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Modified Normalized Fried Mann Fractal Dimension For Characterizing Pore Structure in Khuff Carbonate Reservoirs, Central Saudi Arabia

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Abstract

A regression analysis was performed between the fractal dimensions calculated using the conventional normalized pore-radius method, $Df_{\{\log(R/R_{\max})\}}$, and those obtained from the Modified Normalized Friedmann Method (MNFM), $Df_{\{MNFM\}}$. The analysis yielded a slope of 1.0001, an intercept of -0.00037 , a correlation coefficient (R) of 0.999999984, and a coefficient of determination (R^2) of 0.999999967. These results indicate an almost perfect linear relationship between the two approaches. The slope being essentially equal to unity and the intercept approaching zero demonstrate that the MNFM reproduces the fractal dimensions obtained from the conventional method with negligible deviation. Therefore, the Modified Normalized Friedmann Method can be considered a reliable alternative for characterizing pore-structure fractal behavior in the Khuff carbonate reservoir samples.

Keywords: Modified Normalized Fried Mann Fractal Dimension, Khuff Formation, Carbonate Reservoir, Pore Structure, Permeability, Capillary Pressure, Water Saturation, Pore-Network Heterogeneity, Porous Media

Introduction

Characterization of heterogeneous porous materials remains a major challenge in reservoir engineering and materials science. Fractal geometry provides a quantitative description of pore- network complexity and heterogeneity. Fractal dimensions derived from capillary-pressure data have been widely used to characterize porous media and predict reservoir quality.

Fractal geometry has become an important tool for describing the complexity and heterogeneity of porous materials. The relationship between capillary pressure and wetting-phase saturation in fractal porous media has been extensively investigated and has provided a foundation for reservoir characterization [1]. Experimental verification of permeability estimation from capillary-pressure data demonstrated the applicability of fractal approaches in porous systems [2]. A theoretical relationship between capillary pressure and resistivity index based on fractal scaling theory was later proposed by [3].

Fractal dimensions derived from NMR relaxation times have been shown to characterize both pore-volume geometry and internal pore-surface complexity. reported that heterogeneous pore-throat distributions are associated with higher fractal dimensions in sandstone reservoirs [4]. More recently, fractal approaches have been successfully applied to carbonate reservoirs, tight sandstones, and multiscale pore-network characterization, demonstrating strong correlations between fractal dimension, permeability, and pore connectivity [5-7]. Studies have independently demonstrated the applicability of electric potential energy, electric potential gradient, and electric power for estimating fractal dimensions

in Shajara sandstone reservoirs [8]. However, these investigations were conducted separately despite being based on the same geological formation and petrophysical dataset.

Recent advances in fractal characterization have expanded the understanding of pore-scale heterogeneity in sandstone reservoirs. Deep-learning-assisted fractal analysis has been successfully employed for quantitative characterization of tight sandstone pore systems, revealing strong relationships between pore morphology and fractal dimension [9]. Fractal dimensions have also been shown to be effective indicators of pore-throat heterogeneity, permeability, and reservoir quality in tight sandstone formations [10,11]. Studies integrating high-pressure mercury intrusion, NMR, micro-CT, and digital rock physics have demonstrated that multiscale fractal dimensions provide valuable information regarding pore connectivity and fluid transport behavior [12].

Furthermore, pore-structure evolution and fractal heterogeneity have been investigated under various geological and engineering conditions, including waterflooding, CO₂–water–rock Interactions, and freeze–thaw cycles, confirming the versatility of fractal methods in porous-Material characterization [13–15]. These developments support the application of electrical-property-based fractal dimensions as complementary descriptors of pore architecture and reservoir heterogeneity.

Additional studies have demonstrated the applicability of fractal methods for reservoir characterization using pressure-head, relaxation-time, resistivity, and seismo-electric approaches, as well as multiscale pore-structure analyses based on NMR, digital rock physics, micro-CT imaging, and permeability modeling in sandstone reservoirs [16–28]. These investigations further confirm that fractal parameters provide valuable quantitative descriptors of pore-network complexity, heterogeneity, connectivity, and fluid-flow behavior in porous media.

The novelty of this study lies in introducing and validating a modified normalized Fried Mann (MNFM) fractal approach and demonstrating its equivalence to the conventional pore-radius scaling method for characterization of carbonate pore-network heterogeneity.

Method

Fractal dimensions provide a powerful quantitative framework for reservoir characterization. Two independent approaches were utilized to investigate the internal pore structure of Khartam reservoirs within the Permo-Triassic Khuff Formation, central Saudi Arabia.

The first approach is based on a conventional reservoir parameter, namely normalized pore radius and water saturation. The second approach is based on a statistical parameter termed the Modified Normalized Friedmann(MNFM) index.

