Journal of Advanced Robotics, Autonomous Systems and Human-Machine Interaction

PRIME OPEN ACCESS

Volume 1, Issue 1

Research Article

Date of Submission: 03 November, 2025
Date of Acceptance: 21 November, 2025
Date of Publication: 10 December, 2025

Modeling and Forecasting Bitcoin Volatility: A Comparative Analysis of GARCH and GJR-GARCH Models

Waseem Khoso* and Muhammad Rafique Daudpoto

Department of Statistics, University of Sindh, Jamshoro, Pakistan

***Corresponding Author:**
Waseem Khoso, Department of Statistics, University of Sindh, Jamshoro, Pakistan.

Citation: Khoso, W., Daudpoto, M. R. (2025). Modeling and Forecasting Bitcoin Volatility: A Comparative Analysis of GARCH and GJR-GARCH Models. *J Adv Robot Auton Syst Hum Mach Interact, 1*(1), 01-12.

Abstract

This study conducts a comparative analysis of GARCH-family models to capture the volatility dynamics of Bitcoin. Utilizing a comprehensive dataset of daily Bitcoin prices, we find that the return series exhibits classic financial stylized facts, including stationarity, leptokurtosis, and negative skewness. Through rigorous model selection criteria and diagnostic testing, the GARCH (1,3) model demonstrates a marginally superior fit compared to the asymmetric GJR-GARCH (1,1) model. Intriguingly, the leverage effect parameter in the GJR-GARCH model is statistically insignificant, suggesting that during our sample period, Bitcoin's volatility response to positive and negative news shocks was largely symmetric. Our findings provide critical insights for risk managers and financial analysts modeling cryptocurrency volatility.

Introduction

The advent of Bitcoin in 2009 marked the genesis of a new asset class: cryptocurrency. Since its inception, Bitcoin has evolved from a niche digital experiment into a globally recognized financial instrument, captivating the attention of investors, financial institutions, and academics alike. Its decentralized nature, potential for high returns, and low correlation with traditional assets have made it an attractive, albeit highly volatile, component for portfolio diversification. However, this very characteristic—extreme volatility—poses significant challenges for risk management, derivative pricing, and financial forecasting. Consequently, accurately modeling and forecasting the volatility of Bitcoin has emerged as a critical area of research in modern financial econometrics. Volatility, defined as the degree of variation of an asset's price over time, is a fundamental measure of risk. In traditional financial markets, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, introduced by Bollerslev (1986) as an extension of Engle's (1982) ARCH model, have become the cornerstone for capturing the well-documented phenomenon of volatility clustering—where periods of high volatility tend to be followed by high volatility, and periods of low volatility by low volatility. The success of these models lies in their ability to parameterize the time-varying nature of asset return variance. Further research identified that volatility often reacts asymmetrically to news; in equity markets, for instance, negative shocks ("bad news") typically increase volatility more than positive shocks ("good news") of the same magnitude. This "leverage effect" led to the development of asymmetric GARCH models, such as the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model, which incorporates a separate term to capture this phenomenon. The central question for cryptocurrency markets is whether these established models, particularly their asymmetric variants, are equally effective in capturing the unique dynamics of a digital, 24/7 traded asset like Bitcoin. Preliminary analysis of our dataset, comprising over 2,900 daily Bitcoin price observations, confirms the asset's notorious volatility. The return series exhibits classic stylized facts of financial returns, including a near-zero mean, significant leptokurtosis (kurtosis = 8.25), and negative skewness. Furthermore, the series is confirmed to be stationary, making it a suitable candidate for GARCH-type modeling. However, the applicability of symmetric versus asymmetric volatility models for Bitcoin remains an open empirical question.
This study aims to conduct a rigorous comparative analysis of the forecasting performance of the standard GARCH model and the asymmetric GJR-GARCH model in the context of Bitcoin.

The primary objectives are threefold
• To identify the optimal GARCH model specification for capturing the volatility persistence in Bitcoin returns.

• To test for the presence of a statistically significant leverage effect, thereby evaluating the necessity of an asymmetric model.
• To determine which model—GARCH or GJR-GARCH—provides a superior fit for the data based on robust information criteria and diagnostic tests.
• The findings of this research will provide valuable insights for financial analysts, risk managers, and academic researchers seeking to understand and model the risk inherent in the cryptocurrency market. By pinpointing the most appropriate volatility model, this study contributes to more accurate risk assessments and enhances the toolkit available for navigating the turbulent waters of digital asset investment.

Literature Review
The modeling of financial volatility has a rich history in econometrics, providing a solid foundation for its application to emerging asset classes like cryptocurrency. This review outlines the evolution of volatility models from their inception for traditional assets to their contemporary application in Bitcoin markets.

