

Volume 2, Issue 3

Research Article

Date of Submission: 13 May, 2026

Date of Acceptance: 19 June, 2026

Date of Publication: 09 July, 2026

Nuclear Reaction Calculations for Nuclear Weapon Applications

Malik M. A. Fakron* 

University of Calabria, Italy

***Corresponding Author:** Malik M. A. Fakron, University of Calabria, Italy.

Citation: Fakron, M. M. A. (2026). Nuclear Reaction Calculations for Nuclear Weapon Applications. *J Theor Exp Appl Phys*, 2(3), 01-27.

Abstract

The primary goal of this paper is to offer a comprehensive guide that outlines the steps necessary to design a variety of nuclear weapons, including nuclear fission atomic bombs and nuclear fusion bombs, beginning with first-principles calculations. Comprehensive logarithmic guides for nuclear fission and nuclear fusion reactions derived from mass balance and energy balance calculations. This covers nuclear weapons calculation techniques for neutron flux, nuclear reaction rate, and microscopic and macroscopic cross sections of nuclear reactions. This logarithmic approach, which is determined by client requests, specifies the explosion power and nuclear weapon yield requirements. Any successful bomb design requires a rapid reaction time. This chapter offers a comprehensive examination of the design of various nuclear weapons. From nuclear explosive fusion devices, such as hydrogen bombs and cold fusion bombs, to the design of intensification atomic bombs, the process has evolved from atomic fission bombs. The principal objective of this chapter is to offer a comprehensive comprehension of the steps involved in the design of atomic bombs and to make it more accessible to a broader range of students and professionals who aspire to be involved in international security, arms control, or nuclear non-proliferation issues.

Keywords: Atomic Bomb, Fission, Fusion, Cold Fusion, Nuclear Weapon

Introduction

Each nuclear explosive device must go through a number of steps in its design process. Theoretical and experimental research are part of these steps. The reaction's thermal effects must be computed to select between fission, fusion, or both types of reactions. Next, determine the essential mass and concentration for each nuclear device. A nuclear weapon is a machine that, when it goes off, turns small amounts of matter into a huge amount of energy. The idea behind this nuclear device is to use the heat released by a nuclear reaction. These nuclear reactions can be either fusion, fission, or both, as in the case of a thermonuclear fusion bomb. Nuclear weapons can be divided into two main categories: atomic and hydrogen bombs. A hydrogen bomb uses nuclear fission reactions as initiators and nuclear fusion reactions to produce heat energy waves. The generation of heat energy waves in atomic bombs is achieved through fission nuclear reactions. The atomic bomb, which is a type of fission nuclear weapon, is constructed based on the exponential expansion of nuclear fission chain reactions. The capture of neutrons by the nuclei of very heavy element isotopes is the hallmark of a nuclear fission reaction. Nuclei are split into pieces by the energy of neutrons, which results in the release of energy and several new neutrons. Minimize the number of neutrons lost to fission in the development of the atomic bomb, which is based on the fission chain reaction. The chain fission reaction rate is extremely rapid. A self-sustaining chain reaction is initiated. The nucleus of an atom is influenced by neutron bombardment in two primary ways: it scatters neutrons by deflecting them at various angles and reduces a portion of their kinetic energy, or it absorbs the neutron. The nucleus is affected by neutron absorption by splitting into species. A particular nucleus's probability of scattering or capturing a neutron is measured by its scattering and capture cross-sections. There are two parts to the overall capture cross-section. These are the absorption cross-section and the fission cross-section. The stability of any atom is determined by the binding energy of the nucleus. Nucleus rearranging structure is the energy required to disturb the nucleus by absorbing a proton or neutron.

The binding energy decreases as energy is released through rearrangement. Uranium-235 and plutonium-239 are the products of fission, which is the process by which energy is released. The binding energy of these two isotopes is so low that the energy released by rearrangement through spallation is nearly negligible when a neutron is captured. Thermonuclear reactions, or fusion reactions, are defined as the process of colliding nuclei to produce light elements and energy during reactions between two isotopes of light elements. A significant amount of heat energy is required for nuclei to collide. The rates of every fusion reaction are influenced by both density and temperature. The most significant fusion reaction in weapons is known as the D-T reaction. The process of initiating fusion reactions between deuterium or tritium to produce helium, one neutron, and energy uses the heat and pressure from fission reactions. By using the heat from a fission reaction, thermonuclear devices compress and heat fusion fuel to a plasma state. A substance becomes ionized and highly electrically conductive when it transforms into plasma, and electric and magnetic fields regulate its behavior. The fourth state of matter is called plasma. In the plasma approximation, the plasma parameter, which indicates the number of charge carriers inside the Debye sphere, is greater than unity. According to Einstein's equation of mass and energy, nuclear weapons utilize fission and fusion reactions to generate energy on a large scale, resulting in a reduction in mass. There are three primary types of neutron sources. The first type is produced by the interaction of high-speed D ions with T atoms. The second type is created by the Am-Be source, which uses americium as an alpha emitter and beryllium as a target for neutron production. The third type is the result of spontaneous fission of Cf. The second or third type of neutron sources are frequently utilized in nuclear fission weapons to initiate the fission chain reaction atomic bomb.

