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Matrix Operations Algorithm for Matrix-by-Matrix Division: Concepts, Possibilities, and Limitations

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Abstract
Matrix operations constitute a foundational component of linear algebra with wide applications in economics, engineering, statistics, accounting and finance, and other computational sciences. While operations such as addition, subtraction, and multiplication are well-defined, the notion of matrix-by-matrix division remains conceptually problematic. This paper examines standard matrix operations using a suite of sampled matrix arrays and provides a detailed analytical treatment of the concept of matrix division. It explores whether division between matrices is feasible, the conditions under which it may be interpreted, and the mathematical structures—such as matrix inverses and generalized inverses—that substitute for direct division. The study concludes that true matrix division is not defined in the conventional sense but can be meaningfully represented through multiplication by inverses under strict conditions guided by well-defined algorithmic steps.

Keywords: Matrix Division, Matrix Operation, Matrix Algorithm, Scalar, Vector, Columns and Rows, Matrix Inverse

Introduction
Matrices are rectangular arrays of numbers or functions that facilitate compact representation of linear transformations and systems of equations. Matrix operations—including addition, multiplication, and inversion—are central to solving problems in various scientific disciplines [1]. However, unlike scalar arithmetic, not all operations translate directly into matrix algebra. One such operation is division, particularly matrix-by-matrix division, which raises both conceptual and computational challenges.

This paper aims to clarify the concept of matrix division by first reviewing fundamental matrix operations and then analyzing the theoretical feasibility of dividing one matrix by another.

Fundamental Matrix Operations
Matrix Addition and Subtraction
Matrix addition and subtraction are defined only for matrices of the same dimensions. If A and B are of the same dimensions, then:

$$\text{If } A = [a_{i,j}], \text{ and } B = [b_{i,j}], \text{ then}$$

$$A + B = [a_{i,j} + b_{i,j}]$$

$$A - B = [a_{i,j} - b_{i,j}]$$

Matrix Multiplication
Matrix multiplication is defined when the number of columns of the first matrix equals the number of rows of the second. For matrices A and B, the product is another matrix given by:
$(AB)_{i,j}$ = Sum to n rows with k columns starting from $(a_{i,k}b_{i,j})$

Scalar Multiplication
Each element of a matrix is multiplied by a scalar constant. This operation preserves matrix structure and is distributive over addition.

Matrix Inverse
For a square matrix, the inverse exists if and only if the matrix is non-singular. The inverse satisfies:

$$A^{-1} = \mathit{det(A)} \text{ multiplied by } \mathit{Dual\ of\ A}$$

$$AA^{1} = A^{-1}A = \mathbf{I}$$

Conceptualizing Matrix Division
Matrix "division" isn't defined like scalar division because matrix multiplication l lacks the commutative property needed for a direct inverse operation. Instead, dividing matrix A by matrix B (both n×n) means computing matrix A multiplied by the inverse of matrix B, or $A \times B^{-1}$, but only if B is invertible (square, full rank, non-zero determinant). For example: If A × B = C and you want C / B, solve for $C \times B^{-1}$. Equal size is necessary but not enough—B must be invertible.

Invertible and Non-Invertible Matrices.
Invertible matrices are those matrices are non-zero determinant arrays which the inverse can be found using the formula method or the tabula elimination algorithmic method.

Invertible matrices have their determinants greater or less than zero, but never zero.

Non-invertible matrices, on the other hand, are those matrices that cannot be converted to inverse because their primary determinants are equal to zero.
Assume JJ is a matrix with

[a b]
[c d]

array elements. Then JJ inverse will be given as:

1/(ad – bc)[d -b]
[-c a]

Where (ad – bc) is the value of the determinant. When this formula is applied to matrix J, then J inverse equals to:

1/(2x4 – 8x1)[4 -1]
[-8 2]

The determinant of the matrix is (2x4 – 8x1) = (8 – 8) = 0.
Since any number divided by zero is infinity or undefined, therefore, the inverse of J is undefined and cannot be inverted. It can also be seen that matrices K and L have the same problem. K's determinant is (4x4 – 8x2), which equals zero. In the same vein, L's determinant is (8x4 – 16x2), which also equals zero. Examples of matrices:

A = [3 5]. **B** = [6 2] **C** = [3 2 5]
[4 7] [8 1] [4 1 6]
[9 0 4
D = [4 4 5]

Scalar Analogy and its Limitations
In scalar arithmetic, division is defined as multiplication by the reciprocal:

Matrix-by-Matrix Division: Possibility and Constraints
Non-Existence of Direct Division Operator
Matrix algebra does not define division as a fundamental operation. There is no operator "/" such that:

Division via Matrix Inverse
When matrices A and B are square and invertible, matrix division can be represented indirectly:

$$\mathbf{a/b = a.b^{-1}}$$

$$\mathbf{A/B = AB^{-1}}$$

$$\mathbf{A/B = C}, \text{ implies that } \mathbf{CB = A}$$

Right division

$$\mathbf{A/B = AB^{-1}}$$

Left division

These two expressions are generally not equal due to non-commutativity:
Thus, matrix division is not unique and depends on the side of multiplication [2]. It is the opinion of this paper that both the right side and left side division solutions should be applied, implying that while only the denominator matrix should be inverted, it can be used from the right as the multiplier matrix and can also be used as the matrix to be multiplied upon. Either way, the best solution would be found through comparison.