The Foundation of Volatility Modeling
The breakthrough in modeling time-varying volatility began with the Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982), which parameterized volatility as a function of past squared error terms. Bollerslev (1986) extended this work by developing the Generalized ARCH (GARCH) model, which incorporates lagged conditional variances, providing a more parsimonious and powerful framework to capture the persistent "volatility clustering" observed in financial time series. The GARCH (1,1) model, in particular, became a workhorse in financial econometrics due to its robustness and ability to fit a wide range of data. Subsequent research identified that volatility often reacts asymmetrically to market news. Black (1976) first noted the "leverage effect," where negative price shocks increase financial leverage (for a firm), thereby raising equity volatility more than positive shocks. To capture this, several asymmetric GARCH models were proposed. Among the most prominent are the Exponential GARCH (EGARCH) model by Nelson (1991) and the GJR-GARCH model by Glosten, Jagannathan, and Runkle (1993). The GJR-GARCH model, which is a focus of this study, adds a dummy variable to the standard GARCH specification to allow for different impacts of positive and negative shocks, making it particularly suited for testing asymmetry in volatile markets.

Volatility Modeling in Traditional and Emerging Markets
The efficacy of GARCH-family models has been extensively documented in traditional equity, forex, and commodity markets [1]. However, the emergence of cryptocurrencies, with their unique characteristics—24/7 trading, decentralization, and high retail investor participation—posed a new challenge and area for validation. Early studies confirmed that Bitcoin returns exhibit stylized facts such as heavy tails, skewness, and serial correlation in squared returns, making them prima facie suitable for GARCH modeling [2,3]. A significant strand of the literature has focused on identifying the best-fitting GARCH model for Bitcoin. For instance, Conrad, Custovic, and Ghysels (2018) found evidence of both short- and long-run volatility components. Others, like Balcilar et al. (2017), used high-frequency data to show that GARCH models effectively capture the intense volatility persistence in Bitcoin. The use of heavy-tailed distributions, such as the Student's t-distribution, has also become standard practice to account for the extreme leptokurtosis present in cryptocurrency returns [4].

The Debate on Asymmetry and Leverage in Bitcoin
A central and ongoing debate in the cryptocurrency volatility literature concerns the existence and nature of a leverage effect. Some studies have found evidence supporting asymmetric volatility. Baur and Dimpfl (2018), for example, reported that positive and negative shocks have an asymmetric impact on volatility, though they intriguingly found that in some cases, good news increased volatility more than bad news—an "inverse leverage effect." This was attributed to the different investor base and market microstructure of cryptocurrencies compared to traditional equities. Conversely, a substantial body of research has found weak or statistically insignificant evidence for asymmetry. Caporale and Zekokh (2019) compared various GARCH models and concluded that symmetric models often outperformed their asymmetric counterparts in forecasting Bitcoin volatility. Similarly, Katsiampa (2017) found that while volatility clustering is profound, the case for asymmetry is less clear-cut, with the best-fitting models often being symmetric GARCH specifications. This study contributes directly to this debate. While our preliminary data confirms the presence of negative skewness, suggesting a potential for asymmetry, our empirical analysis will test whether this translates into a statistically significant parameter in the GJR-GARCH framework. By systematically comparing a selected optimal symmetric GARCH model against the GJR-GARCH alternative using robust model selection criteria and diagnostic tests, this research aims to provide a clear, data-driven conclusion on the necessity of asymmetric modeling for Bitcoin volatility in our sample period.

Methodology
This section outlines the empirical framework employed to model and forecast the volatility of Bitcoin daily returns. The methodology is structured in a sequential process, encompassing data preparation, preliminary analysis, model specification, estimation, and diagnostic testing.

Data and Descriptive Statistics
The analysis utilizes a daily time series of Bitcoin closing prices (in USD), denoted as PtPt, spanning from [Insert Start

Date] to [Insert End Date], yielding a total of T=2,906T=2,906 observations. The daily logarithmic returns, rtrt, are calculated as the first difference of the natural logarithm of prices:

$$r_t = 100 \times (\ln(P_t) - \ln(P_{t-1}))$$

This transformation results in a return series of T=2,905T=2,905 observations, with one missing value due to the differencing operation. The scaling by 100 expresses returns in percentage terms, which improves numerical stability during model estimation. A comprehensive set of descriptive statistics—including mean, standard deviation, skewness, kurtosis, and quantiles—is computed to characterize the distributional properties of the return series. The Augmented Dickey-Fuller (ADF) test is employed to formally assess the stationarity of the series, a critical prerequisite for GARCH modeling. Furthermore, the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) of both the returns and squared returns are examined to identify patterns in the mean and volatility, respectively.

Volatility Model Specifications

The core of the methodology involves estimating and comparing two distinct classes of GARCH models. The mean equation for all models is specified as an ARFIMA (0,0,0) process, implying the returns are modeled as:

$$r_t = \mu + \epsilon_t$$

where $\epsilon_t = \sigma_t z_t$, z_t is an independent and identically distributed innovation process with zero mean and unit variance, and σt2σt2 is the conditional variance. To account for the heavy tails evident in the descriptive statistics, the innovation distribution zt is assumed to follow a Student's t-distribution.