Nuclear Weapon Devices Design Steps

The initial step in the design of any nuclear explosive device is to determine the explosive power of the bomb by analyzing the design requirements data. That is, the energy released per unit of time by nuclear reactions. Selection of nuclear material reactants is the second step in the process of establishing a nuclear chain reaction for a fission nuclear bomb or a nuclear fusion reaction for a fusion bomb. This determination is implemented in nuclear explosive devices. The design details complete calculations for energy balance and mass balance for the main nuclear reaction inside the nuclear explosive device. Calculations are predicated on the necessary energy release per unit of time. To achieve the desired effects of these nuclear explosive devices. A large-scale release of energy in nuclear weapons is achieved through the use of two primary types of nuclear reactions, which result in a decrease in mass. (a) The nuclear fission reaction is the spallation of heavy nuclei into light nuclei with the release of energy. (b) the nuclear fusion reaction is the combination of two light nuclei into a heavier nucleus with the release of energy. By using the liquid drop model of nuclei, thermonuclear reactions for the fission of heavy nuclei are examined. This model highlights the analogy of the process that divides a fluid sphere into two smaller droplets due to deformation brought on by an external disturbance. Based on the liquid drop model, the fission reaction operates. L. The critical deformation energy required can be calculated by comparing the potential energy of the nuclear mass and charge-induced drop in the unstable equilibrium state. The degree of freedom between quarks and the shape of the nucleus undergoes a transformation as a result of the excitation energy for the nucleus. A heavy nucleus captures projectiles, primarily high-speed neutrons, and accumulates enough energy to unleash multiple particles. Furthermore, the target nucleus breaks up into lighter parts. This transformation results in critical deformation and, subsequently, a fission probability as a result of neutron capture. The overall amount of energy released during a nuclear fission reaction into smaller pieces is calculated by

$$\Delta E = (M_0 - \Sigma M_i) c^2 \quad (3.1)$$

M_0 and M_i are the masses of the original and product nuclei at rest and unexcited. c light speed.

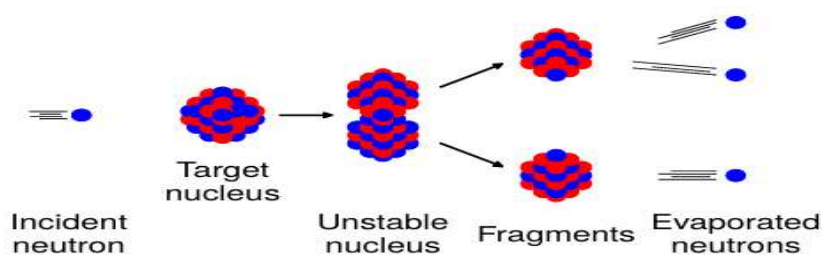


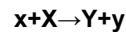
Figure 3.1

The nuclear binding energy E_b is defined as In the nucleus, binding energy is the attractive energy between nucleons. The masses difference between the nucleus and its constituent protons and neutrons. Let us consider an atom that has a nucleus, a mass number A , and an atomic number Z . This atom is composed of Z protons and $(A-Z)$ neutrons.

$$M.D. = [Z(m_p + m_e) + (A-Z)m_n] - M = Zm_H + (A-Z)m_n - M \quad (3.2)$$

$$E_b(\text{in MeV}) = 931 \times M.D \quad (3.3)$$

M.D mass defect ,The difference between the mass of the nucleus as determined by a spectrograph M and the mass of the nucleus's constituent protons Zm_H and neutrons $(A-Z)m_n$ alone is known as the mass defect. The following three fundamental conservation laws are essential for the study of any nuclear reaction: the conservation of mass and energy for nuclear reactions, the conservation of linear and angular momentums, and the conservation of charge and nucleons. Conserving mass and energy in nuclear reactions, Consider a nuclear reaction $X(x, y)Y$ represented by the equation,



In this case, x is the projectile, X is the target nucleus, and Y and y are products of nuclear reaction. The net mass and energy should be equal before and after the reaction, in accordance with the conservation of mass and energy principle.

$$(E_x + m_x c^2) + M_X c^2 = (E_Y + M_Y c^2) + (E_y + m_y c^2) \quad (3.4)$$

Where m_x , M_X , M_Y and m_y are masses of projectile, target nucleus, product nucleus and product particle. Where E_x , E_Y & E_y are kinetic energy of projectile, product nucleus and product particle. The Q value indicates the reaction's energy balance.

$$Q=E_Y+E_y -E_x \quad (3.5)$$

If $Q > 0$, then the reaction is exoergic or exothermic nuclear reaction and $Q < 0$ then it is said to be endoergic or endothermic nuclear reaction. For positive KE of products is greater than that of reactants.

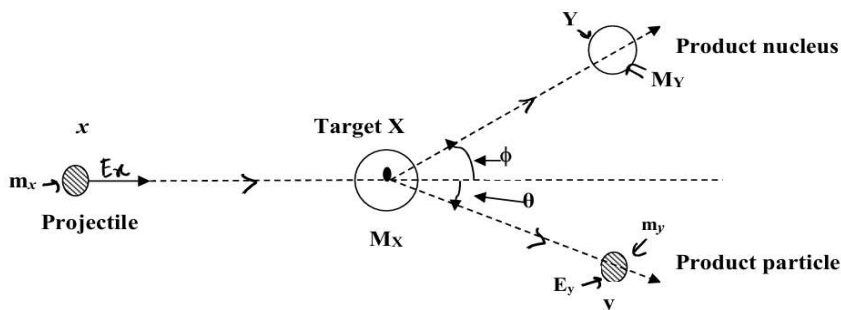


Figure 3.2

In writing nuclear reaction equations, two conservation conditions are invariant: the conservation of charge and the conservation of nucleons. The subscripts on both sides of the equation must add up to the same amount in order for conservation of charge to occur. The superscripts on both sides must be equal in order for nucleon conservation to apply. If a substance will fission when exposed to neutrons of any energy, it is considered fissile. The isotope uranium-235 is a fissile material. A fissile material is achieved through the transmutation of a neutron into a fertile material through radioactive decay. Uranium-238 is a fertile substance.