Conditions for Feasibility

Matrix-by-matrix division via inverses is only possible when:
• The divisor matrix is square.
• The determinant of the divisor matrix is non-zero.
• The inverse exists and is computationally stable.
Failure to meet these conditions renders division undefined.

Illustrative Examples
Given that:

$\mathbf{A} = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$. $\quad \mathbf{A^{-1}} = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$\mathbf{B} = \begin{bmatrix} 6 & 2 \\ 8 & 1 \end{bmatrix}$. $\quad \mathbf{B^{-1}} = \begin{bmatrix} -0.1 & 0.2 \\ 0.8 & -0.6 \end{bmatrix}$

$\mathbf{AB} = \begin{bmatrix} 58 & 11 \\ 50 & 15 \end{bmatrix} \quad \mathbf{BA} = \begin{bmatrix} 26 & 44 \\ 28 & 47 \end{bmatrix}$

AB/A-1 = $\begin{bmatrix} 362 & -257 \\ 500 & -355 \end{bmatrix}$ (Wrong solution – **Right side** multiplication).

$\mathbf{A^{-1}/AB} = \begin{bmatrix} 6 & 2 \\ 8 & 1 \end{bmatrix}$ (Correct solution, same as matrix **B – Left side** multiplication).

$\mathbf{AB/B^{-1}} = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ (Correct solution, same as matrix **A – Right side** multiplication).

$\mathbf{B^{-1}/AB} = \begin{bmatrix} 10.2 & 1.9 \\ -1.6 & -0.2 \end{bmatrix}$ (Wrong solution – **Left side** multiplication).

$\mathbf{BA/A^{-1}} = \begin{bmatrix} 6 & 2 \\ 8 & 1 \end{bmatrix}$ (Correct solution, same as matrix **B – Right side** multiplication).

$\mathbf{A^{-1}/BA} = \begin{bmatrix} 42 & 63 \\ -20 & -35 \end{bmatrix}$ (Wrong solution – **Left side** multiplication).

$\mathbf{B^{-1}/BA} = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ (Correct solution, same as matrix **A – Left side** multiplication).

$\mathbf{BA/B^{-1}} = \begin{bmatrix} 32.6 & -21.2 \\ 34.6 & -22.6 \end{bmatrix}$ (Wrong solution – **Right side** multiplication).

It is evident from the preceding computations that what worked for one matrix division might not necessarily work for another. Reason being that the positions of the multiplying matrices differ and the difference has to be taken into consideration while applying the multiplication of the inverted denominator matrix with the numerator matrix. For instance, a BA matrix can only be divided using B^{-1}/BA or BA/A^{-1} inverse multiplication methods as the name BA denotes that matrix B multiplied matrix A. Conversely, matrix AB can only be divided using A^{-1}/AB or AB/B^{-1} inverse multiplication methods. Hence the need for the Right and Left sides multiplication trial method. Readers should note that the detailed multiplication steps were deliberately omitted from the preceding presentations.

Division in Non-Square and Singular Cases

Moore–Penrose Pseudoinverse

For non-square or singular matrices, the Moore–Penrose pseudoinverse A+ provides a generalized notion of inversion:

$$A^{+} = (A^{T}A)^{-1}A^{T}$$

Application in Linear Systems

If $X = A^{+}B$, solving $AX = B$, can be possible if A is invertible:
That is: X = A-1B

Computational and Practical Considerations

Matrix inversion is computationally expensive and numerically sensitive. In practice, direct inversion is often avoided in favor of solving systems via decomposition methods (e.g., LU, QR, or SVD) [3].

Moreover, interpreting division as multiplication by an inverse can lead to misleading conclusions if the underlying assumptions are not verified, particularly in applied fields such as econometrics and machine learning.

Discussion

The absence of a direct matrix division operation reflects deeper structural properties of matrices, including non-commutativity and the conditional existence of inverses. While inverse-based approaches provide a credible and soothing workaround, they do not fully replicate scalar division. The introduction of generalized inverses expands applicability but introduces approximation and interpretational challenges. To avoid the ambiguity and problem of computational result choice and interpretation, this paper advocates the use of a two pronged approach by performing both right and left divisions and selecting the solution that makes better sense. As deduced from the Illustrative examples in section 4.4, what worked for one matrix division might not necessarily work for another because the positions of the multiplying matrices differ and by implication the difference practically dictates how the multiplication of the inverted denominator matrix with the numerator matrix would be applied. For instance, a BA matrix can only be divided using B^{-1}/BA or BA/A^{-1} inverse multiplication methods as the name BA denotes that matrix B multiplied matrix A. Conversely, matrix AB can only be divided using A^{-1}/AB or AB/B^{-1} inverse multiplication methods. But the usual method of naming a matrix do not always follow this example as most matrix names do not show how the matrix is composed. Hence the need for the Right and Left sides multiplication trial method.

Recognizing the fact that this trial and error approach might be a little laborious for the analyst, the consolation is that it minimizes the chance of hanging on to a costly and deficient solution. As additional check, the computed result can be used to multiply the denominator matrix to see if the numerator matrix can be arrived at.

Conclusion

Matrix-by-matrix division, in the strict algebraic sense, does not exist as a fundamental operation. However, this methodical drawback can be meaningfully resolved using a two pronged algorithmic trial with the multiplication by inverses method when the divisor matrix is square and non-singular. In more complex scenarios, generalized inverses such as the Moore–Penrose pseudoinverse may provide a practical alternative. Scholars and practitioners must exercise caution in interpreting such operations to avoid analytical errors.

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