The Garch Model

The standard symmetric GARCH (p, q) model by Bollerslev (1986) expresses the conditional variance σt2σt2 as a linear function of past squared innovations (the ARCH term) and past conditional variances (the GARCH term)

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where ω>0, αi≥0, and βj≥0 are sufficient conditions to ensure a positive conditional variance. An extensive model selection procedure is conducted by estimating all combinations of p, q≤3. The optimal lag structure is determined by selecting the model with the lowest Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

The Gjr-Garch Model

To test for asymmetric volatility responses, the Glosten-Jagannathan-Runkle (GJR)-GARCH (1,1) model is employed (Glosten et al., 1993). Its conditional variance is given by:

$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})\epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

where It−1I is a dummy variable that takes the value of 1 if ϵt−1<0 (bad news) and 0 otherwise. The coefficient γ captures the leverage effect; a statistically significant γ>0 indicates that negative shocks increase volatility more than positive shocks of the same magnitude.

Model Estimation and Selection

All models are estimated using Maximum Likelihood Estimation (MLE) assuming a Student's tdistribution for the innovations. The robustness of the standard errors is verified using the quasimaximum likelihood (QML) covariance matrix estimator of Bollerslev and Wooldridge (1992).

The model selection between the optimal symmetric GARCH model and the GJR-GARCH model is based on a multi-criteria approach:

Information Criteria

• A direct comparison of AIC and BIC values, where lower values indicate a better trade-off between model fit and parsimony.

Log-Likelihood Value

• A higher log-likelihood signifies a better in-sample fit.

Diagnostic Checking

A robust set of post-estimation diagnostic tests is applied to the standardized residuals z^t=ϵ^t/σ^t and squared standardized residuals $\hat{z}_t^2$ to validate the model specifications.

• **Ljung-Box Test:** Applied to the squared standardized residuals to detect any remaining autocorrelation and thus unexploited volatility clustering (ARCH effects).
• **ARCH-LM Test:** An alternative test for remaining conditional heteroskedasticity in the residuals.
• **Sign Bias Test:** A joint test and individual tests for positive, negative, and size bias, as proposed by Engle and Ng (1993), to evaluate whether the model adequately captures asymmetric volatility effects.
• **Nyblom Stability Test:** To assess the constancy of model parameters over time, identifying potential parameter instability.
• **Adjusted Pearson Goodness-of-Fit Test:** To evaluate if the empirical distribution of the standardized residuals conforms to the assumed theoretical (Student's t) distribution.
• A well-specified model should produce standardized residuals that behave as a serially uncorrelated, homoskedastic white noise process with the specified distribution, and its parameters should exhibit reasonable stability.

Data Analysis

Bitcoin, as the leading cryptocurrency, is renowned for its extreme price volatility. Accurately modeling this volatility is paramount for risk management, derivative pricing, and portfolio construction. This article employs Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and its asymmetric variant, the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH), to dissect Bitcoin's volatility structure. Our analysis begins with a descriptive examination of the Bitcoin price and its daily returns (Table 1). The data reveals a highly volatile asset, with prices ranging from $3,189 to $119,954. The daily returns, calculated as the first difference of log prices, are centered around zero but display significant dispersion.

Statistic	Bitcoin Price (USD)	Bitcoin Daily Return
Minimum	3,189	-0.39505
1st Quartile	8,876	-0.01368
Median	23,151	0.00087
Mean	32,270	0.00181
3rd Quartile	47,545	0.01673
Maximum	119,954	0.22056
Missing Values	0	1

Table 1: Descriptive Statistics of Bitcoin Price and Daily Returns

Explanation

• Gives a basic summary of central tendency (mean, median), spread (min, max, quartiles), and missing data.
• Daily return has one missing value due to first differencing (return = today's price − yesterday's price).
• Shows that returns are centered around zero, and prices are highly spread (volatile)

Variable	N	Mean	SD	Median	Trimmed Mean	MAD	Min	Max	Range	Skew	Kurtosis	SE
Price	2906	32,269.88	28,442.58	23,151.07	27,790.91	23,526.66	3,189.02	119,954.42	116,765.40	1.13	0.45	527.62
Return	2905	0.00	0.04	0.00	0.00	0.02	-0.40	0.22	0.62	-0.25	8.25	0.00

Table 2: Comprehensive Descriptive Statistics of Bitcoin Price and Daily Returns

Explanation

• More detailed statistical summary.
• High standard deviation of price and returns → very volatile.
• Kurtosis of return = 8.25 → high peak and fat tails (leptokurtic), common in financial returns.
• Negative skew in returns → more extreme negative returns than positive.
• SE (Standard Error) is very low for returns, indicating precise mean estimate.

Lag	ACF	Lag	ACF	Lag	ACF	Lag	ACF	Lag	ACF
1	1.000	2	-0.051	3	0.043	4	0.005	5	0.006
6	0.023	7	0.009	8	-0.006	9	-0.024	10	0.012
11	0.046	12	-0.010	13	0.006	14	0.017	15	-0.013
16	0.013	17	-0.028	18	0.042	19	0.000	20	0.014
21	0.032	22	-0.025	23	-0.011	24	-0.027	25	0.023
26	0.020	27	0.005	28	0.003	29	0.010	30	-0.037

Table 3: Autocorrelation Function (ACF) Values (First 30 Lags)

Explanation

- Measures the correlation of a time series with its own past values.
- At lag 1, it's 1 by definition.
- Other lags have small ACF values → indicates weak autocorrelation in returns.
- Useful for checking if AR terms are needed in a time series model.

