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Finite Element Investigation of Entropy Generation and Natural Convection in a Non-Darcy Porous Cavity with Thermal Radiation

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Abstract

A high-resolution numerical investigation using finite elements is presented on the two-dimensional buoyancy-driven flow with thermal radiation in a porous square enclosure. The fluid flow within the permeable medium is modeled by the extended Brinkman–Forchheimer equations of motion. The study considers the combined effects of several dimensionless quantities, including the porous medium, Prandtl number, radiation parameter, and Rayleigh number. The anticipated flow patterns and velocity fields show a significant level of consistency with previous numerical simulations. At higher Prandtl number the velocity is independent of all dimensionless parameters. Furthermore, an increase in Rayleigh numbers leads to a significant increase in entropy generation, particularly entropy due to thermal transmission. High entropy generation occurs along the vertical walls of the cavity.

Keywords: Finite Element Method, Natural Convection, Darcy-Forchheimer, Entropy Generation, Thermal Radiation

Introduction

Today's engineering and scientific researchers routinely confront problems involving numerical solution techniques. Sometimes researchers must use programming skills to devise their own solutions. In other cases, these solutions are automatically generated by a software package. In either case, knowledge of numerical methods is necessary for developing and interpreting such solutions.

Nomenclature

$\bar{u}$	Velocity along x component	ρ	Density
$\bar{v}$	Velocity along y component	μ	Dynamics viscosity
$\bar{y}$	y component	c_p	Specific heat
$\bar{x}$	x component	κ	Thermal conductivity
$\bar{p}$	pressure	d_p	Mean particle diameter
T	Local temperature	K	Permeability
ϵ	porosity	F	Form drag constant
β	Thermal expansion coefficient	g	Gravitational acceleration
$ V $	Vector norm	σ^*	Stefan-Boltzmann constant
K'	Absorption coefficient	T_h	Maximum temperature
T_c	Minimum temperature	ω	Vorticity
ψ	Stream function	ν	Kinematic viscosity
Ra	Raleigh number	γ	Inverse Darcy number
Γ	Forchheimer resistance	Rd	Radiation parameter
Da	Darcy number	Gr	Grashof Number
Ec	Eckert number	Nu	Local Nusselt number
Nu_{avg}	Average Nusselt number	q_r	Thermal radiation
θ	Dimensionless temperature	Φ	Viscous dissipation
S_{total}	Total entropy	S_{fluid}	Entropy due to fluid friction
S_{heat}	Entropy due to heat	$\Delta\theta$	Temperature difference
$q_{r\bar{x}}$	Radiation flux with respect to $\bar{x}$	$q_{r\bar{y}}$	Radiation flux with respect to $\bar{y}$

with wisdom and insight. The Finite Element Method (FEM) is a numerical technique frequently applied in engineering and science-related fields to address complex flow phenomena. The numerical method uses iterations to compute solutions. In a technique like this, an approximation is made based on a prior approximation to generate successively, conceivably better, and better approximations; this process is repeated. Several important advances in recent years have led to further development and use of finite element techniques. This leads to the modeling of complicated phenomena like fractures, interfaces, and material interactions without the use of explicit meshing. The approach provides engineers and researchers with a diverse and powerful numerical tool for simulating and understanding the behavior of real-world systems in a computationally efficient manner [1]. Thomas et al., used a finite element-based commercial program FIDAP to simulate and compute fluid flow distribution in a continuous steel slab-casting machine's mold zone shell [2]. The authors investigated the impact of turbulence simulation, mold width, and casting speed. Puri and Anantheswaranb successfully used the finite-element method to model several food-processing operations [3]. Their work suggests that the finite element method still has considerable potential for food processing. Also, Jones and Clarke applied the FIDAP commercial software that utilizes the FEM approach (fluid dynamic package) to calculate forces of lift and drag on comparatively simple underwater vehicles, comprising of spheres, flat surfaces, cylindrical shapes, and profiles of wings [4]. Hughes et al., pioneered the idea of isogeometric analysis, which combines computer-aided design with FEM to improve simulation accuracy and efficiency [5]. Nguyen et al., developed a characteristic (based) approach using FEM to study free convection in an enclosed porous cavity [6]. Obstacles related to object size, motion, location, and position on flows are professionally managed by the finite element method. Both Nammi et al., and Bhowmick et al., applied the FEM to study the thermal dispersion and fluid flow characteristics within a cavity with embedded cylinders. Recently, Abderahmane et al., employed the Galerkin FEM to investigate buoyancy-induced flow of nanofluid flow in a wavy three-dimensional porous media subjected to magnetic effects [7-9]. The authors found that the average Bejan number is enhanced by only the Hartman number.

Natural convection is increasingly utilized for fluid transport processes owing to its reliability compared to its mechanical counterparts. The applications of natural convection are wide and varied and are well contained in the literature [10]. Solar collectors, gas-filled cavities around nuclear reactor cores, double-glazed windows, electronic instrument cooling systems, and building insulation are some of the most prominent applications for these technologies. Moreover, there is also the advantage of the second rule of thermodynamics on natural convection that can be applied to improve energy systems and thermal designs [11,12]. Experimental and theoretical research on buoyancy-induced flow through a porous medium has shown that the presence of solid matrix potentially affects the efficiency of fluid transport [13]. For instance, such an experiment was conducted by Krane and Jessee on free flow in a vertical enclosure. Assuming a natural convection type flow, De Vahl Davis implemented a benchmark numerical solution during buoyancy-induced flow of air in an enclosed chamber [14,15]. Several graphical representations were shown to prove the accuracy of the benchmark simulations. Fusegi et al., presented a numerical technique to the transport equation's solutions based on natural convection model [16]. For constant boundary temperature along the walls, their three-dimensional data

demonstrate reasonable agreement with experimental measurements. Barakos et al., revisited buoyancy-driven flow in a square enclosure, regarding turbulent and laminar models [17]. However, when the authors contrasted their findings with energy transfer experimental data, It demonstrates certain drawbacks of the conventional $k-\epsilon$ model with logarithmic wall functions, which results in a notable overprediction. Moreover, Baytas examined optimization in a slanted enclosure for low entropy production in free convection [18]. Khanafer et al., explored thermal distribution increase with nanofluids in a 2D enclosure during buoyancy-induced flow [19]. They demonstrated a relationship between the volume fraction, different Grashof numbers, and the average Nusselt number. A numerical analysis of free convection flow of nanofluid within a square hollow, where the impact because of viscosity and energy exchange are unknowns was reported by Ho et al., [12]. They found that buoyancy-induced heat transfer characteristics in the enclosure are significantly impacted by the uncertainties related to viscosity and thermal conduction. Sivasankaran and Bhuvaneswari reported a buoyancy-driven flow in a porous chamber with wavy warm walls. In a porous hollow, Pekmen and Oztop examined how natural convection was impacted by a partially magnetic field [20,21]. Recently, Seyyidi et al., analyses a hydromagnetic-free-convection inside a semi-annulus container that contains a micropolar nanofluid [22]. A detailed study of MHD buoyancy induced flow with entropy generation in a porous chamber utilizing the LBM was given by Charreh et al., [23]. The authors show that the Hartman number influence natural convection phenomena.