The logarithm of fractional water saturation, $X = \log_{10}(S_w)$, was used as the independent variable. A second-order polynomial regression was fitted using the Excel LINEST function according to:

$$\log_{10}(R/R_{\max}) = a[\log_{10}(S_w)]^2 + b \log_{10}(S_w) + c \quad (1)$$

And

$$\log_{10}(\text{MNFM}) = a[\log_{10}(S_w)]^2 + b \log_{10}(S_w) + c. \quad (2)$$

Where a is the quadratic coefficient, b is the linear coefficient, and c is the intercept. The fractal dimension was subsequently calculated from the linear coefficient according to:

$$D_f = 2 + b$$

The coefficients a , b , and c were obtained directly from polynomial regression using the LINEST function in Microsoft Excel.

Fractal dimension is related to logarithm permeability as follow:

$$D_f = m \cdot \log(K) + c \quad (3)$$

D_f = Fractal dimension.

K = permeability (mD).

M = Slope of linear regression.

C = Intercept of Linear regression.

Results and Discussion

Petrophysical Characteristics of the Khuff Carbonate Reservoir Samples

The analyzed carbonate samples from the Permo–Triassic Khuff Formation exhibit porosity values ranging from 2.27% to 11.25%, with an average porosity of 4.86%. Measured permeability ranges from 0.264 to 3.445 mD, with an average value of 1.12 mD. The relatively low permeability values indicate a tight carbonate reservoir characterized by restricted

pore-throat systems despite the presence of measurable pore volume.

The porosity–permeability cross-plot reveals a weak positive correlation ($r = 0.253$; $R^2 = 0.064$), indicating that porosity alone is a poor predictor of permeability in the studied samples. This behavior is typical of heterogeneous carbonate reservoirs, where permeability is controlled primarily by pore-throat geometry, pore connectivity, and diagenetic modification rather than total pore volume. Consequently, samples exhibiting similar porosity may display significantly different permeability values because fluid flow depends largely on the effectiveness of pore interconnections.

The weak porosity–permeability relationship suggests that additional descriptors are required to characterize the complexity of the pore system and explain permeability variations within the Khuff carbonates. The relationships between porosity, permeability, and fractal dimension are illustrated in Figure. 1. Figure 1a shows the weak correlation between porosity and permeability, whereas Figure. 1b and Figure. 1c demonstrate the stronger relationship between permeability and fractal dimension, particularly when permeability is expressed on a logarithmic scale.