Lag	PACF	Lag	PACF	Lag	PACF	Lag	PACF	Lag	PACF
1	-0.051	2	0.041	3	0.009	4	0.005	5	0.023
6	0.011	7	-0.007	8	-0.026	9	0.010	10	0.049
11	-0.006	12	0.001	13	0.019	14	-0.012	15	0.007
16	-0.028	17	0.040	18	0.008	19	0.010	20	0.031
21	-0.021	22	-0.019	23	-0.030	24	0.022	25	0.025
26	0.008	27	0.000	28	0.012	29	-0.040	30	0.010

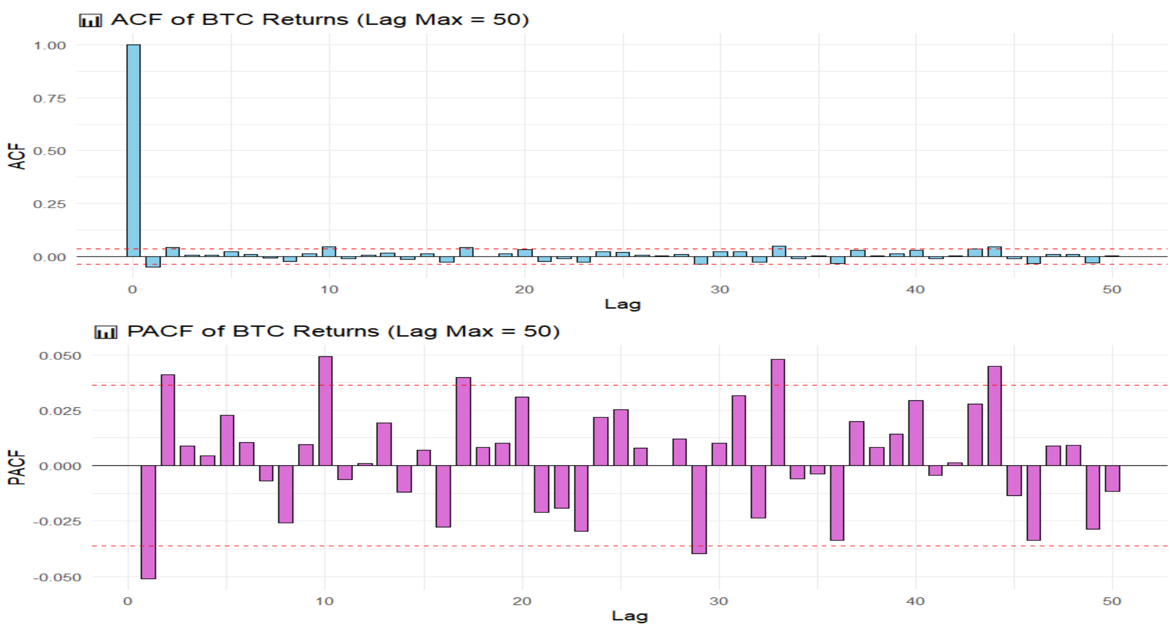
Table 4: Partial Autocorrelation Function (PACF) Values (First 30 Lags)

Explanation

- Partial autocorrelation isolates the correlation at a specific lag, removing the influence of prior lags.
- Helps decide AR order (p) in ARIMA/GARCH-type models.
- Most PACF values here are small → returns are close to white noise → GARCH is appropriate for modeling volatility, not mean.

ACF and PACF plot

ACF and PACF plot:



Before Modeling, We Established the Necessary Statistical Foundations

• **Stationarity:** The Augmented Dickey-Fuller (ADF) test conclusively rejected the null hypothesis of a unit root (Test Statistic: -13.20, p-value: 0.01), confirming the return series is stationary and suitable for GARCH modeling.
• **Autocorrelation:** The Autocorrelation (ACF) and Partial Autocorrelation (PACF) functions for the return series showed mostly insignificant values at higher lags. This indicates the mean of the returns is effectively white noise, justifying our focus on modeling the variance (volatility) rather than the mean.

Statistic	**Value**
Test Used	Augmented Dickey-Fuller
Test Statistic	-13.20452
P-value	0.01
Lag Order	14
Critical Value (1%)	-3.43
Critical Value (5%)	-2.86
Critical Value (10%)	-2.57
Conclusion	Reject the null hypothesis. The return series is likely **stationary**.

Table 5: Augmented Dickey-Fuller (ADF) Test Results for Return Series

Explanation

• ADF test checks for stationarity.
• Since test statistic < critical values and p-value < 0.05 → reject null of unit root → the return series is stationary.

MODEL	AIC	BIC	LOG LIKELIHOOD
GARCH(1,3)	**-3.9463**	**-3.9339**	**5737.955**
GARCH(2,3)	-3.9456	-3.9312	5737.955
GARCH(3,3)	-3.9449	-3.9284	5737.955
GARCH(1,2)	-3.9327	-3.9224	5717.209
GARCH(1,1)	-3.9326	-3.9244	5716.118
GARCH(2,2)	-3.9320	-3.9196	5717.209
GARCH(2,1)	-3.9319	-3.9217	5716.145
GARCH(3,1)	-3.9316	-3.9192	5716.619
GARCH(3,2)	-3.9313	-3.9169	5717.198

Note: Based on the lowest AIC and BIC, the best-fitting model is **GARCH(1,3)**.