The Darcy-Forchheimer model, which accounts for both viscous and inertial forces, is commonly used to describe fluid flows in porous media [24]. In a porous media, the solid matrix can broaden the fluid flow resistance, leading to higher entropy formation. This resistance is quantified by the Forchheimer drag, which represents the nonlinear resistance to flow in the porous medium. The form drag has important implications for the velocity profile, thermal transmission, and entropy production in the system. Many industries, including petroleum engineering, chemical engineering, environmental engineering, and biomedical engineering, utilize non-Darcy models to predict flow behavior in various porous media systems, such as fibrous media, packed beds, and filter cakes [25]. When considering a Darcy-Brinkman flow in a permeable square chamber without thermal radiation and dissipation due to viscosity, Saeid and Pop concluded that enhancing frictional forces reduces the average Nusselt number and the enclosure's free convection process [26]. Singh et al., examined how non-Darcian influences affected a porous system with fluxes in the volume fraction and heat transport [27]. They analyzed thermal distribution in a channel with heat generation/absorption using the extended Darcy-Brinkman Forchheimer model. Nguyen et al., employed a numerical scheme to study natural convection in a non-Darcy permeable cavity with Cu-water nanofluid. Khan focused on the transportation of heterogeneous rotating disc under forced convective non-Darcy flow of nanoparticles [28]. Furthermore, Charreh et al., investigated non-Darcy flow in a permeable square chamber while taking heat radiation and dissipation into account [29]. The authors discovered that the medium's irreversibility and temperature dispersion were greatly impacted by the Forchheimer resistance.

Entropy generation is a natural and irreversible process that occurs in most heat transmission instances, as shown by thermodynamic processes. It is a measurement for the system's energy work value, where a high level of entropy indicates that the energy has no work value, and a low level indicates that the energy has a comparatively high work value. The performance degradation of engineering systems, such as rate and transport process, relied on the amount of irreversibility involved during a procedure. Thermal systems like turbines, diffusers, and pumps are insulated, and their effectiveness can be enhanced by decreasing irreversibility, such as heat transfer effects, in the process [11]. Drost and White conducted numerical predictions of development of irreversibility proving the practicality of irreversibility calculations for thermal design in an impinging jet [30]. Bejan et al., examined entropy formation in thermal designs, while Baytas investigated how to reduce the formation of entropy during buoyancy-induced flow [18,31]. Baytas postulated the production of irreversibility for free convection in a slanted porous cavity [32]. Mahmud and Fraser provided a comprehensive report on entropy generation characteristics in systems with flow and thermal distribution in a channel considering dissipation effects [24]. Saleem et al., investigation of entropy formation in thermocapillary convection flow in an open-ended chamber revealed that the active zone of entropy generation is contained in the middle buoyancy cell's aperture for small Grashof and Prandtl numbers [33]. Seyyedi et al., analyzed irreversibility production within a permeable hollow with an imposed magnetic field with a cardioid geometry at different angles [34]. Cham et al., conducted a Second law analysis numerically using MHD on buoyancy-driven flow in a permeable chamber in the presence of radiation and dissipation effects and discovered that the Lorentz force from the electromagnetic forces mitigates the production of entropy [35].

Thermal radiation is prevalent in daily life and is demonstrated by infrared thermometers and sunlight. A spontaneous radiative cooling process that removes energy from the item happens when an object emits thermal radiation. As a result, controlling thermal radiation or radiative cooling is essential for both the exploration of renewable energy sources as well as the development of useful cooling systems. Radiation affects the temperature distribution by introducing a sink or source of energy that is not related to the thermal gradient. The influence of radiation is only substantial under specific circumstances, such as at high temperatures or in optically thick media, and it is often weaker than other routes of heat transfer, such as conduction and convection. Hossain et al., discussed the influence of surface radiation on a buoyancy-induced flow in a permeable hollow with non-isothermal walls. Their findings are applicable to modelling geothermal and solar systems [36]. Pop and Sherment discussed the impact of radiation and viscous dissipation on a buoyancy-driven flow inside a permeable hollow filled with a non-Newtonian fluid [37]. Amber et al., design a numerical modeling technique used to examine natural convection caused by solar radiation [38]. They revisited previous analytical, experimental, and computational research reported in literature concerning buoyancy-induced flow

driven by absorption of thermal radiation. Mikhailenko et al., investigated thermal radiation for building application [39]. Lim et al., emphasizes the need to understand the impact of both natural convection and thermal radiation for fluid flow problems [40].

In this study, we develop a mathematical model to analyze the flow of a saturated fluid through a non-Darcy porous medium in a square cavity with thermal radiation. The model is solved using the Finite Element Method, which has applications in engineering, physical sciences, and interdisciplinary research. Our main objective is to analyze the flow, thermal transfer, and entropy formation characteristics qualitatively and quantitatively in order to improve energy systems and thermal designs using specific boundary conditions. We find that the flow rate can be increased by buoyancy effects depending on the Raleigh number, contrary to what is observed in flows in open cavities. These findings and other important observations are discussed in detail in the manuscript, along with their physical interpretations. Overall, our analysis demonstrates that harnessing viscous resistance in porous media can enhance the flow rate, which has potential applications in flow control and optimization.

Mathematical Formulation and Problem Statement

Problem Statement

The phenomenon studied considered a Roseland model for thermal radiation in an enclosed hollow containing a saturated non-Darcian permeable medium. Apart from the fluid density change in the buoyancy component, which is modelled using the conventional Oberbeck-Boussinesq approximation, the thermophysical properties of both the solid and fluid are isotropic, uniform, and constant. The enclosure's vertical right and left temperatures are maintained on the walls T_c and T_h , respectively. Furthermore, presumably, the enclosure's horizontal top and bottom surfaces are adiabatic. The flow description is represented by the rectangular coordinates (x,y) as displayed in Figure 1

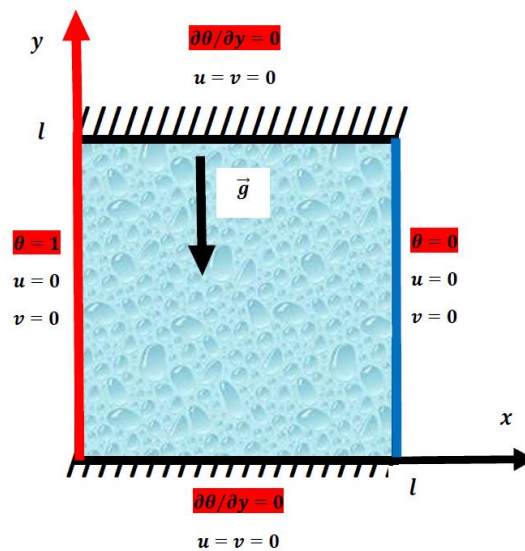


Figure 1: Schematic for the Physical Model

Governing Equations

The dimensional mass, momentum, and energy conservation for the laminar, two-dimensional, free convection flow inside the enclosure are expressed as [29,41].