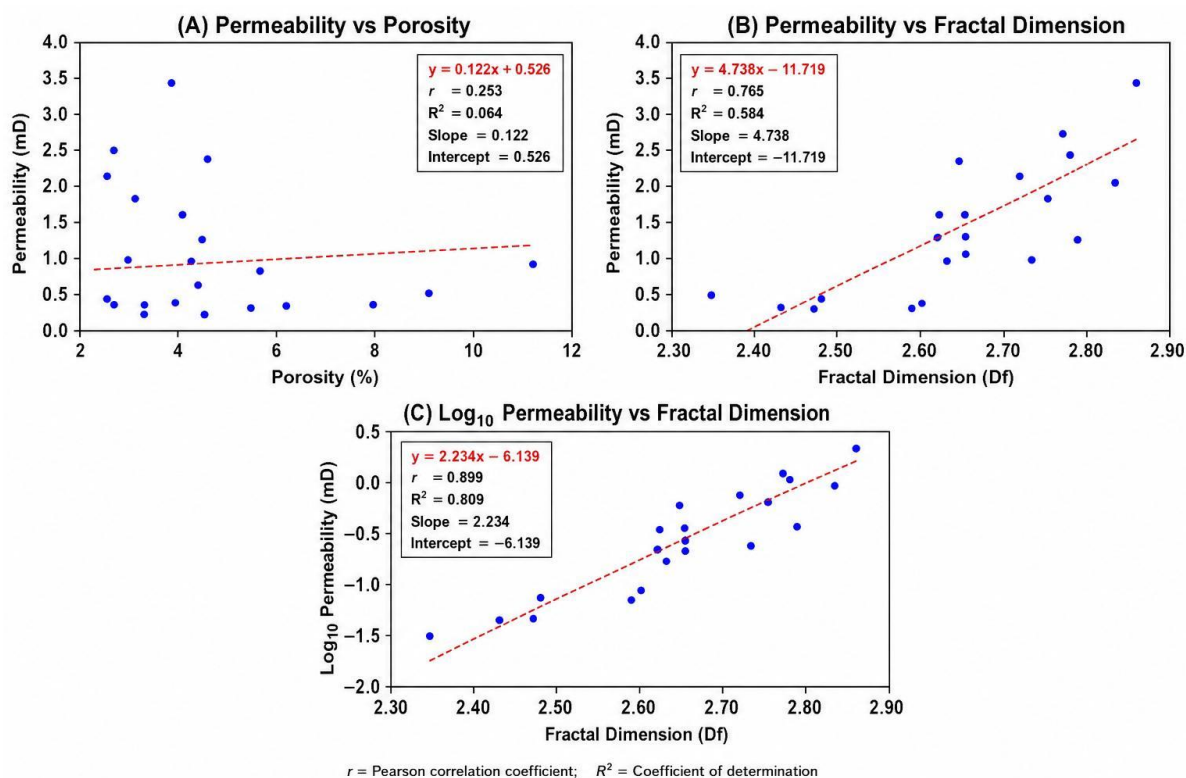


Figure 1: (a) Permeability versus porosity, (b) Permeability versus fractal dimension, (c) Log Permeability versus fractal dimension showing slope, intercept, coefficient of correlation R and Coefficient of determination R² of 26 paired Khuff carbonate samples.

Fractal Characterization of Pore Structure

Fractal analysis was applied to quantify pore-system complexity and heterogeneity. Fractal dimensions derived from the capillary-pressure data range from 2.363 to 2.865, with an average value of 2.710. These values are consistent with highly heterogeneous carbonate pore systems exhibiting multiscale pore-size distributions.

Fractal dimensions approaching 3 indicate increasingly irregular and complex pore geometries. Therefore, the relatively high Df values obtained in this study suggest that the Khuff carbonate pore networks are characterized by significant structural heterogeneity resulting from depositional variations and subsequent diagenetic processes including dissolution, cementation, recrystallization, and pore-throat modification.

The observed variability in fractal dimension reflects differences in pore-size distribution and pore connectivity among samples and demonstrates that pore architecture varies considerably within the studied reservoir interval.

Comparison of Fractal-Dimension Estimation Methods

Fractal dimensions calculated using the Log(MNFM) and Log(R/R_{max}) approaches show an almost perfect linear relationship. Regression analysis produced the following equation:

$$Df(\text{Log}(R/R_{\text{max}})) = 1.000103 \times Df(\text{Log}(\text{MNFM})) - 0.000366$$

with a correlation coefficient of $r = 0.99999998$ and a coefficient of determination of $R^2 = 0.99999997$.

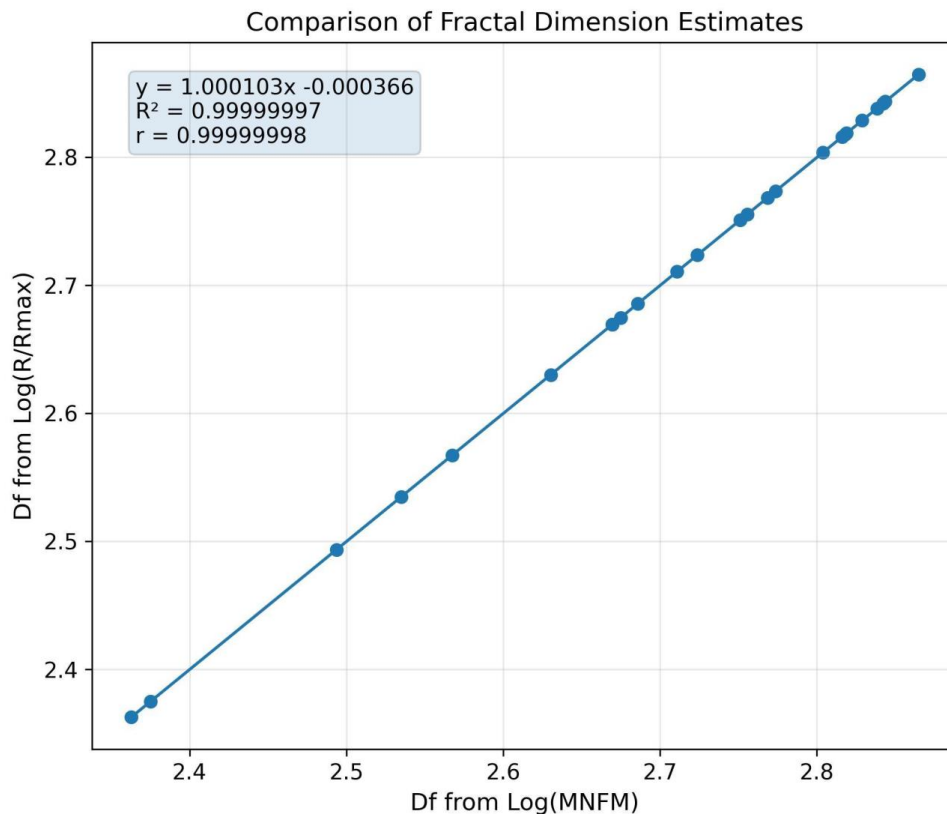


Figure 2: Regression of 26 paired fractal-dimension values showing near-perfect agreement between the Modified Normalized Friedmann Method (MNFM) and the conventional normalized pore-radius method.