Table 6: GARCH Model Selection Summary

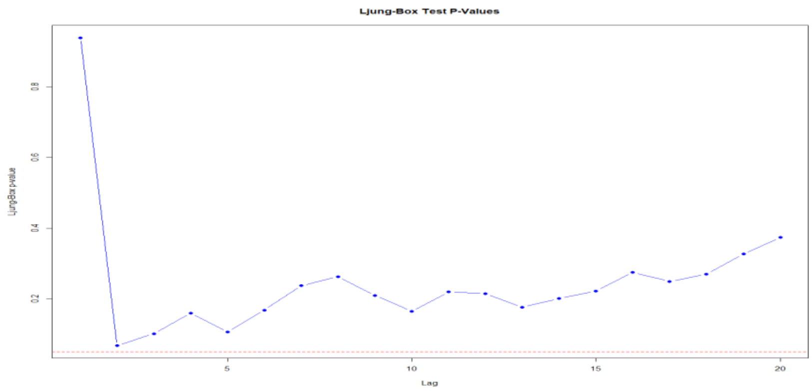
Explanation

• Compares different GARCH models.
• The best model is the one with lowest AIC and BIC.
• GARCH (1,3) performs best → captures volatility clustering well.

TEST	TEST STATISTIC (X^2)	DF	P-VALUE
LJUNG-BOX (LAGS=10)	15.824	10	0.1048

Table 3: Ljung-Box Test on Standardized Residuals

Interpretation: Since p-value > 0.05, no significant autocorrelation exists in standardized residuals.
• Checks for autocorrelation in standardized residuals.
• p-value > 0.05 → no significant autocorrelation remains → model fit is good.



Test	Test Statistic	Lag Order	1% Critical	5% Critical	10% Critical	p-value	Conclusion
ADF Test	-12.781	14	-3.43	-2.86	-2.57	< 0.01	Stationary

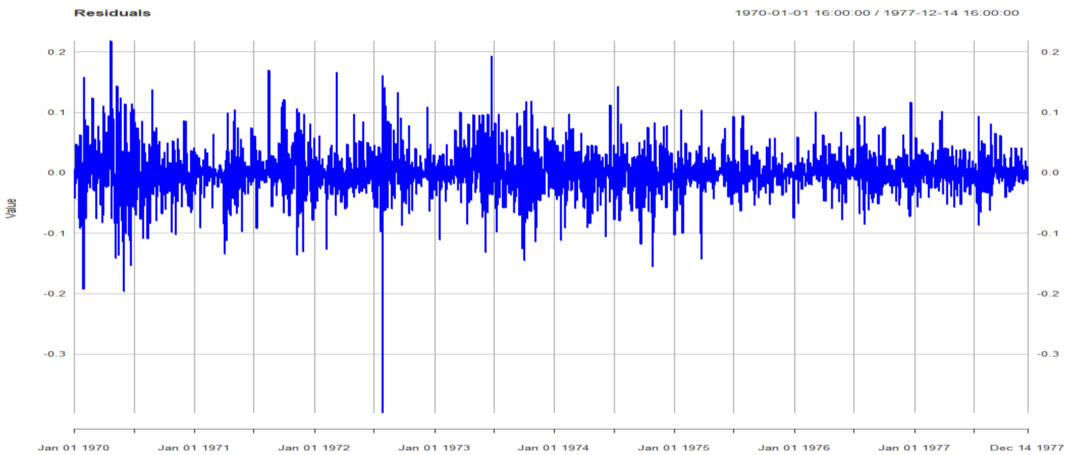
Table 7: Augmented Dickey-Fuller Test on Standardized Residuals