Continuity Equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

x-component of the momentum equations:

$$\rho \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu \left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) - \mu \frac{\varepsilon^+}{K} \bar{u} - \rho \frac{F \varepsilon^{+2}}{\sqrt{K}} |\bar{v}| \bar{u} \quad (2)$$

y-component of the momentum equations:

$$\rho \left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu \left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) - \mu \frac{\varepsilon^+}{K} \bar{v} - \rho \frac{F \varepsilon^{+2}}{\sqrt{K}} |\bar{v}| \bar{v} + g \rho \beta (T - T_c) \quad (3)$$

Energy Equation:

$$\rho C_p \left(\frac{\partial T}{\partial \bar{t}} + \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} \right) = k \left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} \right) - \left[\frac{\partial q_r}{\partial \bar{x}} + \frac{\partial q_r}{\partial \bar{y}} \right] + \frac{\mu}{K} |V|^2 \quad (4)$$

The second and third terms shown to the right of the momentum equations of x, y are referred to as viscous and form drag, respectively. The velocity vector's norm is $|V| = \sqrt{u^2 + v^2}$, where β represents the thermal expansion coefficient and g represents gravitational acceleration. The dimensional velocity components within the $\bar{x}, \bar{y}$ rectangular coordinate is denoted by $\bar{u}, \bar{v}$. The pressure is denoted by $\bar{p}$, while the local temperature is denoted by T . In addition, μ is the dynamic viscosity, ρ is the density, C_p is the specific heat, and k is the thermal conduction. The permeability of the medium is $K = \frac{d_p^2 \varepsilon^3}{150(1-\varepsilon)^2}$ where d_p is the average particle diameter and ε the porosity. The Foechheimer resistance is given by $F = \frac{1.75(1-\varepsilon)}{d_p \varepsilon^3}$, whereas the constant $\frac{F}{\sqrt{K}}$ is the Forchheimer number.

Roseland's approximation of thermal radiation [37], allows for the simplification of q_r in the x and y – coordinates as.

$$q_{r\bar{x}} = \frac{-4\sigma^*}{3K'} \frac{\partial T^4}{\partial \bar{x}}, \quad q_{r\bar{y}} = \frac{-4\sigma^*}{3K'} \frac{\partial T^4}{\partial \bar{y}} \quad (5)$$

where K' is the absorption coefficient and σ^* is the Stefan-Boltzmann constant.

By removing high order terms and expanding the Taylor series on T^4 around T_c , we have

$$T^4 \approx 4TT_c^3 - 3T_c^4 \quad (6)$$

$$q_{r\bar{x}} = \frac{-16\sigma^*T_c^3}{3K'} \frac{\partial T}{\partial \bar{x}}, \quad q_{r\bar{y}} = \frac{-16\sigma^*T_c^3}{3K'} \frac{\partial T}{\partial \bar{y}} \quad (7)$$

As a result, by substituting Eq. (6.5) into Eq. (6.4), we obtain.

$$C_p \left(\frac{\partial T}{\partial \bar{t}} + \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} \right) = \left(1 + \frac{16\sigma^*T_c^3}{3kK'} \right) \left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} \right) + \frac{\mu}{K} |V|^2 \quad (8)$$

We introduce dimensionless variables below to transform the governing equations.

$$x = \frac{\bar{x}}{l}, \quad y = \frac{\bar{y}}{l}, \quad t = \frac{\nu}{l^2} \bar{t}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \bar{u} = \frac{\nu}{l} u, \quad \bar{v} = \frac{\nu}{l} v, \quad (9)$$

$$p = \frac{l^2}{\rho \nu^2} \bar{p}.$$

Also, with the dimensional stream function ψ , $(u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x})$ and vorticity $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.

Thus, employing the dimensionless variables Eq. (9), the governing equations (1) – (8) can be expressed as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (10)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \gamma u - \Gamma |V|u, \quad (11)$$

$$(12)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Gr\theta - \gamma v - \Gamma |V|v, \quad (13)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left(1 + \frac{4}{3} Rd \right) \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \frac{Ec}{Da} |V|^2. \quad (14)$$

Here $Ra = \frac{l^3 g \beta (T_h - T_c)}{\alpha \nu}$ is the Rayleigh number, $\gamma = \frac{l^2 \varepsilon^+}{K}$ is the inverse Darcy number, the Forchheimer resistance is represented by $\Gamma = \frac{l^2 F \varepsilon^+}{\sqrt{K}}$, the Grashof number $Gr = \frac{Ra}{Pr}$, the Prandtl number $Pr = \frac{\nu}{\alpha}$, and the kinematic viscosity $\nu = \frac{\mu}{\rho}$. The formula for Darcy number is $Da = \frac{K}{l^2}$, $Rd = \frac{4\sigma^* T_c^3}{kK'}$ is the radiation parameter, and $Ec = \frac{\nu^2}{C_p (T_h - T_c) l^2}$ is the Eckert

number.

Initial and Boundary Conditions

The dimensionless variables for initial and boundary conditions given are.

$$u = v = \theta = 0, \text{ for } t = 0, \quad 0 \leq x, y \leq l, t > 0, \quad (15)$$

$$\text{Left wall} \quad x = 0, \psi = 0, \theta = 1, \omega = -\frac{\partial^2 \psi}{\partial x^2}, \quad (16)$$

$$\text{Right wall} \quad x = 1, \psi = 0, \theta = 0, \omega = -\frac{\partial^2 \psi}{\partial x^2}, \quad (17)$$

$$\text{Bottom and top walls} \quad \psi = 0, \omega = -\frac{\partial^2 \psi}{\partial y^2}, \frac{\partial \theta}{\partial y} = 0, \text{ at } y = 0, 1. \quad (18)$$

Formulation of the Nusselt Number

Local Nusselt number and average Nusselt number are two physical quantities gaining a lot of attention in our research since they show the prominent form in thermal transfers.

Both the local and average Nusselt numbers can be stated as.

$$Nu = -\left(1 + \frac{4}{3} Rd\right) \frac{\partial \theta}{\partial x} \Big|_{x=0}, \quad Nu_{avg} = \int_0^1 Nu dy \quad (19)$$

Entropy Generation

Convection in a porous media is by its very nature irreversible. Non-equilibrium circumstances result from the transfer of momentum and energy at the solid barriers and inside the fluid. Continuous entropy generation results from this. Heat transport in the finite temperature gradients' direction accounts for a fraction of this entropy creation. In all thermal engineering applications, irreversible finite temperature differential heat transfer is typical. The irreversibility of fluid friction causes a further increase in entropy. The quantity of work value in energy process is measured by entropy generation. Entropy is divided into two components in such models: entropy with heat transfer and entropy with fluid friction or viscous irreversibility. The overall entropy is the sum of the several entropy sections indicated in the cavity.

