or borrowing disagrees with claims
FCFF DCF and EVA	Operating-value convergence	Boundary, date, WACC, or terminal inconsistency
Terminal residual	Independent debt and tax schedule	Arithmetic closure lacks economic identification

Annual FCFE to FCFF Reconstruction

Table 2 applies the same reversal to every explicit year. Values are stated in the workbook's monetary units. After-tax interest and debt movement are financing-boundary items; working capital and operating taxes are not added separately because they are already embedded upstream in net investment, NOPLAT, and after-tax interest.

Year	FCFE	Debt movement	After-tax interest	Reconstructed FCFF	Reported FCFF	Residual
2005	2,790.59	-57.00	519.00	3,252.59	3,252.59	0.00
2006	1,158.75	3,055.00	409.00	4,622.75	4,622.75	0.00
2007	5,541.71	1,445.00	298.00	7,284.71	7,284.71	0.00
2008	8,568.63	978.00	171.00	9,717.63	9,717.63	0.00
2009	11,890.31	0.00	111.00	12,001.31	12,001.31	0.00

Year Specific Reading

In 2005, net borrowing of 57 partly offsets after-tax interest of 519. Under the workbook convention, the debt movement is -57; therefore $2,790.59 - 57 + 519 = 3,252.59$. In 2006, FCFF rises to 4,622.75 while FCFE falls to 1,158.75 because debt repayment of 3,055 and after-tax interest of 409 absorb cash before it reaches equity. This is a central diagnostic result: a lower FCFE does not necessarily indicate weaker operations.

The same logic explains the later years. Debt repayment declines from 1,445 in 2007 to 978 in 2008 and then to zero in 2009, while after-tax interest declines from 298 to 171 and 111. As financing absorption moderates, FCFE approaches FCFF. The annual series therefore separates an operating cash trend from the distribution of that cash between debt and equity claimants.

Interpretation of Exact Closure

The zero residuals prove that the workbook's annual FCFE and FCFF figures are mutually consistent under the declared classifications. They also demonstrate reversibility: the model can move from operations to equity and back without

losing the debt and interest effects. This is stronger than observing that two independently calculated values are “close.” It identifies the exact components that transport cash across the claim boundary.

Debt State Continuity

Cash-flow closure is reinforced by the debt roll-forward. Beginning debt of 7,542 rises to 7,599 after net borrowing of 57 in 2005. It then falls to 4,544 after repayment of 3,055, to 3,099 after repayment of 1,445, and to 2,121 after repayment of 978. With no further movement in 2009, debt remains 2,121. The cash-flow signs and the balance-sheet states therefore tell the same story.

Transition	Opening debt	Net borrowing repayment	Closing debt
2005	7,542	57	7,599
2006	7,599	-3,055	4,544
2007	4,544	-1,445	3,099
2008	3,099	-978	2,121
2009	2,121	0	2,121

This balance-sheet control gives the bridge economic content. A repayment cannot reduce FCFE unless it also reduces the debt claim; a borrowing inflow cannot increase FCFE without creating a corresponding liability. Agreement between cash movement and claim state rules out a class of spreadsheet errors that a valuation output alone would not reveal.

Why Working Capital and Tax are Not Added Again

A common reconciliation error is to append every familiar cash-flow adjustment to FCFE. That would double count. Working capital belongs to the operating investment schedule. It affects FCFF through net operating investment before the financing bridge begins. Once FCFE has been derived from FCFF, the reversal to FCFF requires only the financing components that separated the two cash-flow definitions: after-tax interest, net borrowing or repayment, and any explicitly declared boundary adjustment.

$$\text{Net operating investment} = \text{Change in operating working capital} + \text{Net investment in productive and other operating assets}$$

$$\text{FCFF} = \text{NOPLAT} - \text{Net operating investment}$$

Corporate tax is similarly located. NOPLAT is an after-tax operating measure, while the interest term used in the bridge is after tax. Adding a separate tax amount to the annual reversal would combine operating and financing tax treatments inconsistently. A valid bridge declares whether interest is pre-tax or after-tax and applies one convention throughout.

