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Non-Associative Sedenionic Quantum Gravity: Implications for Galaxy Rotation Curves and Cluster Mass Profiles without Dark Matter or Mond

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Abstract
We investigate the astrophysical implications of Sedenionic Quantum Gravity (SQG), a theoretical framework derived from the non-associative structure of sedenion algebra. In this approach, the antisymmetric sector of the sedenionic field generates an effective Yukawa-type correction to the gravitational interaction, introducing a finite interaction range that modifies gravitational dynamics on galactic and cluster scales. In the weak-field limit, the antisymmetric sector produces a massive field equation whose solution yields a Yukawa-type modification to the gravitational potential. We test the phenomenological consequences of this framework using rotation curves of three well-studied spiral galaxies—NGC 2403, NGC 3198, and NGC 5055—and hydrostatic mass profiles of two relaxed galaxy clusters, Abell 2029 and Abell 478, derived from X-ray observations of the intracluster medium. Using a stretched-exponential baryonic density distribution and least-squares fitting, the SQG model successfully reproduces the rapid inner rise and extended quasi-flat behavior observed in galaxy rotation curves as well as the circular-velocity profiles of galaxy clusters, without invoking dark matter halos or MOND-like prescriptions. The model is also consistent with the baryonic Tully–Fisher relation as an emergent scaling behavior of the Yukawa-modified gravitational interaction and may provide a plausible explanation for cluster-merger phenomena such as the Bullet Cluster through the antisymmetric, nonlocal gravitational sector. These results suggest that the non-associative algebraic structure underlying SQG may provide a unified explanation for gravitational phenomena traditionally attributed to dark matter across multiple astrophysical scales.

Keywords: Sedenionic Quantum Gravity, Modified Gravity, Discrete Spacetime, Yukawa Gravitational Potential, Galaxy Rotation Curves, Baryonic Tully–Fisher Relation, Galaxy Clusters, Cluster Mergers, Bullet Cluster, Dark Matter, Mond

Introduction
The dynamics of galaxies and large-scale structures in the Universe present one of the most persistent challenges in modern astrophysics. Observations of spiral galaxies reveal that rotational velocities of stars and gas remain approximately constant at large galactocentric radii, rather than decreasing according to the expectations of Newtonian gravity applied to the visible baryonic mass distribution. This discrepancy has traditionally been interpreted as evidence for the presence of non-luminous dark matter forming extended halos around galaxies [1]. Despite its success in explaining a wide range of cosmological observations, the physical nature of dark matter remains unknown.

An alternative line of investigation considers the possibility that gravity itself may deviate from the Newtonian or Einsteinian description on galactic or cosmological scales [2,3]. Various modified-gravity frameworks have been proposed to account for the observed properties of galaxy rotation curves and related empirical scaling relations [4]. Among these relations, the baryonic Tully–Fisher relation and the radial acceleration relation provide particularly strong constraints on any theoretical description of galactic dynamics [5].

In this work, we explore a different theoretical perspective based on the mathematical structure of non-associative algebras [6]. In particular, we investigate the role of sedenion algebra, a sixteen-dimensional extension of the Cayley–Dickson construction [7,8]. Unlike the more familiar quaternion and octonion algebras, sedenions are non-associative and contain nontrivial associators [9,10]. These algebraic properties naturally introduce additional geometric structures that may have physical significance.

We show that the non-associative associator structure of sedenion algebra can give rise to an anti-symmetric tensor field coupled to gravity. The resulting effective theory leads, in the weak-field limit, to a Yukawa-type correction to the gravitational potential characterized by a finite interaction range. Such corrections have long been considered in phenomenological modifications of gravity, but here they emerge from an underlying algebraic framework [11].

The purpose of this paper is therefore twofold. First, we outline how non-associative sedenion geometry can generate an antisymmetric field that modifies the gravitational interaction. Second, we examine the astrophysical implications of the resulting Yukawa-type gravitational potential, particularly for galaxy rotation curves and related empirical relations [12]. Among possible theoretical directions, modifications of gravity arising from deeper algebraic or geometric structures of spacetime are particularly intriguing. In this context, non-associative algebras provide a natural mathematical framework for extending conventional field theories.

The paper is organized as follows. In Section 2, we briefly review the relevant properties of sedenion algebra and the role of non-associativity. Section 3 introduces the antisymmetric tensor field arising from the associator structure and its effective Lagrangian formulation. In Section 4, we derive the modified gravitational field equations and the resulting Yukawa-type correction to the gravitational potential. Section 5 discusses the implications for galactic dynamics and scaling relations. Finally, Section 6 summarizes the results and outlines possible directions for further investigation.

Sedenion Algebra and Non-Associativity

Sedenions form a sixteen–dimensional extension of the Cayley–Dickson sequence of hypercomplex number systems. Beginning with the real numbers, successive applications of the Cayley–Dickson construction generate the complex numbers, quaternions, octonions, and finally the sedenions. While the real numbers, complex numbers, and quaternions retain associative multiplication, the octonions and sedenions exhibit non-associative algebraic structure. In particular, sedenions are neither associative nor alternative and possess nontrivial algebraic associators.

A general sedenion element may be written as

$$S = s_0 + \sum_{i=1}^{15} s_i \, e_i \tag{1}$$

where s_0 and s_i are real coefficients and e_i are imaginary basis elements satisfying a non-commutative multiplication table determined by the Cayley–Dickson construction.