We calculate the volumetric rate for the given entropy generation on a homogeneous porous system using the second law of thermodynamics [24, 29, 33].

$$S_{heat} = \frac{k}{T_c^2} \left(\left(\frac{\partial T}{\partial \bar{x}} \right)^2 + \left(\frac{\partial T}{\partial \bar{y}} \right)^2 \right) - \frac{1}{T_c^2} \left(\left[\frac{\partial q_r}{\partial \bar{x}} + \frac{\partial q_r}{\partial \bar{y}} \right] \right), \quad (20)$$

$$S_{fluid} = \frac{\mu}{T_c} \Phi + \frac{\mu}{KT_c} (\bar{u}^2 + \bar{v}^2). \quad (21)$$

Since

$$S_{total} = S_{heat} + S_{fluid},$$

then

$$S_{total} = \frac{k}{T_c^2} \left(1 + \frac{16\sigma^* T_c^3}{3kK'} \right) \left[\left(\frac{\partial T}{\partial \bar{x}} \right)^2 + \left(\frac{\partial T}{\partial \bar{y}} \right)^2 \right] + \frac{\mu}{T_c} \Phi + \frac{\mu}{KT_c} |V|^2 \quad (22)$$

the viscous dissipation is expressed as:

$$\Phi = \left(2 \left[\left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 \right] + \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 \right) \quad (23)$$

The first component of Eq. (22) represents entropy owing to heat, while the subsequent terms reflect entropy due to viscous dissipation of porous media or the non-Darcy flow model. Using the non-dimensional quantities from Eq. (9), the non-dimensional equation for the overall irreversibility is given as.

$$S_{total} = \left(1 + \frac{4}{3} Rd\right) \left[\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2 \right] + \overline{Ec} \Phi + \gamma \overline{Ec} (u^2 + v^2). \quad (24)$$

The Modified Eckert number is $\overline{Ec} = \frac{PrEc}{\Delta \theta}$ and the dimensionless temperature difference is $\Delta \theta = \frac{T_h - T_c}{T_c}$. Neglecting the energy equation's viscous dissipation when the main goal of our research is to determine irreversibility due to fluid friction and heat transfer effects may initially seem inconsistent [30]. However, the evaluation of Eq. (24) only relies on having precise knowledge of the velocity field, temperature, and transport characteristics. Several benchmark results have demonstrated that the finite element method can reliably forecast temperature and velocity fields even without accounting for viscous dissipation in the energy equation. This implies that even in the absence of viscous dissipation in the energy equation, local entropy generation can be predicted using the FEM results in conjunction with Eq. (24).

Numerical Solution and Code Validation

Most real-world issues are too complicated to have analytical solutions. In this instance, the velocity and temperature fields can be predicted using a computational fluid dynamics code. In our research, we employed the finite element method to solve the governing equations (10) – (24) along with their associated boundary and initial conditions. The domain is discretized into finite elements, and the governing equations are approximated over each element using basis functions. To convert the partial differential equations into a set of integral equations, we employ the Galerkin weighted residual technique. Additionally, we apply the Gauss quadrature technique to integrate these equations. The criterion for convergence of the thermal and velocity fields is $\max \left| \frac{\lambda^n + \lambda^{n-1}}{\lambda^n} \right| \leq 10^{-6}$, where n is defined as the value of a transport variable over n iterations.

The numerical algorithm was evaluated for classical buoyancy-driven flow within an enclosure. The average Nusselt number was calculated, and the results obtained from the current approach were compared with those reported by various authors for different Rayleigh numbers, as indicated in Table 1. It is found that the results obtained in the present study show good agreement with the results in previous studies for different Rayleigh numbers. In addition, an extensive grid independence test was carried out at $Ra = 10^6$, $Pr = 10$, $Ec = 10^{-6}$, $Rd = 2$, and $\gamma = \Gamma = 0.5$. For the mesh sensitivity study, we considered five different grid sizes: 18005, 28765, 73065, 184465, and 278525 elements. The grid density with 73063 elements was used for the entire simulations.

Table 2 shows the Average Nusselt Number for the Chosen Grid Systems with different Numbers of Elements.

Ra	Present	Khanafer et al., [19]	Barakos et al., [17]	De Vahl Davis [15]	Fusegi et al., [16]
10^3	1.1178	1.118	1.114	1.118	1.105
10^4	2.2442	2.245	2.245	2.243	2.302
10^5	4.5187	4.522	4.510	4.519	4.646

Table 1: Comparison of the Average Nusselt Number with Previous Works for Different Ra -values

Elements	$Nu_{(avg)}$	$\Delta_{Nu_{(avg)}} = \frac{ Nu_{(avg), Elements} - Nu_{(avg), 73063} }{Nu_{(avg), Elements}} \times 100\%$
18005	22.582	0.0709%
28765	22.575	0.0399%
73065	22.566	0.0000%
184465	22.565	0.0044%
278525	22.565	0.0044%

Table 2: Comparison of Average Nusselt Number for different Number of Elements

Results and Discussion

This section focuses on the physical interpretation of the numerical results obtained from the current investigation. It analyzes the flow field, thermal fields, entropy isolines, velocities, maximum stream functions, and Nusselt numbers. Particular attention is given to the influence of relevant dimensionless parameters on the Brinkman-Forchheimer model within a permeable square cavity. This analysis sheds light on the physical behavior of flow, heat transfer, and entropy generation related to non-Darcian flow, as well as quantifying the impact of inertial forces on flow dynamics and energy transfer along with entropy. The crucial dimensionless parameters emerging from this study include Ra , Pr , Rd , γ and Γ . Consequently, to explore the effects of radiation flux alongside the viscous and form drag within the porous medium subjected to a heated vertical left wall, the ensuing discussions will concentrate on how these dimensionless factors influence flow, thermal behavior, and entropy generation.

Effect of Radiation

Radiation is a distinctive method of thermal transfer, as it does not necessitate a medium for propagation. It involves heat transfer via electromagnetic waves. Figure 2 illustrates the flow fields, thermal field, and entropy contours for varying radiation parameters. An increase in radiation results in diminished fluid circulation within the porous chamber and a reduction in the size of the convective cell core. When the radiation parameter is low, a single convective cell occupies the cavity's center, indicating decreased heat transfer through conduction. The isotherms in Figure 2 demonstrate that the upper horizontal wall experiences a more significant influx of high thermal energy from the left hot wall, while the lower adiabatic wall sees less intense penetration of lower thermal energy from the right cold wall. The temperature gradients and fluid flow resistance contribute to entropy generation. A steeper temperature gradient between the left hot wall and the right cold wall results in an increased heat transfer rate, as well as greater maximum entropy production along the vertical walls.