The Diagnostic Value of the Bridge

The bridge distinguishes three questions that are often conflated. Did operations generate cash after reinvestment? How much of that cash was absorbed or supplied by debt financing? How much remained available to common equity? FCFF answers the first, debt movement and interest answer the second, and FCFE answers the third. The model's value lies less in the final arithmetic than in keeping these questions simultaneously visible.

Observed movement	Economic reading	Required investigation
FCFF rises while FCFE falls	Operations improve but financing absorbs cash	Debt repayment schedule, interest burden, refinancing policy
FCFE rises faster than FCFF	Borrowing or lower financing cost supports equity cash	Sustainability of leverage and covenant headroom
FCFF rises as investment falls	Cash release may reflect efficiency or underinvestment	Maintenance, capacity utilization, and next-state NOPLAT
EVA rises while ROIC falls	Scale can outweigh a narrowing spread	Incremental return and capital intensity
Debt declines and cash accumulates	Resilience may improve, or capital may become idle	Liquidity target and positive-spread deployment

Thus, exact cash-flow equivalence is not the end of analysis. It establishes the reliability of the diagnostic image so that management can investigate causes without confusing operating performance with financing transmission.

Decomposing the Change in FCFF

The annual movement in FCFF can be assigned to two sources: the change in NOPLAT and the change in net operating investment. This decomposition is essential because identical increases in FCFF can carry opposite economic meanings. Higher NOPLAT with stable investment signals stronger operating productivity. Lower investment with stable NOPLAT may signal efficiency, completion of a build-out, asset harvesting, or deferred renewal.

$$\Delta FCFF_t = \Delta NOPLAT_t - \Delta \text{Net operating investment}_t$$

The model therefore refuses to reward current cash release without examining the successor state. If lower investment is followed by declining capacity, margin, or NOPLAT, the apparent improvement was obtained by consuming future productive capability. If operating performance is preserved or strengthened, the same cash release is evidence of improved capital efficiency. This temporal test is one of the main differences between valuation as an endpoint and performance diagnosis as a process.

FCFE is not a Dividend Forecast

FCFE measures cash available to common equity under the model's operating and financing assumptions. It is not necessarily the dividend paid. Undistributed FCFE must appear in cash, investments, retained earnings, or another identified equity movement. A distribution above current FCFE must be financed by opening liquidity, asset sales, new debt, or another explicit source. This distinction connects the cash-flow identity to governance: retention is justified when it supports positive-spread opportunities or necessary resilience, not merely because cash is available.

Continuing Value as a Claim Boundary

Continuing value is often treated as a final formula applied after the explicit forecast. In a closed-loop system, it is another economic state and must obey the same claim boundaries. FCFF continuing value belongs to all providers of capital. FCFE continuing value belongs to common equity after financing. The two cannot be reconciled merely by declaring that each is a perpetuity; the terminal debt policy, interest treatment, growth, and discount rates must be explicit.

The model reports FCFF continuing value of 120,290.0452 and FCFE continuing value of 108,791.1362. It also reports terminal debt movement of 2,121.0000 and a financing and tax-shield component of 9,377.9090. The arithmetic bridge is exact:

$$108,791.1362 + 2,121.0000 + 9,377.9090 = 120,290.0452$$

Terminal component	Amount	Role in closure
FCFE continuing value	108,791.1362	Equity-claim continuation
Terminal debt movement	2,121.0000	Reversal of financing movement
Financing and tax-shield component	9,377.9090	Workbook-reported terminal differential
Reconstructed FCFF continuing value	120,290.0452	All-capital-provider continuation
Arithmetic residual	0.0000	Exact terminal closure

What the Terminal Proof Establishes

The proof establishes that the reported terminal figures form a complete arithmetic reconciliation. It also prevents the 9,377.9090 component from disappearing inside a single unexplained terminal number. This transparency is important because continuing value carries a large share of total present value and small classification errors can dominate the explicit-period evidence.

What it Does not Establish

The 9,377.9090 figure is not a separately observed terminal cash flow. Its independent economic identification as an interest tax shield requires a terminal debt-and-tax schedule that reproduces the amount without using the residual equation itself. The article therefore retains the workbook label but distinguishes label, arithmetic role, and independent verification.