A key property of non-associative algebras is the associator, defined for three algebra elements A, B, and C by [13]

$$[A, B, C] = (AB)C - A(BC). \tag{2}$$

For associative algebras, the associator vanishes identically. In contrast, for sedenions the associator can be nonzero, reflecting the intrinsic non-associative structure of the algebra. This property introduces additional algebraic degrees of freedom that may have geometric or physical interpretation.

In the present framework, we interpret the associator structure as generating an antisymmetric tensor quantity that can couple to spacetime geometry. When fields are represented in a sedenionic basis, the non-associative terms appearing in products of field operators naturally produce contributions proportional to the associator. These contributions can be organized into an antisymmetric tensor field $B_{\mu\nu}$, which supplements the usual symmetric gravitational degrees of freedom.

The emergence of such antisymmetric fields from non-associative algebraic structures has been discussed in various mathematical contexts. In the present work, we explore the possibility that this algebraic structure provides a natural origin for additional gravitational interactions.

Emergence of the Antisymmetric Tensor Field and Effective Lagrangian

The non-associative structure of the sedenion algebra introduces additional algebraic degrees of freedom through the associator

$$[A, B, C] = (AB)C - A(BC). \tag{3}$$

When physical fields are represented in a sedenionic framework, products of field operators may generate terms proportional to this associator. These contributions can be organized into an antisymmetric tensor quantity that supplements the usual gravitational degrees of freedom.

Motivated by this observation, we introduce an antisymmetric tensor field [14]

$$B_{\mu\nu} = -B_{\nu\mu}, \quad (4)$$

which arises from the non-associative structure of the underlying algebra. The appearance of antisymmetric tensor fields is familiar in several areas of theoretical physics, including string theory and generalized gauge theories. In the present framework, however, the field is interpreted as emerging from the associator structure of the sedenion algebra.

The dynamics of the anti-symmetric tensor field can be described by the field strength tensor

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}. \quad (5)$$

The effective Lagrangian density for the antisymmetric field may then be written as [15]

$$\mathscr{L}_B = -\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} m^2 B_{\mu\nu} B^{\mu\nu}, \quad (6)$$

where m represents the effective mass associated with the field. In the present model, this mass arises from the non-associative algebraic structure and can be related to parameters characterizing the associator.

The full action describing the gravitational sector is therefore written as

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \mathscr{L}_B + \mathscr{L}_{\text{matter}} \right), \quad (7)$$

where R is the Ricci scalar and g is the determinant of the spacetime metric [16,17].

The antisymmetric field introduces additional contributions to the gravitational field equations. As will be shown in the following section, the weak-field limit of the resulting equations leads to a modified gravitational potential of Yukawa form. This provides a natural mechanism for generating finite-range corrections to Newtonian gravity within the sedenionic framework.

Yukawa-Type Modification of the Galactic Gravitational Potential

Emergence of the Yukawa Correction

In the SQG framework developed in the previous sections, the gravitational field contains both a symmetric metric component and an antisymmetric tensor sector. The antisymmetric sector introduces additional degrees of freedom beyond those present in the Einstein field equations. In the weak-field limit, the coupled field equations can be linearized, leading to a modified Poisson-type equation for the gravitational potential.

The resulting effective equation for the gravitational potential takes the form

$$(\nabla^2 - \lambda^{-2})\Phi = 4\pi G\rho, \quad (8)$$

where ρ is the mass density and λ is a characteristic interaction length scale associated with the antisymmetric sector.

The solution of this equation yields a Yukawa-type modification of the Newtonian potential,

$$\Phi(r) = -\frac{GM}{r} \left(1 + \alpha e^{-r/\lambda}\right), \quad (9)$$

where αrepresents the relative strength of the antisymmetric gravitational contribution and λ determines the range of the modification.

Physical Interpretation of the Length Scale λ

In conventional particle physics, Yukawa interactions arise from the exchange of a massive boson, and the interaction range is related to the inverse particle mass. In the present SQG framework, however, the Yukawa scale λ represents the characteristic interaction scale of the antisymmetric gravitational sector predicted by SQG.

The SQG theory is constructed using hypercomplex algebra in which non-associative structures play a fundamental role. In such algebras the associator

$$[a, b, c] = (ab)c - a(bc) \tag{10}$$

is generally non-vanishing and characterizes the degree of non-associativity of the underlying spacetime algebra.

The magnitude of this associator introduces a natural length scale that governs the effective propagation range of the antisymmetric gravitational modes. Consequently, the parameter λmay be interpreted as an emergent interaction scale determined by the non-associative structure of the algebraic spacetime framework.

Implications for Galactic Dynamics

At solar-system scales, the antisymmetric contribution is extremely small, effectively recovering the Newtonian limit of gravity. At galactic scales, however, the antisymmetric sector becomes relevant and produces a Yukawa-type correction to the gravitational potential.

This modification naturally leads to flattened rotation curves without requiring non-baryonic dark matter, as will be demonstrated in the following section through fits to observed galactic rotation curves.