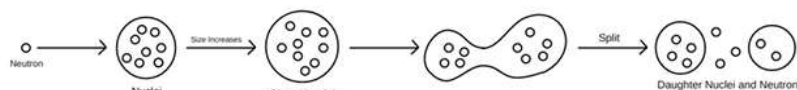
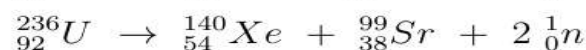
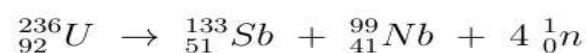
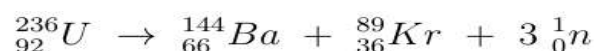
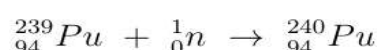


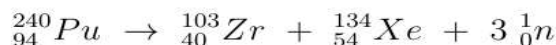
Figure 3.3

Considering two nuclear fission reactions for designing an atomic bomb The first reaction is fission of uranium 235 (39.029×10^{-22} g) by a slow neutron with energy 1eV ,Releasing energy of 201.5 MeV per atom.



The second nuclear fission reaction is the fission of plutonium-239 induced by a thermal neutron with energy of about 0.025 eV bombardment,Releasing energy of 207.1 MeV per atom.

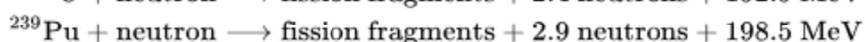
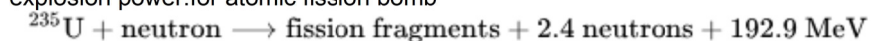




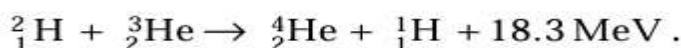
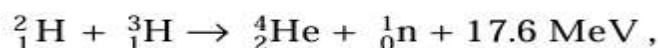
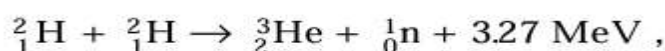
The second nuclear fission reaction involves the splitting of a plutonium-239 nucleus after it absorbs a slow (thermal) neutron. This absorption produces an unstable plutonium-240 nucleus, which quickly undergoes fission. The process yields zirconium-103 and xenon-134 as fission products, along with three fast neutrons that continue the reaction chain. To create a calculation for an atomic weapon, the initial step is to select a reactant charge, such as plutonium-239 or uranium-235, and a reaction equation. In an approximate sense, the Q-value of any nuclear reaction is equivalent to the difference in binding energies of the nuclei of the reactant and product. The heat released from nuclear reactions is commonly referred to as the q-value. Compute the requirement for the nuclear reactant charge based on the specified requirement for the explosion size, which is the energy released from any nuclear reaction. The nuclear reaction, momentum, and energy balance equations serve as the foundation for this

Step 1: Select a nuclear charge for the atomic bomb, such as plutonium-239, uranium-235, deuterium-D, or tritium-T, based on explosion power requirements for the client.

Step 2: Write the reaction equation of nuclear charge and balance the equation, then balance the energy and mass balance. Calculate the Q-value of the explosion reaction based on equation number (5) in this text from the calculation of the Q-value estimate of the number of moles required for getting the required explosion power for atomic fission bomb



for hydrogen bomb



The three primary components of a bomb are the explosive charge, initiators or detonators, and casing. An explosive charge is the fundamental component of any bomb design. Chemical composition, explosive charge concentration, and metallurgical structure Based on the nuclear reaction equation and reaction kinetics calculation, the explosive charge's structure and chemical composition are determined.

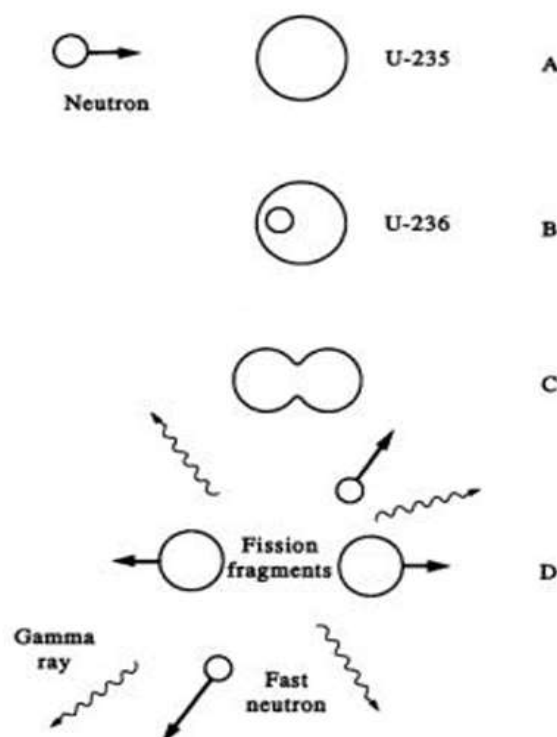


Figure 3.4

Two key considerations in the design of nuclear explosive devices are the cross-sectional area that a nucleus provides to the projectile in order for a reaction to take place.