In Figure 3 (a – c), we depict the variation of radiation with the average Nusselt number, maximum stream function, and vertical velocity profile at the vertical midpoint. Radiation can influence the average Nusselt number by altering the fluid's temperature distribution, which subsequently affects the convective heat transfer coefficient. The average Nusselt number rises linearly with increasing radiation parameters. At sufficiently large radiation parameters, significant changes occur in the maximum stream function. This is attributed to the fact that the impact of radiation on the maximum stream function relies on the relative significance of radiative heat transfer in comparison to other heat transfer modes such as conduction and convection. Figure 3 (c) illustrates how radiation affects the vertical velocity profile. The vertical velocity increases along the heated surface with rising thermal radiation, as flow resistance is minimized under high radiation conditions. To evaluate the effect of thermal radiation on entropy, we present Figure Figure 4 (a – c). The level of entropy within the porous hollow is influenced by the temperature gradient and viscous effects. As the radiation parameter increases, the temperature gradient between the vertical walls escalates, leading to an increase in entropy production along the vertical walls, thus resulting in a non-linear distribution of total entropy production.

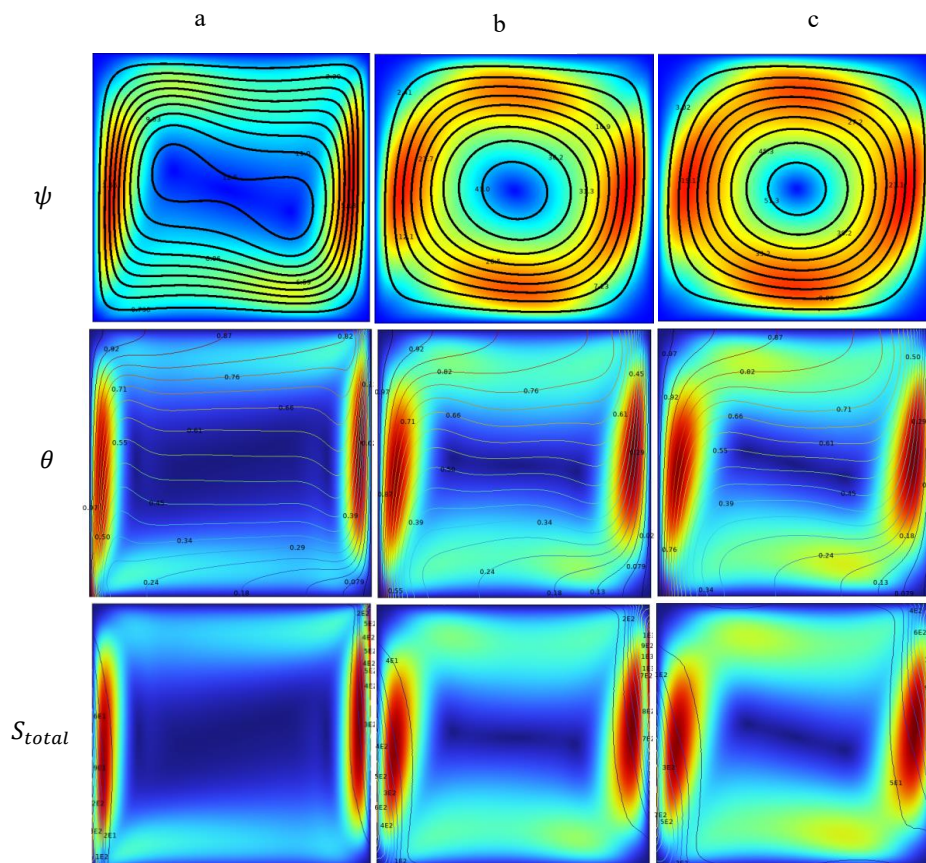


Figure 2

Effect of Prandtl Number

To grasp the implications of the Prandtl number, it is important to note that it indicates the ratio of momentum diffusivity to thermal diffusivity. An increase in the Prandtl number enhances momentum diffusivity, which ultimately surpasses thermal diffusivity. Figure 5 (a) Average Nusselt number, (b) Maximum stream function $\psi_{\max}$ and, (c) Velocity at vertical mid-section for various Prandtl number illustrates how the Prandtl number affects the flow fields, heat lines, and entropy isolines. A higher Prandtl number leads to reduced convective flow within the enclosure. Additionally, as the Prandtl number rises, the size of the convective core decreases and shifts towards the upper left corner. This shift is a result of increased viscous forces within the fluid layers, which hinder fluid motion. The heightened flow resistance then reduces the flow rate. On the other hand, at sufficiently low Prandtl numbers, the flow rate of the system can be improved while taking into account the absolute maximum values of the stream function. A decline in the Prandtl number accelerates energy transfer and thickens the thermal boundary layer, as thermal diffusivity increases when the Prandtl number decreases. Regarding entropy isolines, entropy production is more pronounced at lower Prandtl numbers. The total irreversibility is more significant at the upper corner of the cold wall and the left corner of the heated wall, a situation attributed to the clustering of isotherms in this region of the cavity.