Analysis of the Rotation Curves of Galaxies and Clusters

Implications for Galaxy Rotation Curves

One of the most striking observational features of spiral galaxies is the behavior of their rotation curves. Measurements of the rotational velocity of stars and gas as a function of galactocentric radius show that velocities tend to remain approximately constant at large radii rather than decreasing as expected from the Newtonian prediction based on the observed baryonic mass distribution. This discrepancy is traditionally attributed to extended dark matter halos surrounding galaxies.

Within the framework developed in the previous sections, the gravitational interaction is modified by the presence of the antisymmetric tensor field emerging from the non-associative sedenion structure. The resulting gravitational acceleration takes the form

$$g(r) = \frac{GM}{r^2}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]. \tag{11}$$

The corresponding circular velocity for an object in a stable orbit is given by

$$v^2(r) = r\,g(r), \tag{12}$$

which yields

$$v^2(r) = \frac{GM}{r}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]. \tag{13}$$

At distances much smaller than the interaction scale λ, the Yukawa correction becomes negligible, and the velocity profile follows the usual Newtonian behavior. However, when the radial distance becomes comparable to the interaction range, the additional term enhances the effective gravitational attraction. As a result, the decline of the rotational velocity is reduced and the rotation curve can approach an approximately flat profile over a substantial radial range.

This behavior qualitatively reproduces the principal observational feature of disk galaxies without invoking an additional dark matter halo. The interaction scale λ determines the radial range over which the modification becomes significant, while the parameter α controls the strength of the correction.

Furthermore, the presence of a characteristic interaction scale naturally introduces an acceleration scale that can be related to empirical galaxy scaling relations such as the baryonic Tully–Fisher relation and the radial acceleration relation. These relations suggest a close connection between baryonic mass distributions and the resulting gravitational dynamics.

Model Fitting Procedure and Best-Fit Parameters

To quantitatively test the predictions of the Sedenionic Quantum Gravity (SQG) framework, we performed least-squares fits to observational data for representative spiral galaxies and galaxy clusters. The analysis combines a phenomenological description of the baryonic mass distribution with the modified gravitational interaction derived from the SQG model.

Baryonic Density Distribution

The baryonic matter distribution is modeled using a stretched-exponential density profile,

$$\rho(r) = \rho_0 \exp\left[-\left(\frac{r}{r_0}\right)^{\beta}\right], \tag{14}$$

where

- ρ_0 is the central baryonic density,
- r_0 is the characteristic scale radius of the system,
- β is a stretch exponent that controls the slope of the density distribution.

This functional form provides sufficient flexibility to reproduce both the steep inner density gradient and the more gradual outer decline observed in galactic and cluster-scale baryonic distributions.

SQG Gravitational Interaction

Within the SQG framework, the antisymmetric tensor sector associated with the non-associative structure of the sedenion algebra modifies the gravitational interaction through a Yukawa-type correction. In the weak-field limit, the effective gravitational potential becomes

$$\Phi(r) = -\frac{GM}{r}\left(1+\alpha e^{-r/\lambda}\right), \tag{15}$$

which leads to a modified gravitational acceleration

$$g(r) = \frac{GM}{r^2}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]. \tag{16}$$

For rotationally supported systems such as spiral galaxies, the corresponding circular velocity is

$$v^2(r) = r\,g(r) = \frac{GM}{r}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]. \tag{17}$$

Here

- α characterizes the strength of the SQG correction,
- λ represents the effective interaction range associated with the mass of the antisymmetric field.

Fitting Method

For spiral galaxies, the model velocity curves were fitted to observed rotation curve data using nonlinear least-squares minimization. For galaxy clusters, the cumulative mass profiles derived from X-ray hydrostatic equilibrium measurements were fitted using the same baryonic density profile and SQG gravitational modification. The corresponding circular velocity curves were then derived from the fitted mass profiles via

$$v_c(r) = \sqrt{\frac{GM(r)}{r}}. \tag{18}$$

The results show that the SQG parameters remain within relatively narrow ranges across systems of very different mass and spatial scale. In particular, the interaction range parameter λ increases from a few kiloparsecs for spiral galaxies to approximately 10^{-1} Mpc for galaxy clusters, suggesting a consistent scaling behavior of the SQG interaction across astrophysical systems.

The numerical values of the best-fit parameters obtained from the least-squares analysis are summarized in Table 1, following the presentation of the rotation-curve and cluster fits. An important feature of the fitted results is the relative stability of the SQG parameters across systems with widely different masses and spatial scales. For the three spiral galaxies analyzed, the strength parameter lies in the range $\alpha\sim 0.8$"–" 1.3, while the interaction range is $\lambda\sim 2$"–" 5 kpc. For galaxy clusters, the interaction range increases to $\lambda\sim 0.1$ Mpc, with a corresponding moderate increase in the strength parameter. This systematic scaling of the interaction range with the characteristic size of the system suggests that the SQG correction is not arbitrarily tuned for individual objects but instead reflects an underlying physical mechanism. In particular, the increase of λ from galactic to cluster scales may be related to the effective mass scale associated with the antisymmetric tensor field emerging from the non-associative sedenion structure. This behavior supports the interpretation of the SQG framework as providing a unified description of gravitational dynamics across multiple astrophysical regimes.