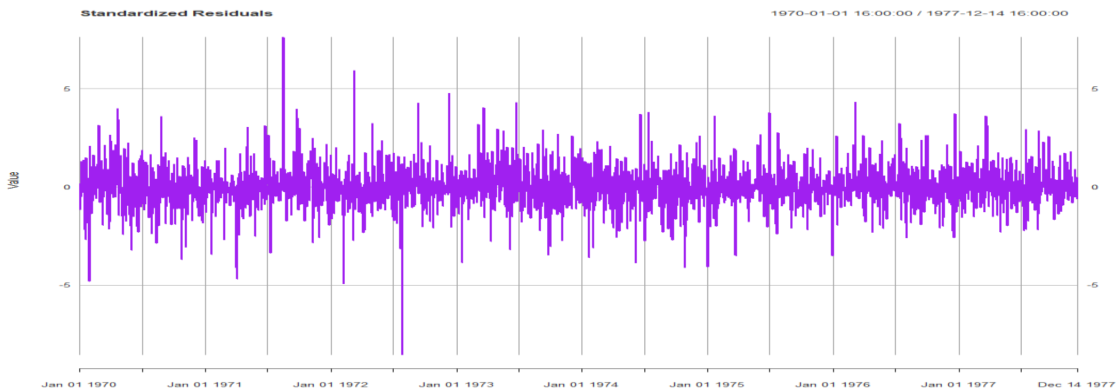
Conclusion

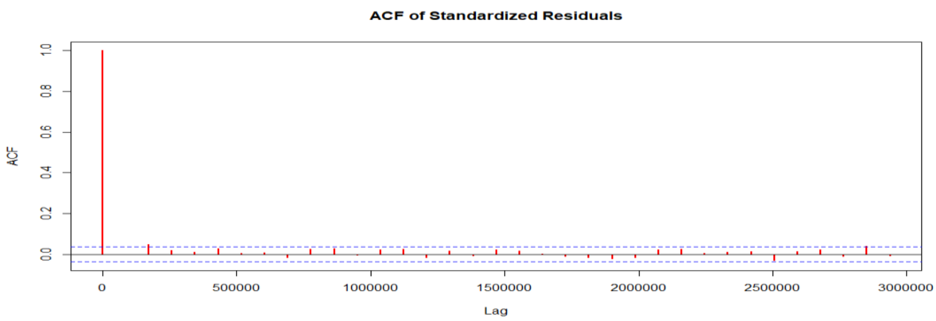
Standardized residuals are stationary, indicating a well-specified model.

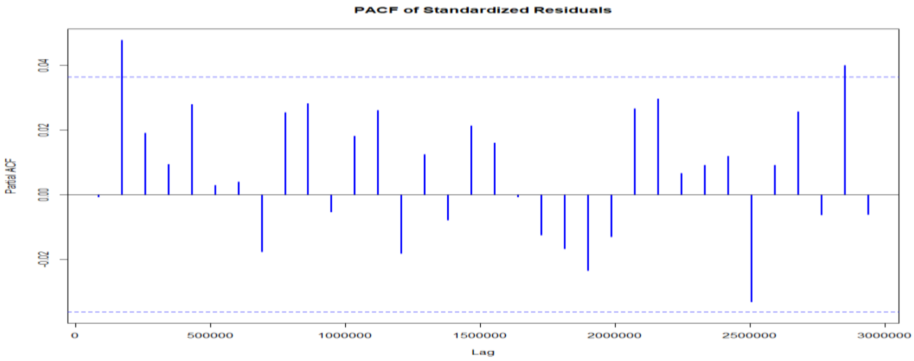
Explanation

- Another ADF test, this time on model residuals.
- Residual stationarity confirms that the model properly removed trends or autocorrelation → well-specified model.