Step 3: After the quantity of atoms required to generate the required energy for an explosion. The number of atoms necessary for uranium-235 to generate the necessary energy is determined by the energy requirements for explosions. Although there are various isotopes of uranium, this atom is pure uranium-235. To produce a chain fission reaction, a nuclear explosion charge requires knowledge of the nuclear reaction's cross section. This step is dedicated to the development of the nuclear fission reaction rate, the nuclear reaction cross sections, the geometry, and the chemical composition of a nuclear explosion charge. The number of fission events per unit of time is calculated by dividing the necessary explosion power by the power of a nuclear fission reaction. This quantity corresponds to the nuclear reaction rate for a nuclear explosion charge.

$$R = \Phi N \sigma \quad (3.6)$$

The rate of nuclear fission reactions, or R , is expressed in reactions per second. Φ is the neutron flux (neutrons/cm²-sec), N is the atom density. The atom density is the number of atoms of a given type per unit volume of the material.

$$N = \rho N_A / M \quad (3.7)$$

N_A Avogadro's number 6.022×10^{23} atoms/mole, M uranium 235 is gram atomic weight, ρ density, σ is the microscopic cross section barn, N . Atom density is contingent upon the chemical composition, the percentage of uranium 235 in charge, and the microstructure for charge. The microscopic cross section is determined with the assistance of the nuclear reaction handbook. This cross section is fluctuated in accordance with the neutron's energy. The microscopic cross section can also be interpreted as the effective area that the nucleus presents to the neutron for the specific reaction. The likelihood of a reaction increases as the effective area increases. This equation can be used to calculate the neutron flux for a detonation chain fission nuclear reaction.

$$\Phi = n v \quad (3.7)$$

Φ is the neutron flux (neutrons/cm²-sec), n is the neutron density and v is the neutron velocity. Cross-section in nuclear dimensions is expressed in terms of a smaller unit known as a barn. The macroscopic cross section Σ is the probability of a particular reaction occurring in relation to the neutron's unit travel. A bombarding particle is presented with an effective target area by a single nucleus, as illustrated by the microscopic cross section σ . The effective target area presented by each nucleus contained in a single unit volume of the material is represented by the macroscopic cross section Σ .

$$\Sigma = N \sigma \quad (3.8)$$

There are two possible ways for neutrons to interact with an atom. A scattering reaction or an absorption reaction. The absorption cross section is the microscopic probability of a neutron being absorbed by a specific atom. The microscopical cross section for scattering is the probability of a neutron being scattered by a specific atom. The sum of the microscopic cross section for absorption and the microscopic cross section for scattering is the total microscopic cross section.

$$\sigma_T = \sigma_s + \sigma_A \quad (3.9)$$

$$\sigma = \frac{\text{Number of products nuclei of one type per second per target nucleus}}{\text{Number of projectiles per unit area per second}}$$

Due to the products of the initial fission reaction event, the neutron intensity is elevated in the chain fission nuclear reaction.

$$I = I_0 e^{-N\sigma x} \quad (3.10)$$

Reaction	Projectile energy	Cross-section in barns
${}^6\text{Li} (n,\alpha){}^3\text{H}$	0.025 eV	940
${}^3\text{He}(n,p){}^3\text{H}$	0.025 eV	5327
${}^2\text{H}(n,\gamma){}^3\text{H}$	0.025 eV	0.0053
${}^{10}\text{B}(n,\gamma){}^{11}\text{B}$	0.025 eV	0.5
${}^{10}\text{B}(n,\alpha){}^7\text{Li}$	0.025 eV	3837
${}^{127}\text{I}(n,\gamma){}^{128}\text{I}$	0.025 eV	6.2
${}^{54}\text{Fe}(\alpha,p){}^{57}\text{Co}$	20 MeV	0.6
${}^{54}\text{Fe}(\alpha,pn){}^{56}\text{Co}$	35 MeV	0.65
${}^{51}\text{V}(\gamma,\alpha){}^{47}\text{Sc}$	18 MeV	0.0003
${}^{63}\text{Cu}(p,n){}^{63}\text{Zn}$	16 MeV	0.38
${}^{63}\text{Cu}(p,n){}^{63}\text{Zn}$	20 MeV	0.50
${}^{235}\text{U}(n,f)$	0.025 eV	583
${}^{235}\text{U}(n,f)$	10 keV	4

Table 3.1

$$P = ((\phi_{\text{th}} \Sigma_f V) / 3.12 \times 10^{10}) \quad (3.11)$$

where P The bomb explosion's power in watt, ϕ_{th} thermal neutron flux, Σ_f macroscopic cross-section for fission, V volume of bomb core.

The excitation triggers the separation of the two fragments, and the powerful electrostatic force provides them a large amount of kinetic energy. The mass of the nuclear products is less than the mass of the compound nucleus from which they originate due to the principle of mass-energy conservation. The fission reaction has a high cross section for slow neutrons, whereas the neutrons produced by fission are fast. Because of this fact, reflectors—which slow down and reduce neutron energy through a series of collisions to a fission level—were used in the internal casings of atomic bombs.

The usefulness of the fission process in nuclear explosive devices is demonstrated by figuring out how much uranium is needed to produce a given amount of energy. Fission is the process by which heavy elements' nuclei absorb neutrons, releasing fast neutrons, heavy fragments, and other radiations. Manmade isotope Pu-239 and natural U-235 are fissile materials for atomic bombs. The fission process in atomic bombs generates an excess of neutrons, which enables a chain reaction. A single fission can yield up to 190 MeV of energy under certain conditions. These are the two main types of nuclear bombs that have been used:

- The fission explosive ("atom bomb") utilized plutonium or highly enriched uranium.
- The hydrogen bomb, which is a fusion or thermonuclear explosive.

There are two main configurations known for atomic fission bombs; these configurations are gun type and implosion type. The gun system is illustrated in Figure. 5, which involves the firing of a plug of highly enriched uranium into a hollowed-out cylinder of uranium to generate a supercritical mass.. The materials are temporarily fixed together by a natural U-shaped "tamper." Little Boy was the moniker given to this atomic bomb.