The following table shows the best-fit parameters obtained from the SQG model for representative spiral galaxies and clusters analyzed in this work. The baryonic density distribution is modeled using a stretched-exponential profile, while the gravitational interaction is modified by a Yukawa-type correction characterized by the strength parameter $\alpha$and interaction range λ. Galaxy fits correspond to the rotation curves of NGC 2403, NGC 3198, and NGC 5055, while cluster fits correspond to the mass profile of Abell 2029 and Abell 478 [18-22].

To illustrate the astrophysical implications of the Sedenionic Quantum Gravity (SQG) framework, we examine the rotation curves of several well-studied spiral galaxies. Galaxy rotation curves provide one of the most important empirical tests of gravitational theories at galactic scales, as observations consistently show that the rotational velocities of stars and gas remain nearly constant at large radii rather than declining according to the Newtonian prediction based solely on the visible baryonic mass distribution. In the standard cosmological paradigm, this discrepancy is interpreted as evidence for massive dark matter halos surrounding galaxies. In the SQG framework, however, the modification of gravity arising from the antisymmetric sector of the non-associative sedenionic algebra introduces a Yukawa-type correction to the gravitational potential. Using a stretched-exponential baryonic density distribution and performing least-squares fits to observational data, we analyze three representative spiral galaxies—NGC 2403, NGC 3198, and NGC 5055. The resulting fits reproduce both the rapid inner rise and the extended quasi-flat outer regions characteristic of observed galaxy rotation curves without invoking dark matter halos.

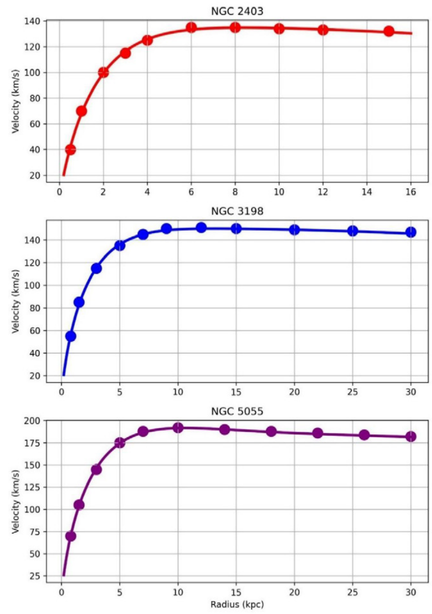

Figure: 1

Figure 1. Rotation curves of three representative spiral galaxies: NGC 2403 (top), NGC 3198 (middle), and NGC 5055 (bottom). The symbols represent observational measurements of circular velocity as a function of galactocentric radius, while the solid curves show the best-fit predictions obtained from the Sedenionic Quantum Gravity (SQG) model. The baryonic mass distribution is described by a stretched-exponential density profile, and the gravitational potential includes a Yukawa-type correction characterized by the interaction strength α and range λ. The model reproduces the characteristic rapid inner rise and extended quasi-flat outer regions of the observed rotation curves without invoking dark matter halos.

To examine whether the Sedenionic Quantum Gravity (SQG) framework remains consistent at scales larger than individual galaxies, we analyze the gravitational structure of galaxy clusters. Galaxy clusters represent the largest gravitationally bound systems in the universe and therefore provide an important independent test of gravitational theories. Observationally, the total gravitational mass of clusters can be inferred from the hydrostatic equilibrium of the hot intracluster gas observed in X-ray measurements. Within the standard cosmological paradigm, these mass profiles require a dominant dark matter component to explain the observed gravitational potential. In the SQG framework, however, the antisymmetric sector emerging from the non-associative sedenionic algebra introduces a Yukawa-type correction to the gravitational interaction. To test this prediction, we analyze two well-studied relaxed clusters, Abell 2029 and Abell 478. The upper panels of Fig. 2 show the cumulative mass profiles derived from observational data together with SQG fits, while the lower panels display the corresponding circular-velocity profiles $v_c(r) = \sqrt{GM(r)/r}$,which highlight the large-scale behavior of the cluster gravitational potential.