The study tests two benchmark terminal policies using the reported tax rate of 20.056 percent, pretax debt rate of 8 percent, and terminal debt of 2,121. Under permanent level debt, the standard estimate is $T \times D = 425.3878$. Under debt and shield growth of 7 percent discounted at the 8 percent debt rate, the estimate is 3,641.3192. Neither

benchmark reproduces 9,377.9090. Accordingly, the larger amount requires a different explicitly stated terminal financing convention before it can be identified independently as a tax shield.

Terminal Controls and the Double Counting Risk

A tax benefit can enter an enterprise valuation through more than one route. When FCFF is discounted at an after-tax WACC, the benefit of debt tax deductibility may already affect the discount rate. Adding a separately capitalized shield above the same FCFF stream can double count unless the valuation is explicitly structured as adjusted present value or otherwise separates the financing benefit. The terminal reconciliation must therefore be read together with the discount-rate architecture.

The correct control is not to force all methods to use one rate. FCFF belongs to the enterprise boundary and is discounted at WACC. Standalone FCFE belongs to the equity boundary and is discounted at the required return on equity. Their continuing values may differ even when both are internally coherent. Convergence is expected only after the financing boundary has been neutralized and the same asset-and-claim bridge has been applied.

Control question	Failure indicated	Required response
Is the interest term after tax?	Tax may be omitted or counted twice	Rebuild the annual bridge under one convention
Is debt movement signed consistently?	Borrowing may be mistaken for repayment	Translate to net borrowing before interpretation
Is the shield inside WACC or added separately?	Potential double counting	Choose WACC or APV architecture explicitly
Do terminal growth and reinvestment agree?	Unsustainable continuation	Link growth to incremental return and capital needs
Does closing debt support the terminal policy?	Claim state discontinuity	Roll forward debt and interest into the terminal state
Can the 9,377.9090 amount be independently reproduced?	Economic label remains provisional	Provide a separate debt-and-tax schedule

Interpretation

The disciplined conclusion is two-part. The terminal bridge is mathematically exact within the workbook. The economic characterization of one component remains conditional on an independent schedule and non-double-counting test. This is not a weakness in reporting; it is the hierarchy of proof working correctly. Exact arithmetic receives an exact conclusion, while an incompletely identified economic component receives a qualified conclusion.

Convergence of FCFF DCF EVA and the Transformed FCFE Route

After annual and terminal reconstruction, the resulting FCFF stream is discounted at the declared fixed WACC of 17.6753652604 percent. The operating value is 74,268.4586. The model's declared asset-and-claim bridge then produces common-equity value of 73,718.4586, or 335.0839028 per share. These figures equal the direct FCFF/DCF and EVA routes.

Representation	Boundary and rate	Result
Direct FCFF DCF	Operating cash flow discounted at WACC	Operating value 74,268.4586
EVA economic profit	Opening OIC plus present value of economic profit	Operating value 74,268.4586
Transformed FCFE to FCFF	FCFE reversed through financing boundary, then WACC	Operating value 74,268.4586
Asset and claim bridge	Operating value adjusted for declared nonoperating assets and claims	Common equity 73,718.4586
Per-share result	Common equity divided by shares	335.0839028

$$PV(FCFF) = \text{Opening OIC} + PV(EVA)$$

This identity explains why DCF and EVA converge when the same operating boundary, timing, WACC, and terminal assumptions are used. FCFF represents cash after reinvestment. EVA represents income after charging the capital

employed. The opening capital term connects the stock of productive resources to the stream of future surplus. A mismatch is therefore diagnostic: it points to an inconsistent capital base, discount date, tax treatment, terminal condition, or claim adjustment.

Meaning of Multi-Path Agreement

Agreement across routes does not mean that three independent economic realities exist. It means that one reconstructed reality survives three representations. The strength of the result lies in its redundancy. Each representation exposes different errors: FCFF exposes investment and cash conversion; EVA exposes the spread and capital charge; the transformed FCFE route exposes financing classification and claim transmission.