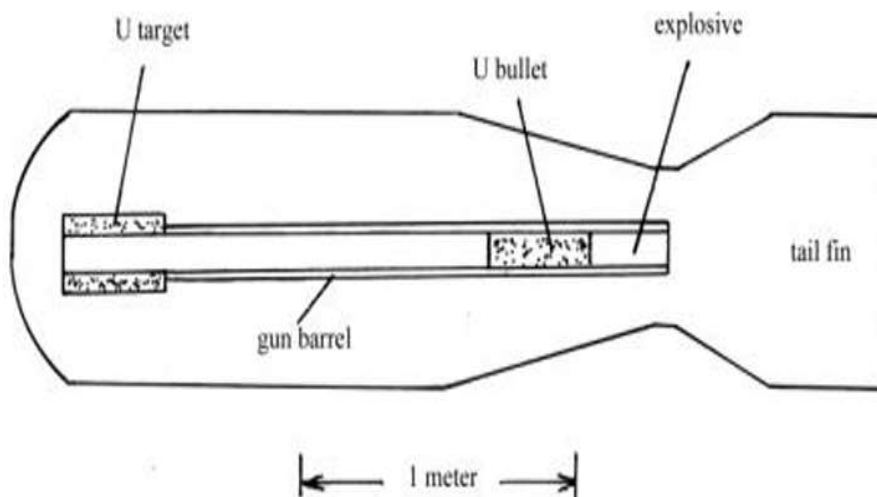


Figure 3.5

In Figure 6, high-pressure chemical explosives in the shape of lenses compress a plutonium metal sphere to supercriticality, demonstrating the implosion method. Additionally, an uranium tamper is implemented. In the implosion configuration, the fissile material is compressed, resulting in a higher ratio of surface to volume and a greater rate of neutron leakage. However, the mean free path is reduced, which mitigates the leakage. This latter effect is the most significant, resulting in a net positive increase in the multiplication factor. Fat man was the moniker given to this atomic bomb.

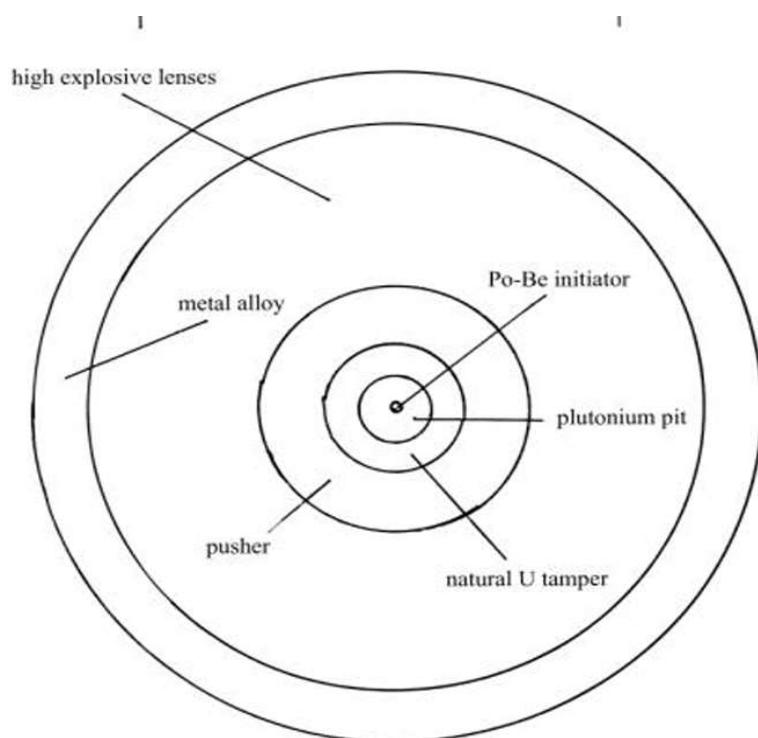


Figure 3.6

An initial supply of neutrons is required for any atomic fission bomb. One potential source is the use of polonium-beryllium as a neutron source. There are two criteria that must be met to design an atomic bomb that utilizes a chain reaction involving neutrons in a mass of nuclear charge, such as uranium or plutonium: the size, shape, and arrangement of the materials, as well as nuclear properties, such as cross sections and neutrons per absorption in Figure.7 The thermonuclear explosive device, hydrogen bomb, the Sausage configuration was a hollow steel cylinder 20 ft long and 6 ft 8 in. in diameter. Lead was employed to line the cavity. There was a primary at one end of the cavity. The fusion reaction between tritium and deuterium is caused by fission implosion with a sphere of plutonium and enriched uranium. It was a cylindrical container of liquid deuterium, resembling a big thermos bottle, in the center of the cavity. The secondary fission source was the sparkplug, which was a stick of Pu positioned along its axis. There was a U-shaped natural pusher in the tritium and deuterium container. Lastly, polyethylene was used to line the interior of the casing. This was the order of events. The high-explosive shell of the primary was detonated by an electrical discharge to detonators. Neutrons were released when a Po-Be source inside was triggered by a uranium tamper and shell, which also compressed and vaporized the central plutonium ball. The polyethylene was heated by X-rays from the ensuing supercritical fireball, creating a plasma that heated the U pusher by reradiating X-rays. Deuterium fission occurred in the sparkplug, resulting in the release of neutrons and energetic alpha particles. Fast neutron fission in the uranium resulted in the production of additional energy and radiation 238 in the tamper. A crater that was 200 feet deep was formed as a result of the explosion one mile in both directions.