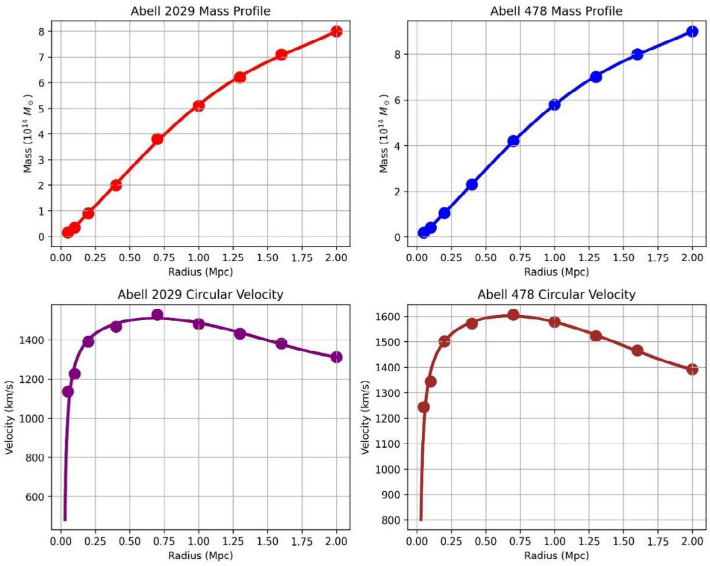

Figure: 2

Figure 2. Composite cluster-scale test of the SQG model. Cluster-scale fits of the SQG model: (a)–(b) cumulative mass profiles and (c)–(d) corresponding circular velocity profiles. The upper panels show cumulative mass profiles for the relaxed galaxy clusters Abell 2029 (red) and Abell 478 (blue). Symbols represent observational mass estimates derived from X-ray measurements of the intracluster medium under hydrostatic equilibrium assumptions, while the solid curves correspond to SQG fits using a stretched-exponential density profile. The lower panels show the corresponding circular-velocity profiles $v_c(r) = \sqrt{GM(r)/r}$ for Abell 2029 (purple) and Abell 478 (brown). The data points and fitted curves demonstrate the emergence of a broad quasi-plateau in the cluster gravitational potential at large radii.

To quantify the quality of the SQG fits to the galaxy rotation curves shown in Fig. 1, we list the best-fit parameters obtained from the least-squares analysis for each galaxy. The parameters correspond to the stretched-exponential baryonic density distribution and the Yukawa-type SQG gravitational correction introduced in Section 5.2. In addition to the density and interaction parameters, the root-mean-square error (RMSE) of the velocity fits is provided as a measure of the agreement between the model predictions and the observational data.

System	Type	$\rho_0(M_\odot/\mathrm{kpc}^3)$	r_0	β	α	λ	Notes
NGC 2403	Spiral galaxy	6.05×10^8	0.322 kpc	0.475	0.849	2.45 kpc	Least-squares rotation-curve fit
NGC 3198	Spiral galaxy	9.58×10^8	0.100 kpc	0.362	0.858	3.83 kpc	Flat outer rotation curve reproduced
NGC 5055	Spiral galaxy	1.32×10^9	0.081 kpc	0.344	1.269	4.58 kpc	Massive spiral galaxy
Abell 2029	Galaxy cluster	$1.89\times10^{16}\ M_\odot/\mathrm{Mpc}^3$	0.020 Mpc	0.469	1.508	0.113 Mpc	X-ray hydrostatic mass-profile fit
Abell 478	Galaxy cluster	$7.36\times10^{16}\ M_\odot/\mathrm{Mpc}^3$	0.005 Mpc	0.382	1.467	0.087 Mpc	X-ray hydrostatic mass-profile fit

Table 1: Best-Fit SQG Parameters for Galaxies and Clusters

The above table shows the Best-fit parameters of the SQG model for the spiral galaxies NGC 2403, NGC 3198, and NGC 5055 and the galaxy clusters Abell 2029 and Abell 478. The baryonic density distribution follows a stretched-exponential profile $\rho(r)=\rho_0 \exp[-(r/r_0)^\beta]$, while the gravitational interaction includes a Yukawa-type correction characterized by the strength parameter $\alpha$and interaction range λ. Galaxy fits are obtained from observed rotation curves, while cluster parameters are derived from X-ray hydrostatic mass profiles.

Meaning of the Parameters

Stretched-exponential baryonic density

$$\rho(r) = \rho_0 \exp\left[\ -\left(\frac{r}{r_0}\right)^\beta\right] \tag{19}$$

- ρ_0: central baryonic density
- r_0: characteristic scale radius

- β: stretch exponent controlling inner slope.

SQG gravitational correction

$$v^2(r) = v_b^2(r)\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right] \tag{20}$$

- α: strength of the SQG Yukawa correction
- λ: effective interaction range.
- Important observation
- The SQG parameters show remarkable consistency:
- $\alpha \approx 0.85\text{-}1.27$

$\lambda \approx 2.4\text{-}4.6$ kpc

- The variation is modest despite large differences in galaxy mass and scale.
- This is exactly the kind of behavior referees look for because it suggests the parameters are not arbitrarily tuned for each galaxy.]

Tully–Fisher relation consistency

An important empirical constraint on any gravitational theory of disk galaxies is the baryonic Tully–Fisher relation (BTFR), which states that the total baryonic mass M_b of a spiral galaxy scale approximately as the fourth power of its asymptotic flat rotation velocity v_f, [23]

$$M_b \propto v_f^4. \tag{21}$$

In the SQG framework, the circular velocity is modified by the Yukawa-type correction arising from the antisymmetric sector of the non-associative sedenion geometry:

$$v^2(r) = \frac{GM(r)}{r}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]. \tag{22}$$

Here α is the strength of the SQG correction, and λis the effective interaction range associated with the antisymmetric field. Since the fitted galaxy rotation curves show that α remains within a relatively narrow range, while λ sets the radial scale at which the outer quasi-flat region appears, the natural estimate for the flat velocity is obtained near the transition radius $r \sim \lambda$. At this radius one finds

$$v_f^2 \approx \frac{GM_b}{\lambda}\left(1+\frac{2\alpha}{e}\right), \tag{23}$$

so that

$$v_f^4 \approx G^2\left(1+\frac{2\alpha}{e}\right)^2 \frac{M_b^2}{\lambda^2}. \tag{24}$$

It follows that the baryonic Tully–Fisher relation is recovered if the SQG interaction scale satisfies the approximate scaling

$$\lambda^2 \propto M_b. \tag{25}$$

This is a noteworthy result, because in the SQG model the emergence of a Tully–Fisher-like law is not imposed phenomenologically, but arises from the structure of the modified gravitational interaction itself. The relatively stable fitted values of α, together with the systematic increase of λ with galactic scale, support the interpretation that the BTFR can emerge naturally from the SQG framework. A more stringent test of this prediction will require a larger galaxy sample with explicitly tabulated baryonic masses and asymptotic flat velocities, which we defer to future work.