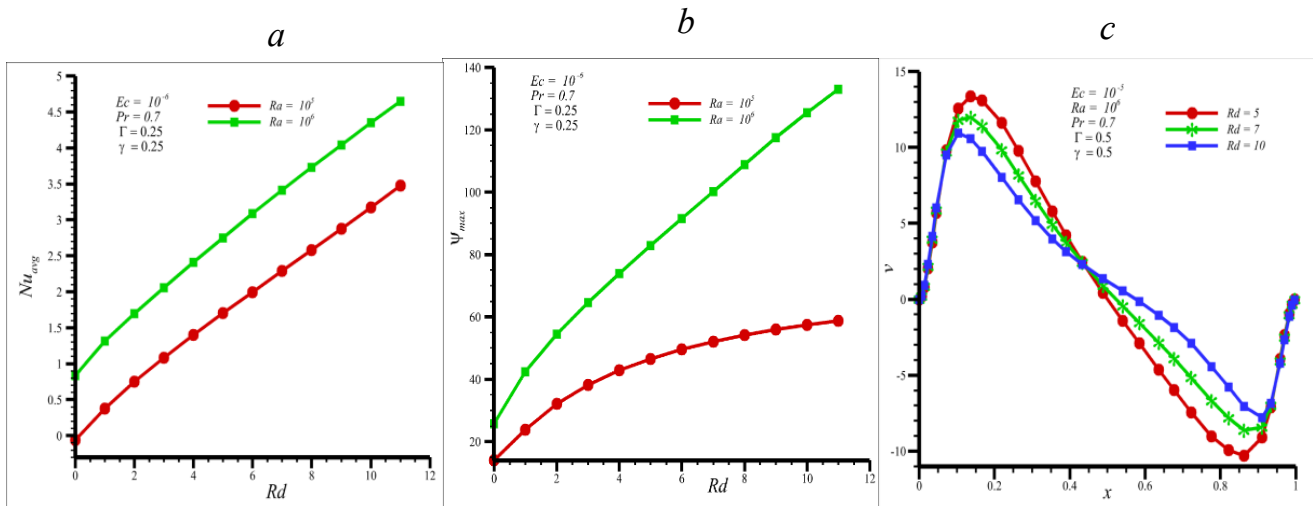


Figure 3

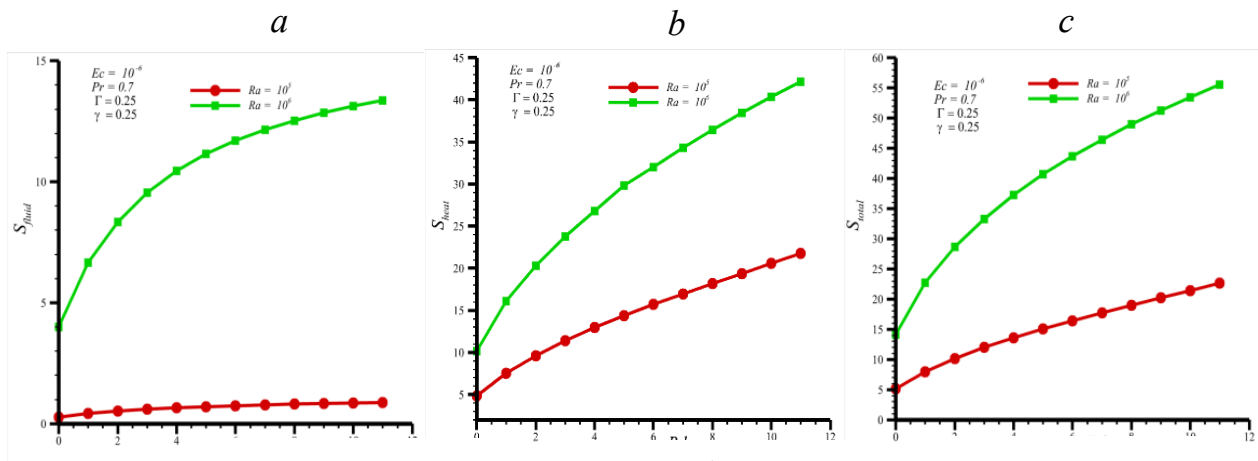


Figure 4

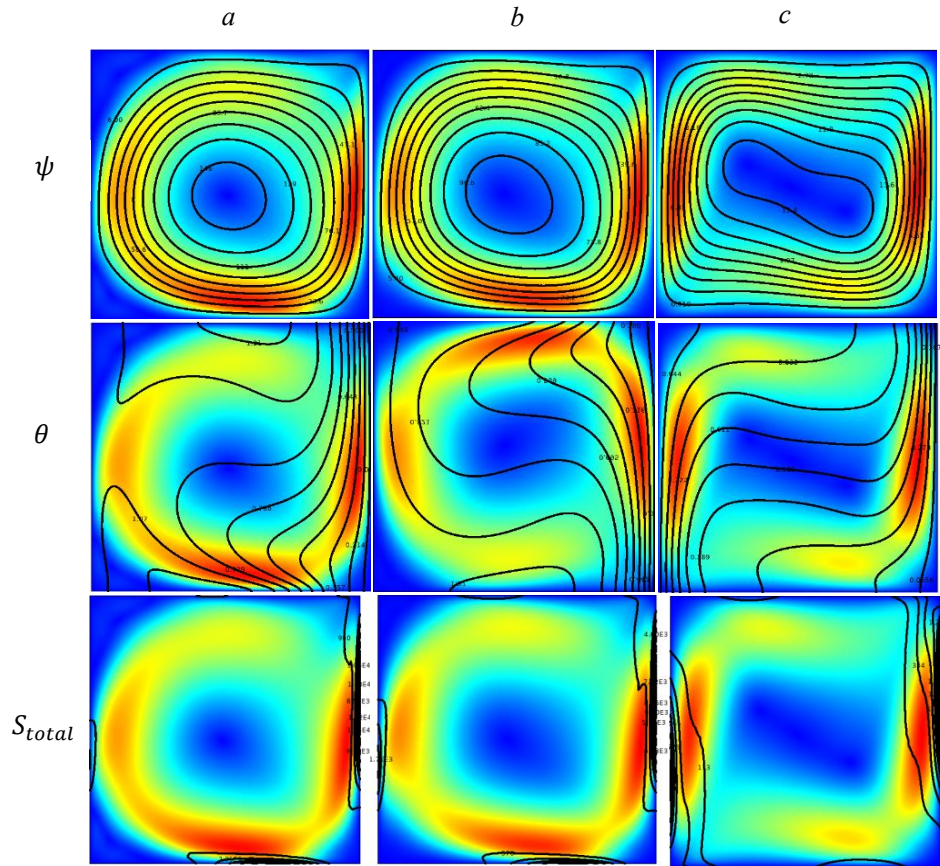


Figure 5

In Figure 6 (a), it is observed that an increase in the Prandtl number corresponds with a rise in the convective heat transfer of the medium. Conversely, Figure 6 (b) and (c) illustrate opposite effects on temperature and velocity distribution, respectively. This phenomenon arises because the flow remains laminar; thus, as the Prandtl number increases, the Nusselt number also tends to rise. However, the variation in the mean Nusselt number with an increase in the Prandtl number is minimal. On the other hand, both the maximum stream function and velocity decrease as the Prandtl number increases. With a higher Prandtl number, viscous effects within the fluid layers become more significant, resulting in a reduction of the fluid's flow strength, which in turn diminishes the maximum stream function. Additionally, fluid velocity increases from the heated wall before descending downstream, with a noticeable spike in velocity observed from the hot wall toward the cold wall. At elevated Prandtl numbers, velocity becomes less dependent on relevant parameters. Figure 6 (c) illustrates the influence of the Prandtl number on entropy generation, as shown in Figure 7 (a) through (c). It is important to highlight that the effect of the Prandtl number on entropy resulting from fluid friction is markedly different from its impact on entropy generation associated with heat transfer, as evidenced in Figure 7 (a) and (b). With an increase in the Prandtl number, the velocity gradient diminishes, which reduces the irreversibility from fluid friction that contributes to total entropy generation. However, it should be noted that the temperature gradient rises with an increase in the Prandtl number, leading to a rise in thermal entropy generation. As a result, total entropy generation is predominantly governed by thermal entropy production