The convergence is a high-level internal validation of the model. External validity remains a separate question requiring observed company data, peer evidence, and prospective outcomes. The system is designed to make that distinction visible rather than to substitute internal agreement for empirical truth.

Structural Meaning of the DCF EVA Identity

The DCF–EVA identity is more than a second way to calculate the same number. FCFF recognizes that part of operating income must be reinvested to preserve or expand the capital base. EVA recognizes that the capital already employed carries an opportunity cost. When opening OIC, tax treatment, timing, WACC, and continuing assumptions are aligned, the present value of future FCFF equals opening operating capital plus the present value of future economic profit. The equality states that value above invested capital arises from expected surplus returns, not from cash-flow notation itself.

This interpretation places EVA and FCFF in their correct relationship. EVA is not an additional cash flow to be added to FCFF. It is an income representation of the same operating economics. Likewise, FCFF is not the original source of value independently of operations; it is the cash representation of NOPLAT after the reinvestment required by the operating state. Adding the two would double count the same economic process.

Conditions for Genuine Convergence

Condition	Why it matters	Diagnostic consequence if violated
Matched asset boundary	Both methods must value the same operating assets	Nonoperating assets or claims create a false difference
Matched tax convention	NOPLAT, interest, and WACC must use compatible taxes	Tax benefit is omitted or double counted
Matched discount date	Cash flows and capital charges must be timed consistently	Present values differ despite identical economics
Matched opening capital	EVA requires the capital already invested	Value added is confused with total operating value
Matched continuing state	Growth, return and reinvestment must agree	Terminal value absorbs an unexplained residual

Why Standalone FCFE Does not Equal the Enterprise Routes

The standalone FCFE route discounts explicit and continuing FCFE at the required return on equity of 18.6945480492 percent. It produces a present value of future FCFE inflows of 62,030.1606. After deducting opening equity of 6,206, the workbook reports FCFE NPV or value added of 55,824.1606, equal to 253.7461845 per share.

The transformed route begins from the same FCFE cash flows but reverses the financing boundary. It adds the annual debt movements and after-tax interest, applies the terminal bridge, discounts the resulting FCFF at WACC, and completes the asset-and-claim bridge. It produces 335.0839028 per share. The difference is 81.3377183 per share.

Item	Standalone FCFE route	Transformed operating route
Claim boundary	Common equity	All capital providers, then equity bridge
Discount rate	Required return on equity 18.6945480492%	WACC 17.6753652604%
Terminal treatment	FCFE continuing value 108,791.1362	FCFF continuing value 120,290.0452
Per-share result	253.7461845	335.0839028
Primary interpretation	Value of declared equity cash-flow claim	Enterprise economics after financing neutralization

The Difference is not the Tax Shield Alone

The 81.3377183 per-share difference must not be described as a tax-shield value. It is the combined consequence of financing neutralization, the terminal debt and financing component, different claim-consistent discount rates, and the declared asset-and-claim bridge. Attributing the entire difference to one line item would confuse a valuation comparison with a causal decomposition.

The two outputs answer different questions. Standalone FCFE asks what the declared equity cash-flow stream is worth at the equity required return. The transformed route asks what operating cash flow is worth after reversing financing effects and then assigns the resulting enterprise value across claims. Their divergence is informative precisely because the boundaries are different.

From Proof to Management Diagnosis

The model becomes a performance system only when mathematical closure leads to a managerial question. Table 8 translates the reconstructed history into a disciplined diagnostic sequence. Candidate interventions are hypotheses, not prescriptions; each must be confirmed in a later operating state or controlled scenario.

Observed condition	Evidence carried by the loop	Management question
Rapid operating expansion	Sales, NOPLAT and OIC rise	Did growth require proportionate capital?
Positive but narrowing productivity	ROIC declines while remaining above WACC	Which assets or decisions caused the decline?
Rising economic surplus	EVA rises in every explicit year	Is the increase driven by productivity or scale?
Stronger cash release	FCFF increases as net investment moderates	Is cash released through efficiency or underinvestment?
Deleveraging and liquidity accumulation	Debt declines and cash rises	Has resilience improved, or has idle capital accumulated?
Recovery signal in 2009	ROIC and NOPLAT improve after the 2008 low	Is recovery operationally attributable and repeatable?