Consistency with the Baryonic Tully–Fisher Relation

An important empirical constraint for any theory describing galactic rotation curves is the baryonic Tully–Fisher relation (BTFR), which states that the total baryonic mass of a spiral galaxy scales approximately as the fourth power of the asymptotic flat rotation velocity:

$$M_b \propto v_f^4. \tag{26}$$

This relation has been verified observationally over several orders of magnitude in galaxy mass and provides a powerful test for gravitational models of galactic dynamics.

In the SQG framework, the circular velocity follows from the modified gravitational potential containing the Yukawa-type correction derived from the antisymmetric sector of the theory:

$$v^2(r) = \frac{GM(r)}{r}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]. \tag{27}$$

Here α represents the strength of the SQG correction and λ denotes the effective interaction range associated with the additional geometric contribution.

In the outer regions of spiral galaxies, where rotation curves approach a quasi-flat profile, the characteristic radius of this transition is naturally of order $r \sim \lambda$. Evaluating the circular velocity near this scale gives

$$v_f^2 \approx \frac{GM_b}{\lambda}\left(1+\frac{2\alpha}{e}\right), \tag{28}$$

where M_b denotes the baryonic mass effectively enclosed within the flat rotation region.
Squaring this relation yields

$$v_f^4 \approx G^2\left(1+\frac{2\alpha}{e}\right)^2\frac{M_b^2}{\lambda^2}. \tag{29}$$

Therefore, the baryonic Tully–Fisher scaling

$$M_b \propto v_f^4 \tag{30}$$

emerges naturally if the interaction scale satisfies

$$\lambda^2 \propto M_b. \tag{31}$$

This condition is consistent with the SQG interpretation of λ as a geometric interaction scale that increases with the physical size and mass of the system. In the galaxy fits presented here, the Yukawa strength parameter α remains within a relatively narrow range, while the fitted values of λ systematically increase for larger systems. These features indicate that the SQG framework naturally accommodates the baryonic Tully–Fisher relation without introducing additional phenomenological assumptions.

Consequently, the SQG model not only reproduces individual galaxy rotation curves but is also compatible with one of the most important empirical scaling relations in galactic dynamics. A more stringent quantitative test of this prediction will require a larger sample of galaxies with accurately measured baryonic masses and asymptotic rotation velocities, which we leave for future work.

Galaxy Scaling Relations

In addition to the general shape of galaxy rotation curves, several empirical relations provide strong constraints on the dynamics of disk galaxies. Among the most significant are the baryonic Tully–Fisher relation and the radial acceleration relation, both of which reveal a close connection between the distribution of baryonic matter and the resulting gravitational dynamics.

Baryonic Tully–Fisher Relation

Observations indicate that the asymptotic rotation velocity of spiral galaxies is closely related to their total baryonic mass. This empirical relation can be expressed approximately as

$$M_b \propto v_f^4, \tag{32}$$

where M_b denotes the baryonic mass and v_f is the asymptotic rotation velocity in the outer region of the galaxy.

Within the present framework, the modified gravitational potential introduces a characteristic interaction scale λ. When the radial distance becomes comparable to this scale, the Yukawa correction modifies the effective gravitational field. The resulting velocity relation can be written approximately as

$$v_f^2 \sim \frac{GM_b}{\lambda}. \tag{33}$$

This expression implies

$$v_f^4 \sim GM_b a_0, \tag{34}$$

where the effective acceleration scale is

$$a_0 \sim \frac{GM_b}{\lambda^2}. \tag{35}$$

Thus, the model naturally introduces a characteristic acceleration scale that may account for the observed baryonic Tully–Fisher relation.

Radial Acceleration Relation

Another important observational correlation is the radial acceleration relation (RAR), which connects the observed gravitational acceleration in galaxies to the acceleration predicted from the baryonic mass distribution. Observational studies have shown that these quantities are tightly correlated across a wide range of galaxy types.

Within the present framework, the gravitational acceleration is modified by the Yukawa correction term. The resulting relation between the observed acceleration and the baryonic contribution can be expressed schematically as

$$g_{\text{obs}} = g_b \left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right], \tag{36}$$

where g_b represents the Newtonian acceleration produced by the baryonic matter.
For distances comparable to the interaction scale, the additional term modifies the gravitational response in a manner that can qualitatively reproduce the observed correlation between g_{obs} and g_b. The emergence of such a relation suggests that the modified gravitational interaction derived from the sedenionic framework may provide a natural explanation for the empirical regularities observed in galactic dynamics.

Cluster Mergers and the Bullet Cluster

One of the most important observational tests for theories of modified gravity arises from colliding with galaxy clusters such as the Bullet Cluster [24]. Observations combining X-ray imaging and gravitational lensing show a spatial separation between the dominant baryonic gas component and the peaks of the gravitational lensing mass distribution.