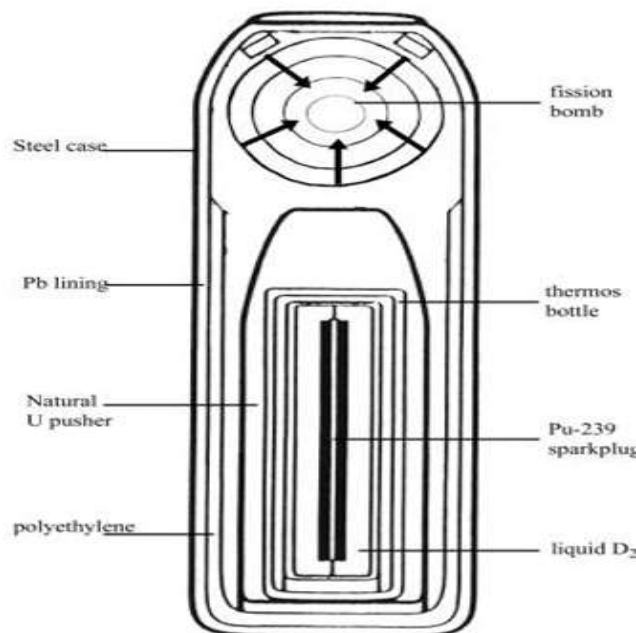


Figure 3.7

From an atomic bomb, the deuterium and tritium nuclear fusion reaction (D-T reaction) generates more nuclear energy. Tritium was the primary explosive material employed in hydrogen bombs and thermonuclear devices. Nuclear fusion explosives generate energy in various ways. A blast effect-generated shock wave is released into the air, water, or rock during the first stage of a nuclear fusion reaction. The second is thermal radiation, which is typically around 6000°C from the hot material surrounding it. Finally, the nuclear radiation is primarily composed of neutrons and γ rays. There are 50, 30, and 10% of the energy that goes into each of these stages. The energy released due to a nuclear fusion reaction or nuclear fission is called the energy yield of weapons. Similar tons of chemical explosives are used to measure this amount of energy. Typically, one ton of TNT is equivalent to 109 calories of energy. 20,000 tons of force were generated by the initial atom bomb. During the Gnome test, a smaller device containing three kilotons was detonated underground. There was a huge hollow made.

Table.3.1

Distance-Yield Relation for Nuclear Explosion	
Yield (tons)	Radius (meters)
1	120
2	450
10,000	1050
1,000,000	2000

Critical Mass

Critical mass refers to the minimum quantity of fissile material necessary for a prolonged nuclear chain reaction in a particular configuration. Density, shape, enrichment, purity, temperature, environment, and nuclear properties (i.e., nuclear fission cross-section) all influence the critical mass of a fissionable material. The critical mass calculation step comes after the final step of figuring out how many moles are needed to produce the bomb's explosive power. Reducing neutron leakage from the bomb core depends on the volume-to area ratio. For the bomb core explosive charge, the sphere is the ideal shape. The requirements for a chain fission reaction that can sustain itself. In one act of fission, the neutrons must trigger another act of fission. The percentage of neutrons that escape from a system where fission is taking place can be calculated. It is clear that the escape probability will decrease as the system's volume-to-area ratio increases because neutrons are produced by fission throughout the entire volume, while loss by escape occurs only from the exterior surface. This can be achieved by increasing the mass, that is, by increasing the size at a constant density of fissile material. By increasing the dimensions of the fissile material while maintaining a constant density, this can be accomplished for a specific geometry. Uranium has three types of crystals. The first crystal is face-centered orthorhombic, the second crystal is body-centered tetragonal (bct), and the third crystal is body-centered cubic. Under high temperatures, uranium changes from the first type to the second type at about 935 K and from the second type to the third type at about 1045 K. Uranium's crystal microstructure is crucial to nuclear fission reactions, particularly when it comes to raising the fission cross section. This serves as a scientific foundation for the compression of critical mass and the reduction of the mean free path for neutrons by increasing the atomic density of the crystal. Since I is the number of neutrons lost through escape and other processes, like nonfission capture, and v is the average number

where Σ_a is the macroscopic absorption cross-section, Σ_f macroscopic fission cross-section, D is the diffusion coefficient, v is the neutron multiplicity, v is the speed of mono-energetic neutrons, ϕ is the neutron flux distribution, and β is the delayed neutron fraction. The concept of critical mass is qualitatively derived from the observation that specific nuclei undergo fission when exposed to a bombarding neutron, resulting in the release of secondary neutrons that may trigger further fissions and a chain reaction. The criticality concept is linked to the fact that the number of free neutrons in a bomb core increases over time. A thorough understanding of criticality requires knowledge of time-dependent diffusion theory and the mean free path for neutron travel. Atomic weapons are designed to enclose the fissile in a tamper. A heavy metal shell known as a tamper increases the effectiveness of a weapon in two techniques: by reflecting neutrons back into the fissile core to trigger fissions, and additionally, the core's violent expansion is postponed to allow the chain reaction to operate more effectively. The rate at which incoming particles bombard the fissile core's surface area multiplied by the percentage of surface area available for reactions is known as the rate of reactions, or R (reactions/s). The probability of an incident neutron initiating a reaction is denoted as P_{react} .

$$P_{\text{react}} = (\text{reactions/second}) / (\text{incident neutron flux per second}) \quad (3.16)$$

The likelihood that a neutron will escape P_{escape} after passing through the fissile core.

$$P_{\text{escape}} = 1 - P_{\text{react}} \quad (3.17)$$

According to the theory of critical mass, some neutrons cause fission while the rest escape.