ROIC WACC and Economic Spread

ROIC and WACC are not isolated valuation inputs. They are diagnostic coordinates of managerial effectiveness. ROIC measures the after-tax operating productivity of invested capital. WACC represents the required return on the financing boundary under the stated risk and capital-structure assumptions. Their difference is the economic spread. Positive spread indicates that the capital base earns more than its charge; negative spread indicates destruction of economic value.

$$Economic\ spread_t = ROIC_t - WACC_t$$

$$EVA_t = Economic\ spread_t \times Operating\ invested\ capital_t$$

Management can improve EVA by increasing spread, deploying more capital at a positive incremental spread, or releasing capital that earns below its required return. These routes are economically different. Growth raises value only when the return on incremental capital exceeds its incremental required return. Cutting investment raises current cash but can destroy value if it impairs later NOPLAT or capacity.

Five Year Diagnostic Narrative

Across 2005–2009, the model records operating expansion, positive economic spread, rising economic profit, increasing FCFF, substantial deleveraging, and a recovery signal after the 2008 low. These movements should not be compressed into a single favorable label. Expansion asks whether additional sales required proportionate capital. A narrowing ROIC that remains above WACC asks whether scale is masking weaker productivity. Rising EVA asks how much improvement comes from spread and how much from a larger capital base.

The financing history adds another layer. Debt repayment absorbs cash that would otherwise appear as FCFE, especially in 2006. The lower equity cash flow is therefore consistent with stronger operations and a safer claim structure. By 2009, debt movement is zero and after-tax interest has fallen to 111, allowing FCFE to approach FCFF. The trajectory illustrates why equity cash flow cannot diagnose operations without the financing bridge.

The 2009 recovery must also be treated as a hypothesis until its driver and persistence are verified. If margin improved, the analyst should identify pricing, mix, or cost changes. If turnover improved, the analyst should identify working-capital conversion, utilization, or asset release. If the improvement arose from a one-time accounting or nonoperating event, it should not be projected as a continuing operating state.

ROIC to ROE Transmission

ROIC belongs to the operating boundary, while ROE belongs to the residual equity boundary. Moving between them requires the income bridge, financing cost, debt movement, nonoperating income, and the equity base. Leverage can redistribute and amplify an operating result, but it cannot convert weak operating productivity into sustainable economic performance. Conversely, strong operating performance can be transmitted poorly to equity through expensive or fragile financing.

The diagnostic order is therefore margin and turnover, then ROIC, economic spread and EVA, then income and claim bridges, and finally ROE and the equity state. This order keeps the operating cause separate from the financing consequence while preserving their mathematical connection.

Intervention and Confirmation Protocol

Observed condition	Candidate intervention	Required confirmation
Rapid operating expansion	Improve pricing, mix, utilization and working-capital conversion	Higher or stabilized ROIC without impaired capacity
Positive but narrowing productivity	Reallocate or release low-return productive capital	Wider spread and sustainable NOPLAT in the next state
Rising economic surplus	Preserve positive-spread expansion; reject capital below its hurdle	Incremental return above incremental required return
Stronger cash release	Protect maintenance and positive-spread investment; distribute true surplus	No later deterioration in productive capacity or operating income
Deleveraging and liquidity accumulation	Optimize debt, liquidity reserves and excess-cash deployment	Lower financing vulnerability without dilution of capital productivity
Recovery signal in 2009	Act on the verified margin, turnover or capital-intensity driver	Persistence across later states and controlled scenarios

Incremental Return as a Treatment Test

Average ROIC describes the inherited capital base; it cannot by itself validate a proposed intervention. The decision must be tested on the capital changed. For an expansion, the screening rule is incremental NOPLAT divided by incremental operating invested capital. A full cohort analysis should match investment date, ramp-up period, incremental income stream, terminal recovery, and risk-adjusted required return.

$$\text{Incremental ROIC} = \Delta \text{NOPLAT} / \Delta \text{Operating invested capital}$$

$$\text{Accept only if Incremental ROIC} > \text{Incremental required return}$$

For capital release, the logic reverses. Releasing capital improves economic profit when the NOPLAT surrendered is smaller than the capital charge avoided. This allows the system to diagnose expansion and contraction without treating growth or cash maximization as inherently desirable.