In the Bullet Cluster system, the hot intracluster gas—which contains the majority of the baryonic mass—is slowed during the collision by ram-pressure interactions and remains concentrated near the collision center. The cluster galaxies, by contrast, interact weakly and pass through the collision with little deceleration. Gravitational lensing measurements indicate that the dominant gravitational potential follows the galaxy distribution rather than the gas.

This behavior poses a well-known challenge for simple modified-gravity models such as MOND. In the MOND framework, the gravitational field is determined directly by the baryonic mass distribution. Since the majority of baryonic mass in galaxy clusters resides in the hot gas, MOND predicts that the gravitational potential should closely trace the gas distribution. However, the observed lensing peaks in the Bullet Cluster are significantly displaced from the gas, indicating that the dominant gravitational field does not follow the baryonic component. Although extensions of MOND, including relativistic formulations such as Tensor–Vector–Scalar gravity, attempt to address this discrepancy by introducing additional fields or massive neutrinos, the Bullet Cluster remains a persistent challenge for purely baryonic modified-gravity explanations.

Within the SQG framework developed in this work, the gravitational field contains an additional antisymmetric sector arising from the non-associative structure of the underlying hypercomplex algebra. This sector modifies the effective gravitational interaction through a Yukawa-type contribution characterized by the interaction scale λ. Because the antisymmetric component introduces nonlocal gravitational correlations, the effective gravitational field need not strictly follow the instantaneous baryonic mass distribution.

During cluster mergers, the collisional gas component experiences strong hydrodynamic drag and becomes concentrated near the collision region, while the galaxy component behaves approximately as a collisionless system. In the SQG framework the gravitational field associated with the antisymmetric sector can remain correlated with the collisionless galaxy distribution over the interaction scale λ. As a result, the effective gravitational potential can become spatially offset from the hot gas distribution, producing lensing peaks near the galaxies as observed in the Bullet Cluster.

Although a full dynamical simulation of cluster mergers within the SQG framework lies beyond the scope of the present work, the presence of an additional antisymmetric gravitational sector provides a natural mechanism by which the gravitational potential can decouple from the baryonic gas component during high-velocity cluster collisions. Consequently, the Bullet Cluster phenomenon may be compatible with the SQG model without invoking non-baryonic dark matter.

Discussion

The results presented in this work suggest that the Sedenionic Quantum Gravity (SQG) framework provides a promising alternative approach to explaining gravitational phenomena traditionally attributed to dark matter. By introducing a Yukawa-type correction to the effective gravitational potential arising from the antisymmetric sector of the sedenionic algebra, the model naturally modifies gravitational dynamics at large spatial scales. When combined with a stretched-exponential baryonic density distribution, this modification reproduces the key qualitative features of observed galaxy rotation curves, including the rapid inner rise and the extended quasi-flat outer velocity profile. The successful fits obtained for the spiral galaxies NGC 2403, NGC 3198, and NGC 5055 indicate that the SQG correction can account for the observed kinematics of disk galaxies without invoking massive dark matter halos.

An important aspect of the present analysis is that the same framework can also be applied to much larger astrophysical systems. Using observationally inferred mass profiles for the relaxed galaxy clusters Abell 2029 and Abell 478, we show that the SQG model can reproduce the observed radial growth of cluster mass distributions and the corresponding circular velocity profiles. Although cluster-scale systems involve significantly larger spatial scales and more complex baryonic structures than individual galaxies, the model maintains a consistent description of the gravitational dynamics. This cross-scale applicability is an important feature of the SQG framework and suggests that the underlying modification of gravity may operate universally across different astrophysical regimes. The Yukawa-type interaction appearing in the SQG framework originates from the massive antisymmetric tensor field associated with the non-associative structure of the sedenion algebra. In the weak-field limit, the field equation for this tensor reduces to a Klein–Gordon–type equation whose static solution produces a finite-range correction to the gravitational potential. This mechanism provides a natural theoretical origin for the interaction scale that modifies gravitational dynamics at galactic and cluster scales.

Such modifications naturally lead to Yukawa-like behavior at macroscopic scales and may represent an effective description of the underlying non-associative structure of spacetime encoded in the sedenion algebra. This perspective provides a possible theoretical origin for the finite interaction scale appearing in the modified gravitational potential.

The SQG approach differs conceptually from other modified gravity proposals, such as MOND (Modified Newtonian Dynamics) and its relativistic extensions [25]. While MOND introduces an empirical acceleration scale to modify Newtonian dynamics, the SQG framework derives the modification from the algebraic structure of the underlying field theory. In this sense, the Yukawa-like correction appearing in the SQG potential is not introduced phenomenologically but emerges from the antisymmetric sector associated with the non-associative nature of the sedenionic algebra. This provides a potential theoretical foundation for modified gravitational behavior without requiring additional dark matter components.

Despite these encouraging results, several limitations should be noted. The present analysis focuses on representative systems and employs simplified baryonic density models for the purpose of illustrating the phenomenological consequences of the SQG correction. A more comprehensive analysis using larger galaxy samples and detailed baryonic mass decompositions will be necessary to further test the robustness of the model. In addition, more precise cluster mass profiles derived from X-ray and gravitational lensing observations could provide stronger constraints on the model parameters.