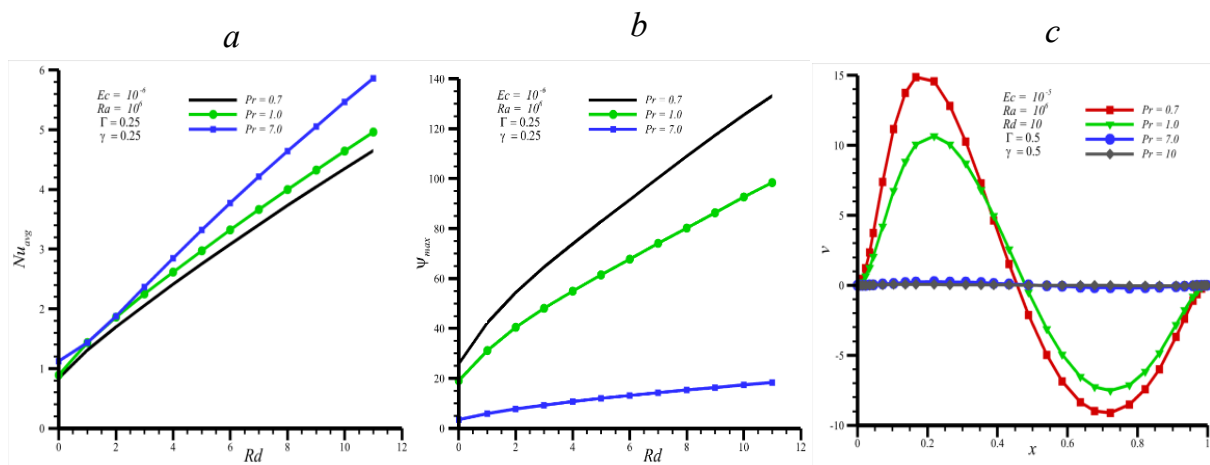


Figure 6

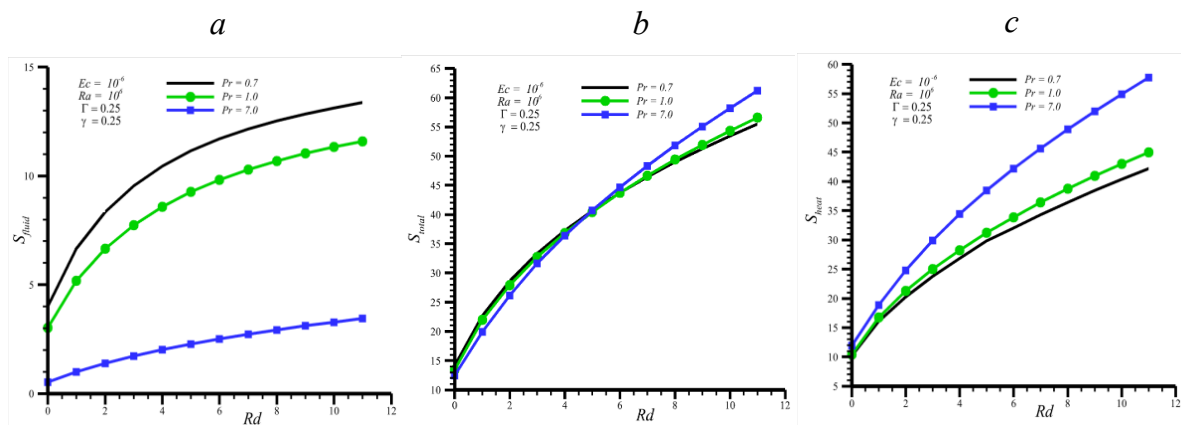


Figure 7

Effect of Rayleigh Number

The effect of the Rayleigh number on entropy generation, heat transfer, and fluid flow is illustrated by the streamlines, isotherms, and total irreversibility contours shown in Figure 8. As the fluid approaches the left heated wall, it becomes less dense, which enhances mixing and leads to chaotic motion. The heated molecules ascend and descend, forming a convective cell core in the center of the cavity. At lower Rayleigh numbers, there is some development of minor convective cell cores near the center of the porous hollow. Figure 8 indicates that an increase in the Rayleigh number results in more vigorous flow and improved heat transfer. Moreover, the isotherms are densely packed around the bottom heated surface and the upper cold wall of the enclosure. The Rayleigh number significantly influences fluid flow dynamics and temperature distribution, which can dramatically change the temperature profiles within the fluid, resulting in steeper thermal gradients. These phenomena, along with the placement of isotherms on the lower heated wall and the upper cold corner, contribute to elevated entropy production at the base of the left vertical wall and the upper corner of the right wall.

Figure 9 (a – c) illustrates the results for the average Nusselt number, maximum stream function, and velocity profiles across various Rayleigh number values. As the Rayleigh number increases, fluctuations in convective heat transfer are observed. Additionally, Figure 9 (a) indicates a marked improvement in thermal transmission rate. This improvement is attributed to the fluid's increased tendency to convection at relatively high Rayleigh numbers. However, excessive increases in the result in a decline in the average Nusselt number, as indicated in $Ra = 10^6$. The effects of Ra on the maximum stream function are contrary to the impact of Ra on convective thermal transmission, as shown in Figure 9 (b). It is noted that increases ψ_{max} with an increase in Ra . This rise in causes more chaotic behavior within the cavity, enhancing flow strength. It is well understood that the irreversibility caused by fluid friction and heat transfer encompasses the total amount of irreversibility in this study.

For effective thermal system design, analyzing the unique contributions of each component to entropy generation is crucial. Figure 10 (a – c) illustrates how the Rayleigh number influences entropy generation. At low Rayleigh numbers, the entropy produced from fluid friction and irreversibilities in heat transfer remains stable across all thermal radiation parameters. However, as the Rayleigh number increases, the irreversibility linked to fluid friction and thermal transfer also escalates. Importantly, the entropy generated from fluid friction exceeds that arising from thermal transmission effects at various Rayleigh numbers, mainly due to the heightened velocity gradient as the Rayleigh number rises.