Attribution and Persistence

An intervention is not confirmed because the forecasted valuation rises. It is confirmed when the targeted operating driver changes, the capital roll-forward remains coherent, the economic spread improves or is protected, and the result persists into subsequent states. Valuation then provides a final image of the cumulative consequences. This order protects management from optimizing a sensitive terminal output while weakening the productive system that must support it.

Implications for Governance and Capital Allocation

For boards, the framework changes the content of performance review. Instead of receiving a share-value estimate as a summary verdict, directors can ask where capital entered, which assets absorbed it, what after-tax operating income resulted, how much was reinvested, what cash was released, which claimant received it, and what state remained for the next period. The same questions apply to historical review and strategic forecasts.

For management, the loop disciplines capital allocation. A project should not be approved because it increases sales, EBITDA, EPS, FCFE, or even enterprise value under one assumption set. It should be traced through incremental operating capital, NOPLAT, spread, cash conversion, financing resilience, and subsequent capacity. Conversely, a project should not be rejected merely because it reduces current FCFF during a productive investment phase.

For lenders, annual FCFF-to-FCFE reconciliation clarifies whether debt service is supported by operations, asset sales, cash reserves, or new borrowing. For equity investors, it distinguishes operating improvement from leverage-supported

distributions. For auditors and model reviewers, the zero-residual bridges create an explicit control surface: every material claim can be tied to a source, identity, state transition, and tolerance.

The Medical Imaging Analogy

The closed-loop model functions as an economic diagnostic imaging system. It does not merely report a final corporate value; it reveals the internal anatomy of value creation. By tracing capital from its sources into productive employment, operating income, reinvestment, cash release, financing claims, and the next operating state, the model identifies where economic performance strengthens, where it deteriorates, and which managerial intervention may improve it.

The analogy has limits. A financial image can localize an inconsistency or performance change, but causal diagnosis may require operational data, commercial evidence, governance records, and management testimony outside the spreadsheet. The model should therefore guide investigation, not replace it. Its strongest contribution is to make unsupported explanations harder to sustain.

Economic Consequence

When firms allocate capital only to uses expected to earn more than their required return, productive resources are less likely to remain trapped in low-return activities. Better diagnosis can therefore improve not only shareholder outcomes but the allocation of savings across the wider economy. That broader benefit depends on disciplined measurement, transparent assumptions, and the willingness to act on verified drivers rather than on the attraction of a single value number.

Hierarchy of Proof Reproducibility and Limitations

The evidence should be read through a hierarchy. Level one verifies formulas and arithmetic. Level two verifies annual identities. Level three verifies capital and debt state continuity. Level four verifies cross-method convergence. Level five evaluates economic assumptions and external evidence. A claim may pass one level and remain open at another. The terminal 9,377.9090 component is the clearest example: it passes arithmetic closure but requires an independent schedule for full economic identification.

Proof level	Question	Status in this study
Formula integrity	Do cells implement their stated equations?	Reconstructed for the reported proof nodes
Annual identity	Does FCFE reverse to FCFF each year?	Exact; zero residual in 2005–2009
Terminal identity	Do claim components close at continuing value?	Exact arithmetic closure
Method convergence	Do FCFF, EVA and transformed FCFE agree?	Exact under declared boundaries and rates
Economic identification	Are all labels independently reproduced?	Conditional for the 9,377.9090 terminal component
External validity	Do outcomes persist in observed companies and later periods?	Not established by the internal model

Reproducibility Protocol

A reproducible test should freeze the controlling workbook; record file checksum, calculation mode, date convention, units, signs, and named scenarios; extract source cells and formulas; reconstruct NOPLAT, net investment, FCFF, FCFE, debt movement, and continuing value independently; calculate residuals at full precision; and report status as exact, within declared rounding tolerance, structurally reproduced under a clarified convention, explained difference, or unresolved difference.