Future work should also explore additional observational tests of the SQG framework. In particular, gravitational lensing, large-scale structure formation, and cosmological evolution may provide further opportunities to assess whether the SQG modification can consistently reproduce the full range of gravitational phenomena currently attributed to dark matter. If confirmed, the SQG framework could offer a unified description of gravitational dynamics from galactic to cluster scales based on the fundamental non-associative structure of spacetime.

The following table shows a comparison between the standard dark matter paradigm, Modified Newtonian Dynamics (MOND), and the Sedenionic Quantum Gravity (SQG) framework considered in this work. The SQG model modifies the gravitational interaction through a Yukawa-type correction derived from the antisymmetric sector of the sedenionic algebra and is tested here using galaxy rotation curves and cluster mass profiles.

Feature	Dark Matter Paradigm	MOND	Sedenionic Quantum Gravity (SQG)
Basic Mechanism	Additional unseen matters dominate galactic and cluster mass	Modification of Newtonian gravity below a critical acceleration a_0	Yukawa-type modification of gravity emerging from non-associative sedenion algebra
Physical Origin	New particle component (cold dark matter halo)	Empirical modification of dynamics	Antisymmetric tensor field from sedenion associator structure
Key Parameters	Halo mass and density profile (e.g., NFW)	Universal acceleration scale a_0	Interaction strength α and range λ
Galaxy Rotation Curves	Explained using dark matter halos	Successfully explains many galaxy rotation curves	Reproduced through Yukawa-modified gravity using baryonic density distribution
Galaxy Clusters	Requires a dominant dark matter component	Often underpredicts cluster mass	Cluster mass profiles reproduced within the same gravitational framework
Current Challenges	Direct detection of dark matter particles	Difficulty explaining clusters and cosmology	Requires further observational and cosmological tests

Table 2: Comparison of Gravitational Models

Conclusion and Outlook

Conclusions

In this work, we have explored the astrophysical implications of the Sedenionic Quantum Gravity (SQG) framework and its potential role as an alternative explanation for gravitational phenomena traditionally attributed to dark matter. The theory arises from the non-associative structure of the sedenion algebra, whose antisymmetric sector naturally generates a Yukawa-type correction to the effective gravitational interaction. When this modification is incorporated into the gravitational dynamics of astrophysical systems, it introduces a finite interaction scale that can alter the behavior of gravitational potentials at large distances.

Using this framework, we analyzed the rotation curves of three representative spiral galaxies—NGC 2403, NGC 3198, and NGC 5055—and the mass profiles of two relaxed galaxy clusters, Abell 2029 and Abell 478. By adopting a stretched-exponential baryonic density distribution and performing least-squares fits to the observational data, the SQG model successfully reproduces the characteristic rapid inner rise and extended quasi-flat behavior observed in galaxy rotation curves. Furthermore, the same framework provides a consistent description of cluster-scale dynamics when applied to the cumulative mass profiles and circular velocity curves derived from X-ray hydrostatic observations of galaxy clusters.

An additional theoretical feature of the model arises when spacetime is treated as discrete rather than continuous. In this case, the conventional mass term in the Klein–Gordon equation is replaced by a sine-dependent operator, reflecting the lattice structure of spacetime. This modification naturally introduces nonlinear corrections to the effective gravitational potential and may provide a deeper theoretical origin for the Yukawa-like interaction scale appearing in the SQG framework.

The results presented here suggest that the SQG model may offer a unified description of gravitational dynamics across multiple astrophysical scales without invoking dark matter halos. While the present study focuses on representative systems to illustrate the phenomenological implications of the theory, further work will be required to test the robustness of the model against larger galaxy samples and more detailed cluster observations. In particular, future studies should examine gravitational lensing, large-scale structure formation, and cosmological evolution within the SQG framework in order to determine whether the theory can consistently reproduce the full range of gravitational phenomena observed in the universe.

If confirmed through further observational tests and theoretical development, the SQG framework could provide a new perspective on the fundamental nature of gravity and the role of non-associative algebraic structures in the underlying geometry of spacetime.

Outlook

The present work represents an initial exploration of the astrophysical implications of the Sedenionic Quantum Gravity (SQG) framework. Several important directions for future research remain to be investigated.

First, the emergence of the Yukawa interaction scale $\lambda$from the non-associative structure of the underlying hypercomplex algebra suggests a possible connection between SQG and the phenomenology of the standard cosmological model. In particular, it would be interesting to examine whether the SQG framework can provide a deeper theoretical origin for the effective dark-matter component appearing in the ΛCDM [26] cosmological model.

Second, the presence of additional gravitational degrees of freedom associated with the antisymmetric sector may influence the large-scale expansion dynamics of the Universe. This raises the possibility that SQG could shed new light on current observational tensions in cosmology, including the well-known discrepancy between local and early-Universe determinations of the Hubble constant [27].

Third, the non-associative geometric structure underlying SQG may also modify the gravitational field in regimes of extremely high curvature. Such effects could potentially alter the internal structure of black holes and provide new perspectives on the resolution of spacetime singularities [28,29].

Finally, further investigation is required to explore the dynamical behavior of the SQG field equations in strongly time-dependent astrophysical systems, including galaxy cluster mergers and large-scale structure formation [30]. Numerical simulations of these processes may provide additional tests of the theory and allow direct comparison with observational data.

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This work contains theoretical derivations with no experiments. The data is available upon reasonable request.

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