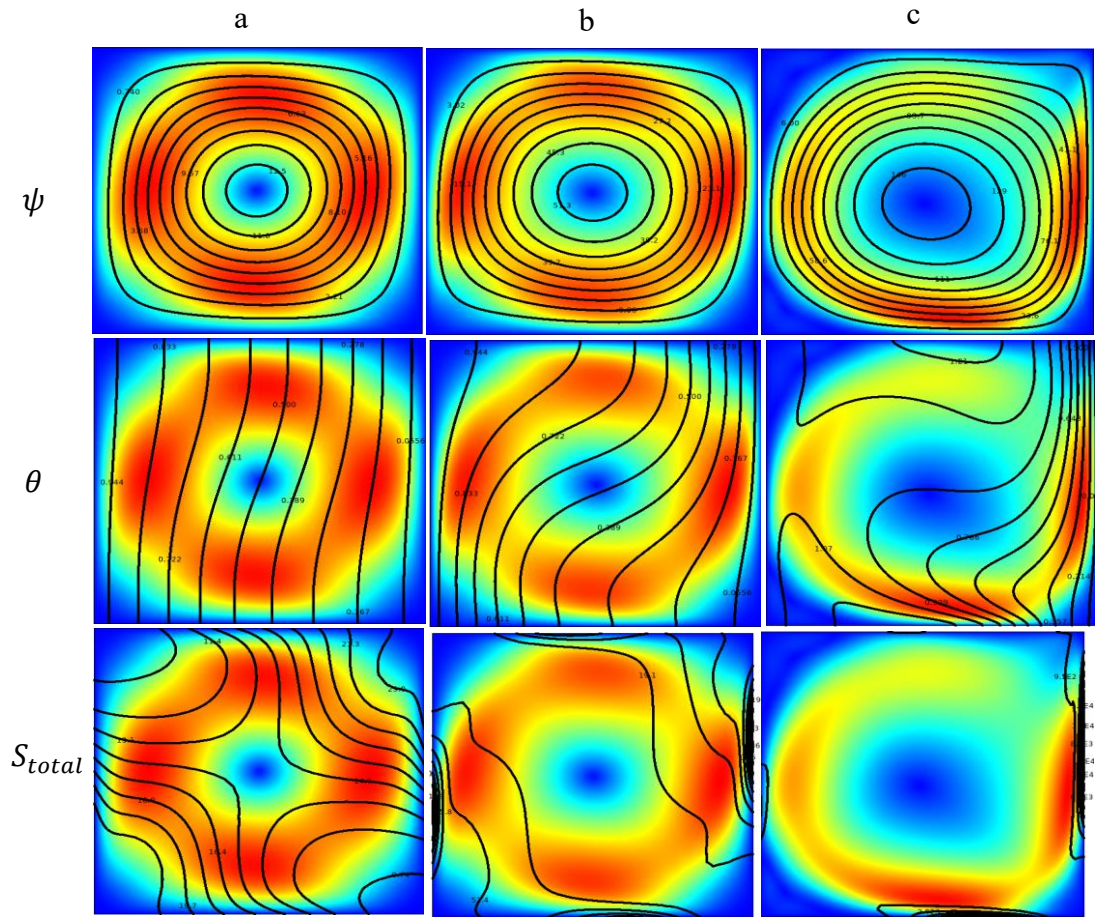


Figure 8

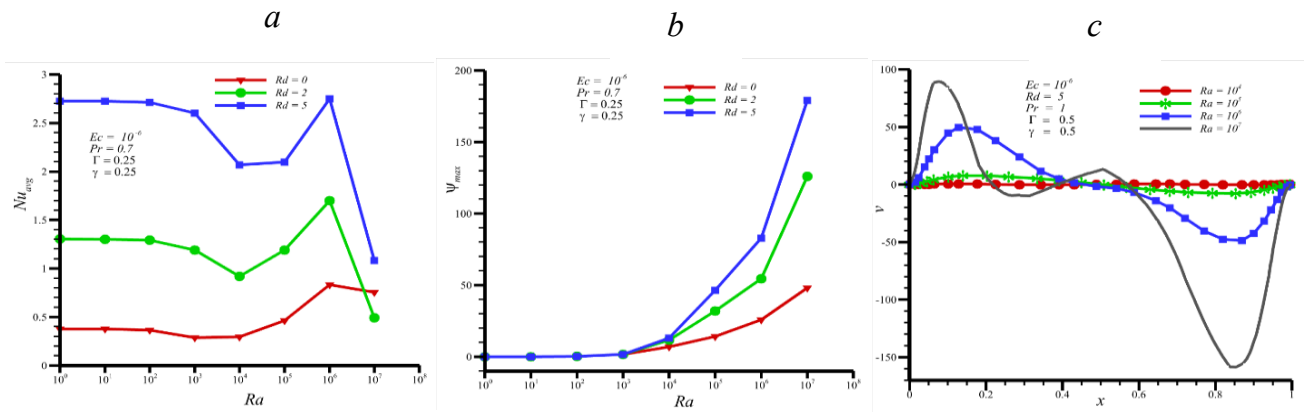


Figure 9

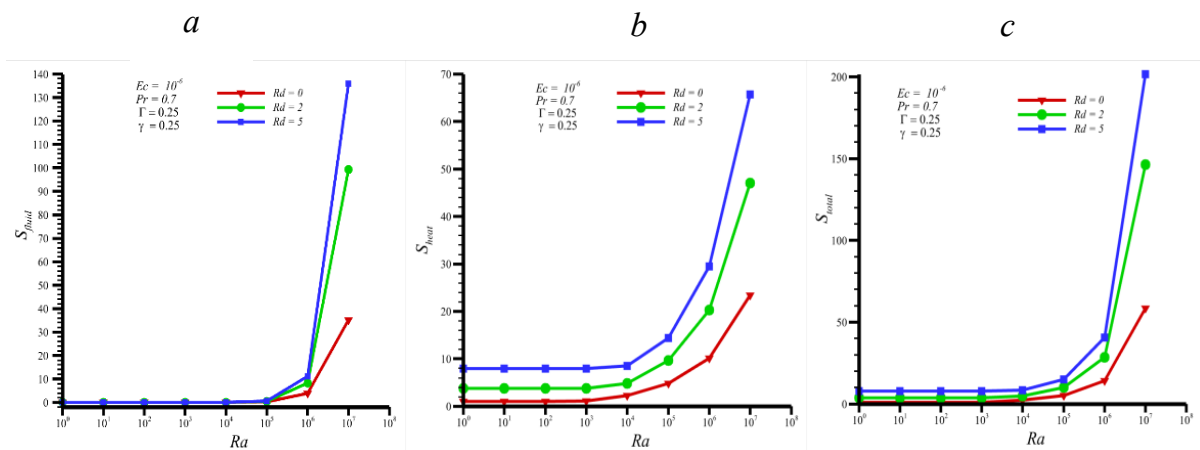


Figure 10

Conclusion

Based on the numerical analysis conducted using the Finite Element Method (FEM) in this study, it can be concluded that FEM serves as a robust numerical approach for efficiently simulating and understanding the behavior of real-world systems. Natural convection has gained popularity for fluid transport processes due to its reliability over mechanical alternatives. The findings indicate that a high Prandtl number improves convective heat transfer, whereas a low Prandtl number facilitates an increase in fluid velocity. Moreover, an increase in the Rayleigh number results in a notable rise in entropy generation, particularly attributed to thermal transmission. The application of thermal radiation on the vertical walls of the enclosure modifies the fluid flow characteristics, with the cold wall acting as a thermal sink and the heated wall serving as an energy source, especially at elevated radiation parameters. The solid matrix present in the porous media influences the efficiency of fluid transport by introducing inverse Darcy and Forchheimer resistance, which diminishes the overall energy level of both the fluid and the flow. Significant entropy generation is observed along the vertical walls of the cavity.

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