The comparison between fractal dimensions obtained from the conventional normalized pore-radius method and the Modified Normalized Friedmann Method is presented in Figure. 2. The near-perfect agreement between the two datasets confirms the validity and reproducibility of the proposed MNFM approach.

The regression slope is essentially equal to unity and the intercept is negligible, indicating that both methods produce virtually identical fractal-dimension values. The near-perfect agreement confirms the robustness and reproducibility of the fractal analysis and demonstrates that the pore systems exhibit consistent self-similar scaling behavior regardless of the normalization procedure employed.

These results validate the use of either fractal model for pore-structure characterization within the Khuff carbonate reservoir.

Relationship Between Fractal Dimension and Permeability

A significantly stronger relationship is observed between fractal dimension and permeability than between porosity and permeability. Linear regression analysis yielded:

$$k = 4.738Df - 11.719$$

with $r = 0.765$ and $R^2 = 0.584$.

The positive correlation indicates that permeability generally increases with increasing fractal dimension. This suggests that samples with more complex pore systems tend to possess better-connected pore networks capable of transmitting fluids more effectively.

An even stronger relationship is obtained when permeability is expressed on a logarithmic scale:

$$\log_{10}(k) = 2.234Df - 6.139$$

with $r = 0.899$ and $R^2 = 0.809$.

The high coefficient of determination indicates that approximately 81% of the variation in log-transformed permeability can be explained by variations in fractal dimension. This result demonstrates that fractal dimension is a powerful quantitative descriptor of pore-network architecture and a reliable predictor of permeability in the studied carbonate samples.

The improvement obtained using log-transformed permeability suggests that permeability responds exponentially rather than linearly to changes in pore complexity. Such behavior is consistent with flow through heterogeneous carbonate pore systems where relatively small changes in pore-throat connectivity can produce large changes in permeability.

Reservoir Implications

The combined petrophysical and fractal analyses demonstrate that pore-network architecture exerts a stronger influence on permeability than porosity alone. The weak porosity–permeability relationship ($R^2 = 0.064$) indicates that pore volume does not adequately describe reservoir quality in the Khuff carbonates. In contrast, fractal dimension captures the geometric complexity and connectivity of the pore network and shows a strong relationship with permeability, particularly when permeability is analyzed on a logarithmic scale ($R^2 = 0.809$).

These findings indicate that fractal analysis provides critical information regarding pore-system organization and hydraulic behavior that cannot be obtained from conventional porosity measurements alone. The strong correlation between fractal dimension and permeability suggests that fractal parameters can be incorporated into reservoir characterization workflows to improve permeability prediction and reservoir-quality assessment.

Overall, the results demonstrate that the Khuff carbonate reservoir is characterized by heterogeneous multiscale pore systems whose flow properties are governed primarily by pore-network complexity and connectivity. Fractal dimension therefore represents an effective quantitative parameter for evaluating reservoir heterogeneity and predicting fluid-flow performance in tight carbonate reservoirs.

Cumulative saturation-distribution characteristics. Small differences in sensitivity to large pore throats may therefore become amplified at higher fractal dimensions.

Increasing fractal dimension reflects greater pore-size variability, improved pore-throat connectivity, and increased geometric complexity of the pore network.

Conclusions

The Modified Normalized Friedmann Method (MNFM) produces fractal-dimension values that are practically identical to those obtained from the conventional normalized pore-radius method. The near-unity slope, negligible intercept, and extremely high R^2 demonstrate that the MNFM approach preserves the fractal characteristics of the pore system while yielding statistically indistinguishable fractal dimensions.

Beyond carbonate reservoirs, the proposed methodology has potential applications in the characterization of a wide range of porous materials, including sandstones, ceramics, catalysts, nanomaterials, and engineered porous media. The observed increase in permeability with increasing fractal dimension suggests that pore-network connectivity exerts a stronger control on fluid-flow capacity than porosity alone in the investigated Khuff carbonate sample.

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Conflict of Interest

The author declares that there are no conflicts of interest regarding the publication of this manuscript.

Author Contributions

Khalid Elyas Mohamed Elameen AlKhidir conceived the study, performed the data analysis, developed the methodology, interpreted the results, prepared the figures, wrote the manuscript, and approved the final version of the manuscript.

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