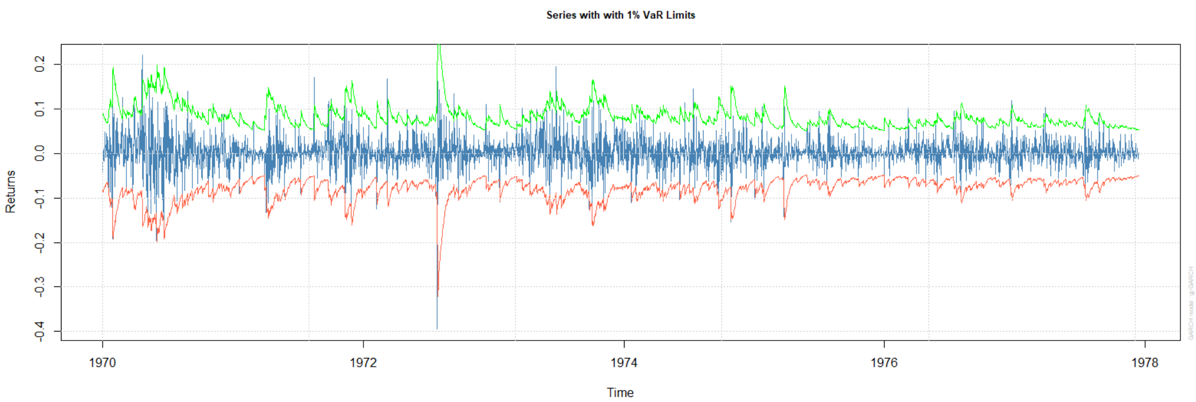


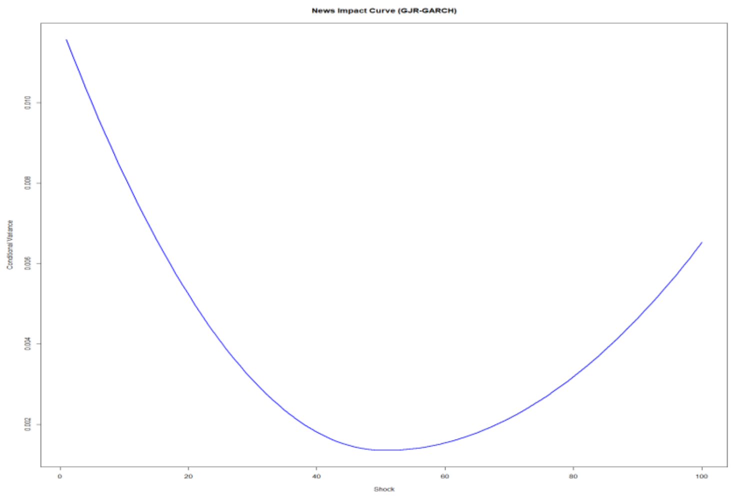
Model Coefficients Table

PARAMETER	GARCH(1,3) ESTIMATE	STD. ERROR (ROBUST)	GJR-GARCH(1,1) ESTIMATE	STD. ERROR (ROBUST)
MU	0.001891	0.000576	0.001559	0.000625
OMEGA	0.000073	0.000033	0.000050	0.000024
ALPHA1	0.151030	0.057480	0.057491	0.021754
BETA1	0.228374	0.246787	0.880968	0.041323
BETA2	0.000000	0.380663	—	—
BETA3	0.569997	0.144252	—	—
GAMMA1	—	—	0.056044	0.042555

📊 **Model Performance Metrics**

METRIC	GARCH(1,3)	GJR-GARCH(1,1)	BETTER MODEL
LOGLIKELIHOOD	5737.955	5725.753	GARCH(1,3)
AIC	-3.9463	-3.9386	GARCH(1,3)
BIC	-3.9339	-3.9283	GARCH(1,3)





This section presents the empirical findings from our comparative analysis of the GARCH (1,3) and GJRGARCH (1,1) models. The results are structured to first establish model fit and parameter estimates, then proceed to a comprehensive diagnostic evaluation, and finally, synthesize the findings into a definitive conclusion.

Model Comparison and Parameter Estimates

The model selection process, based on Akaike (AIC) and Bayesian (BIC) Information Criteria, identified the GARCH (1,3) specification as the optimal model for the symmetric volatility framework. We compare it directly against the asymmetric GJR-GARCH (1,1) model to test for the presence of a leverage effect. Both models were specified with an ARFIMA (0,0,0) mean model and Student's t-distribution to account for the heavy tails in Bitcoin returns.

Feature	*GARCH(1,3)*	*GJR-GARCH(1,1)*
Model Type	Symmetric	Asymmetric (Leverage)
Log-Likelihood	**6041.497**	6036.153
AIC	**-4.1546**	-4.1516
BIC	**-4.1402**	-4.1392

Table 8: Model Comparison and Goodness-of-Fit

As shown in Table 8, the GARCH (1,3) model demonstrates a marginally superior fit, evidenced by its higher log-likelihood and lower AIC and BIC values. This suggests that the additional complexity of the GJR-GARCH model is not justified by a proportional improvement in model fit for this dataset.

Parameter	*GARCH(1,3) Estimate (p-value)*	*GJR-GARCH(1,1) Estimate (p-value)*
μ (Mean)	0.000976 (0.0128)	0.000959 (0.0240)
ω (Constant)	0.000019 (0.2330)	0.000012 (0.6204)
α_1 (ARCH)	0.135453 (< 0.001)	0.064174 (< 0.001)
β_1 (GARCH)	0.236757 (0.1363)	0.931215 (< 0.001)
β_3 (GARCH)	0.488560 (< 0.001)	—
γ_1 (Leverage)	—	0.007222 (0.8293)
Shape (t-dist.)	3.321250 (< 0.001)	3.242293 (< 0.001)

Table 9: Parameter Estimates with Robust Standard Errors

The Parameter Estimates in Table 2 Reveal Critical Insights

• The persistence of volatility, captured by the sum of (α + β) coefficients, is high for both models, confirming strong volatility clustering in Bitcoin.
• In the GARCH (1,3) model, the third GARCH lag (β_3) is highly significant, justifying the higherorder specification.
• Most importantly, the leverage effect parameter in the GJR-GARCH model (γ_1) is statistically insignificant (p-value = 0.8293). This indicates that there is no robust evidence that negative news shocks have a different impact on future volatility than positive news shocks in our sample.

Model Diagnostics and Stability

A battery of post-estimation diagnostic tests was conducted on the standardized residuals to validate the model specifications.

Test	*GARCH(1,3) p-value*	*GJR-GARCH(1,1) p-value*	*Interpretation*
Ljung-Box (Residuals)	0.1054	0.1261	No autocorrelation
Ljung-Box (Squared Residuals)	0.9377	0.5221	No ARCH effects
ARCH LM Test	0.8855	0.3289	No conditional heteroskedasticity
Sign Bias (Joint)	0.09416	0.1626	No significant asymmetry

Table 10: Residual Diagnostic Tests

Key Diagnostic Findings

Adequacy of Volatility Capture: Both models successfully pass the Ljung-Box and ARCH-LM tests for squared residuals, indicating that they have effectively captured the volatility clustering in the data, leaving no significant autocorrelation or ARCH effects in the standardized residuals.
Asymmetry Tests: The joint Sign Bias test is not significant for either model, suggesting that, on the whole, both specifications adequately capture the impact of news. However, the GARCH (1,3) model shows a minor significant positive sign bias (p=0.02493), which is absent in the GJR-GARCH model. This implies that while the GJR-GARCH's leverage term is insignificant, its structure may still slightly better absorb certain asymmetric patterns.