Limitations

The study is a single-model proof and does not estimate population-level effects. Internal convergence cannot validate forecast realism, market risk premiums, capital structure, or terminal growth. Spreadsheet complexity creates model risk, including hidden links, sign reversals, stale ranges, and inconsistent timing. The medical analogy is conceptual and does not imply the certainty of clinical science. Finally, candidate interventions require operating evidence; financial identities locate questions but do not establish managerial causation on their own.

Falsification Tests and Robustness

A useful diagnostic framework must specify how it can fail. The annual claim bridge fails when any residual differs materially from zero after applying one documented sign convention. State continuity fails when investment does not reconcile opening and closing operating capital or when debt cash flows do not reproduce closing debt. Method convergence fails when independently reconstructed FCFF and EVA produce different operating values under matched assumptions. Economic identification fails when a labeled component cannot be generated from its own causal schedule.

Robustness should be tested by changing one controlled assumption at a time and propagating the change through all linked representations. Examples include fixed versus floating WACC, alternative growth rates, debt policies, tax rates, reinvestment requirements, and operating-margin or turnover scenarios. Structural robustness does not require the value to remain unchanged. It requires all representations to respond coherently and to converge again under the revised common assumptions.

Publication Standard for Model Based Claims

A publication-ready claim should disclose its boundary, date, units, source, sign convention, formula, independently reconstructed amount, residual, tolerance, and limitations. Screenshots may support traceability, but they do not replace a machine-readable schedule or a textual derivation. A reviewer should be able to reproduce the proof in another computational environment without depending on formatting, hidden names, or the original spreadsheet interface.

This standard is especially important for terminal value because a small change in growth or discount rate can dominate the explicit forecast. The continuing state should therefore receive at least the same evidential discipline as the annual years: operating return, reinvestment, debt, tax, claim ownership, and discount date must all be explicit.

Conclusion

The evidence supports a precise conclusion. FCFE, FCFF, and EVA can be organized as a closed-loop diagnostic system in which annual cash-flow identities are reversible, capital states are continuous, continuing value respects claimant boundaries, and matched valuation representations converge. In the controlling model, FCFE plus declared debt movement and after-tax interest reproduces FCFF exactly for 2005–2009. At continuing value, 108,791.1362 plus 2,121.0000 plus 9,377.9090 reproduces 120,290.0452. The reconstructed FCFF route then converges with direct FCFF/DCF and EVA at operating value of 74,268.4586 and common-equity value of 335.0839028 per share after the declared bridge.

The deeper result is methodological. Valuation should not dominate the analysis of a company's economic life. The persistent object is the firm's ability to employ capital productively, earn more than its required return, reinvest without destroying spread, release genuine surplus cash, transmit that cash coherently across claims, and preserve or improve the next operating state. Value is the changing financial reflection of those capabilities under a particular set of rates, risks, timing conventions, and terminal assumptions.

The framework's distinctiveness lies in integration, not in claiming invention of its components. It turns established equations into an auditable circulation of capital and a sequence of management tests. Exact closure gives confidence that the image is internally coherent; qualified reporting prevents arithmetic agreement from being mistaken for complete economic validation. That combination—reversibility, transparency, diagnosis, and restraint—is what makes the closed-loop model useful for boards, managers, lenders, investors, researchers, and reviewers [1-19].

Research Agenda

Future research should test the architecture across industries and business cycles; compare stable and distressed capital structures; reproduce terminal tax effects from independent debt schedules; examine whether intervention signals predict later ROIC and EVA; formalize uncertainty around operating states; and publish machine-readable proof ledgers. A particularly useful extension would separate accounting restatements, genuine operational change, financing transmission, and market repricing within one longitudinal dataset.

Author Declaration

The author declares that the numerical analysis is derived from the author's controlling closed-loop model and broader research project. The article is a focused, independently structured study. No claim of priority is made for the established FCFF, FCFE, EVA, residual-income, or discounted-cash-flow identities. No external funding was received for this study.

Appendix A Reproducible Proof Ledger

The proof ledger below separates the result claimed, the evidence used, the calculation required, and the conclusion permitted. It is designed to prevent a local equality from being promoted into a broader economic claim without the additional evidence that broader claim requires.