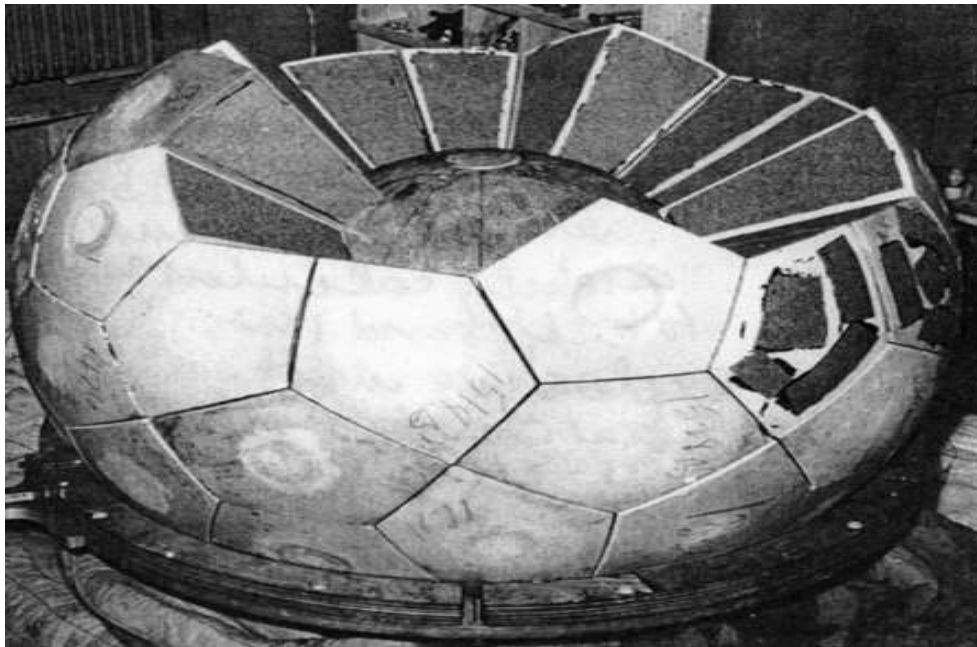


Figure 3.9

Critical mass refers to the minimum mass of fissile material required for a sustained nuclear chain reaction in a specific configuration. Nuclear properties (i.e., nuclear fission cross-section), density, shape, enrichment, purity, temperature, and environment all affect a fissionable material's critical mass. It is an indispensable parameter of a nuclear weapon. In a specific configuration, the critical size is the minimum size of the fissile material required for a sustained nuclear chain reaction. The chain reaction is not sufficiently maintained when the bomb core's size is less than a specific minimum, as an excessive number of fission neutrons escape through its surface. The minimum critical size is generated by a perfect sphere, which has the lowest surface-area-to-volume ratio.

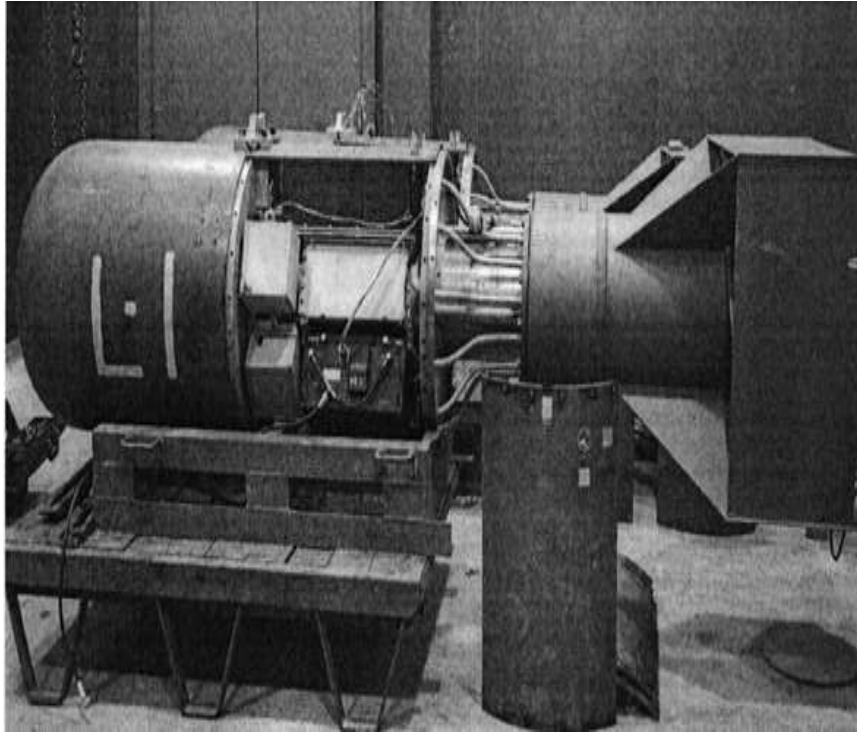


Figure 3.10

The effective neutron multiplication factor k , is the average number of neutrons released per fission event that cause another fission event instead of being absorbed or escaping the core. It is used to calculate a critical mass numerically. A mass that is unable to support a fission chain reaction is known as a subcritical mass. When a subcritical assembly is exposed to a population of neutrons, the population will rapidly decline. This is referred to as subcriticality, where k is less than 1. A critical mass is a quantity of fissile material that is capable of self-sustaining a fission chain reaction. Known as criticality, $k = 1$ in this instance. Neutron activity is proportionately steady when spontaneous fission occurs at a constant rate. If fission starts in a supercritical mass, the rate of fission will go up. This is referred to as supercriticality, where $k > 1$. The proportionality constant increases as k increases. At elevated temperatures or power levels, the substance may either self-destruct or achieve equilibrium. In pure uranium-235 (^{235}U), 15 spontaneous fission events per second are produced, with a spherical critical mass of about 52 kilograms.