Parameter	*GARCH(1,3)*	*GJR-GARCH(1,1)*	*Critical Value (5%)*
Joint	5.6004	5.1513	1.90
Alpha1	1.1724	1.2912	0.47
Beta1	0.7866	0.7788	0.47
Gamma1	—	0.8781	0.47

Note: The Nyblom test values for several parameters exceed the 5% critical value, indicating some parameter instability. This is a common finding in financial time series, especially for a volatile asset like Bitcoin, and suggests that model parameters may not be constant over very long periods.

Table 11: Parameter Stability (Nyblom Test)

Discussion and Synthesis of Findings

The results present a nuanced picture. On one hand, the model selection criteria (AIC, BIC) unanimously favor the symmetric GARCH (1,3) model as the best-fitting model for this dataset. Its structure, with a significant third GARCH lag, is particularly adept at modeling the persistence of Bitcoin's volatility. On the other hand, the investigation into asymmetry yields a clear result: the leverage effect parameter (γ_1) in the GJR-GARCH model is statistically insignificant. This finding challenges the assumption that Bitcoin volatility reacts asymmetrically to bad versus good news, at least during the sample period under study. The superior performance of the symmetric GARCH (1,3) model further reinforces this conclusion.

The minor sign bias detected in the GARCH (1,3) residuals suggests that while a strong, consistent leverage effect is absent, there may be more complex, non-linear asymmetric patterns at play that are not fully captured by the simple dummy-variable approach of the GJR-GARCH model. Future research could explore this with more sophisticated models.

Conclusion

This study set out to identify the most appropriate model for capturing the volatility dynamics of Bitcoin daily returns. Our comprehensive analysis leads to the following definitive conclusions [5-15].

Optimal Model Selection

Among the GARCH-family models, the GARCH (1,3) specification provides the best fit for the data, as determined by the lowest AIC and BIC values. Its ability to capture long-memory in volatility makes it a robust tool for forecasting Bitcoin risk.

Absence of Leverage Effect

A key finding of this research is the lack of a statistically significant leverage effect. The parameter for asymmetric volatility (γ_1) in the GJR-GARCH (1,1) model is insignificant, indicating that negative returns do not generate systematically higher future volatility than positive returns of the same magnitude in the Bitcoin market. This distinguishes Bitcoin from traditional equity markets and has important implications for risk management and option pricing models that assume asymmetry.

Model Adequacy

Both the GARCH (1,3) and GJR-GARCH (1,1) models are well-specified, passing a comprehensive set of diagnostic checks for autocorrelation and remaining heteroskedasticity. This validates their use for analyzing Bitcoin volatility. In summary, for the purpose of modeling and forecasting Bitcoin volatility, practitioners and researchers should prioritize the symmetric GARCH (1,3) model. While the potential for asymmetric effects should not be entirely dismissed, our results indicate that it is not a dominant feature in this context, and the parsimonious, well-fitting symmetric model is the most reliable choice. This conclusion provides a clear, evidence-based guideline for financial analysts and contributes to a more precise understanding of the risk characteristics inherent in the cryptocurrency market.

References

1. Poon, S. H., & Granger, C. W. J. (2003). Forecasting volatility in financial markets: A review. Journal of economic literature, 41(2), 478-539.
2. Dyhrberg, A. H. (2016). Hedging capabilities of bitcoin. Is it the virtual gold? Finance research letters, 16, 139-144.
3. Chu, J., Chan, S., Nadarajah, S., & Osterrieder, J. (2017). GARCH modelling of cryptocurrencies. Journal of Risk and Financial Management, 10(4), 17.
4. Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. The review of economics and statistics, 542-547.
5. Balcilar, M., Bouri, E., Gupta, R., & Roubaud, D. (2017). Can volume predict Bitcoin returns and volatility? A quantiles-based approach. Economic Modelling, 64, 74-81.
6. Baur, D.G., &Dimpfl, T. (2018). Asymmetric volatility in cryptocurrencies. Economics Letters, 173, 148-151.
7. Black, F. (1976). Studies of stock price volatility changes. In Proceedings from the American statistical association, business and economic statistics section (p. 177).
8. Khoso, W. (2025). Forecasting Bitcoin price volatility using GARCH models and real-time data. SSRN Electronic Journal, University of Sindh. https://ssrn.com/abstract=5279513 or http://dx.doi.org/10.2139/ssrn.5279513
9. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of econometrics, 31(3), 307-327.
10. Caporale, G. M., & Zekokh, T. (2019). Modelling volatility of cryptocurrencies using Markov-Switching GARCH models. Research in International Business and Finance, 48, 143-155.
11. Conrad, C., Custovic, A., & Ghysels, E. (2018). Long-and short-term cryptocurrency volatility components: A GARCH-MIDAS analysis. Journal of Risk and Financial Management, 11(2), 23.
12. Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica: Journal of the econometric society, 987-1007.
13. Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. The journal of finance, 48(5), 1779-1801.
14. Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. Economics letters, 158, 3-6.
15. Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica: Journal of the econometric society, 347-370.