Claim	Evidence and reconstruction	Residual or test	Permitted conclusion
Annual FCFE FCFF reversibility	FCFE plus workbook debt movement plus after-tax interest for 2005–2009	Zero in every explicit year	Annual financing-boundary closure is exact
Debt state continuity	Opening debt plus net borrowing or repayment equals closing debt	Zero for each transition	Financing cash agrees with the balance-sheet claim
Terminal numerical closure	108,791.1362 plus 2,121.0000 plus 9,377.9090	Zero against 120,290.0452	The terminal differential exhausts the numerical gap
Terminal tax-shield identification	Independent level-debt and growing-debt benchmarks	425.3878 and 3,641.3192 do not equal 9,377.9090	A different explicit terminal financing convention is required
Operating-value convergence	Direct FCFF DCF, EVA, and transformed FCFE to FCFF	Each produces 74,268.4586	Global internal closure holds under declared assumptions
Common-equity convergence	Apply the same asset-and-claim bridge	73,718.4586 or 335.0839028 per share	The transformed route agrees with FCFF and EVA
Standalone FCFE value	Discount FCFE at the required return on equity	253.7461845 per share	A distinct equity-claim representation, not a failed convergence
Management diagnosis	Observed driver, candidate intervention, successor-state confirmation	Requires later operating evidence	Causal improvement remains a testable managerial proposition

Core Calculation Sequence

A reviewer can reproduce the central result without relying on the workbook's presentation layer. First, extract annual FCFE, debt movement, and after-tax interest at full precision. Second, translate debt movement into a single sign convention. Third, reconstruct FCFF and calculate the annual residual. Fourth, verify that the same debt movements reproduce closing debt. Fifth, reconstruct the continuing-value bridge and report the financing residual separately. Sixth, discount the reconstructed FCFF stream at the declared WACC. Seventh, apply the declared asset-and-claim bridge. Eighth, compare operating and equity results with the independently constructed FCFF and EVA routes.

At no point should a reconciliation residual be treated automatically as an economically identified cash flow. Identification requires a causal schedule appropriate to the label: investment for operating cash, debt and interest for financing cash, and debt policy, tax rate, utilization, growth, and shield discount rate for a terminal tax benefit.

Appendix B Interpretation Boundaries

Statement	Classification
The annual FCFE to FCFF residual is zero in every explicit year.	Established by direct arithmetic reconstruction
The debt roll-forward agrees with the annual cash-flow signs.	Established by state continuity
The terminal amounts reconcile exactly to 120,290.0452.	Established by arithmetic closure
The entire 9,377.9090 is an independently proven tax shield.	Not established under the two benchmark policies tested
The reconstructed FCFF route converges with direct FCFF DCF and EVA.	Established internally under the model's declared assumptions
The closed-loop architecture improves real-world decisions in all firms.	Research proposition requiring external testing
Economic performance is more persistent than any one valuation output.	Conceptual conclusion supported by the distinction between operating states and assumption-sensitive present values

These boundaries are part of the contribution. They permit the article to state exact results firmly while preserving uncertainty where the evidence remains conditional. Such restraint strengthens rather than weakens the diagnostic claim: the framework is intended to expose what is known, what is inferred, and what must be tested next.

Author Biography

Mr. Hany Hassanien Badr is an independent researcher in corporate finance and valuation systems and a banker based in Cairo, Egypt. He holds a B.A. in Economics from the Faculty of Economics and Political Science, Cairo University, and has more than fifty years of professional experience in banking, finance, treasury, investment, corporate governance, and valuation systems. He began his professional career at the Central Bank of Egypt in 1974 and subsequently served with Suez Canal Bank, Delta International Bank, and Faisal Islamic Bank of Egypt. His governance experience includes board service with Al-Ofok Porcelain Company, Helwan Fertilizers Company, Ibn Sina Pharma, and Takaful insurance. His research develops closed-loop corporate-finance and valuation frameworks linking operating capital, ROIC, NOPLAT,

WACC, the author-defined WACE diagnostic, economic profit, FCFF, FCFE, DCF, capital continuity, claim conservation, model validation, and institutional decision protocols.

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