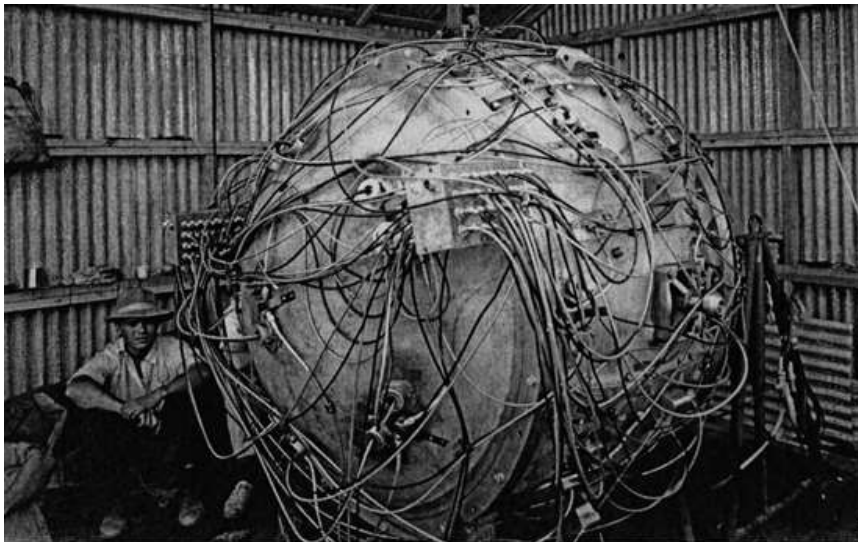


Figure 3.11

Diffusion theory elucidates the neutron bombardment of materials, which results in the fission of certain nuclei, the release of secondary neutrons, and the subsequent induction of a second fission generation chain reaction. This concept anticipates that a specific number of neutrons will escape from the mass and reach the surface. Since the number of neutrons in the mass is growing over time, the criticality state has been reached. The number of neutrons in a bomb core is time-dependent, as determined by diffusion theory-based calculations. The number density of neutrons N within a volume as a function of time and space within the volume.

$$\frac{dN}{dt} = (\text{neutron density gain from fission per unit volume}) + (\text{rate of neutron density gain or loss by})$$

transport through volume boundary per unit volume)

The criticality situation arises when the product of the probability of prompt neutron generation fission and the number of prompt neutrons generated in a nuclear fission is greater than or equal to unity.

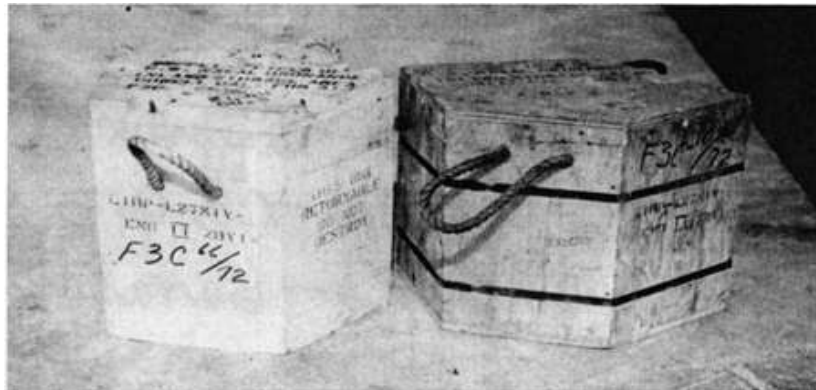


Figure 3.12

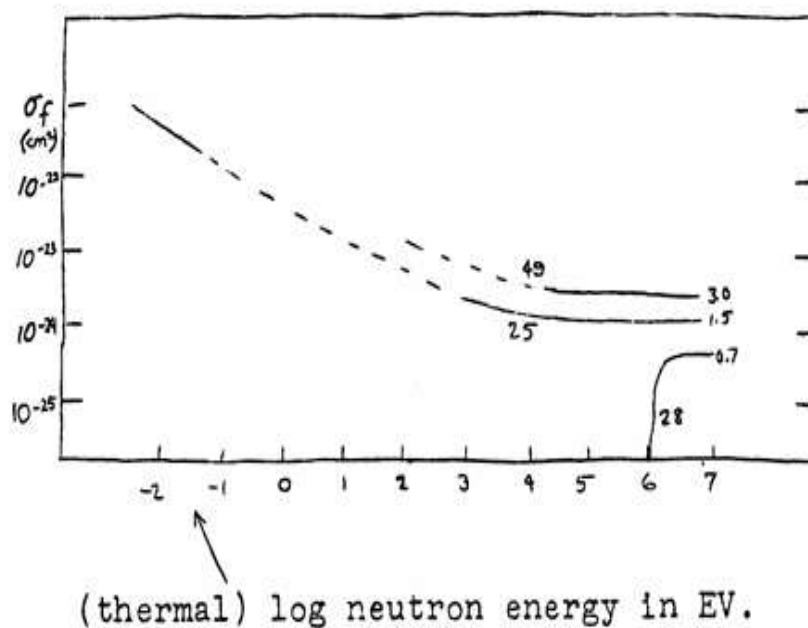


Figure 3.13

The accurate calculation of the critical mass of uranium or plutonium required for a nuclear fission weapon has been widely recognized and accessible to the public for an extended period. To perform the calculation, the radial component of a diffusion equation with a source term must be solved in spherical coordinates. Various methods are employed to determine the critical mass and critical sizes of this mass. Equating the neutron production rate with the neutron loss rate is the foundation of the first calculation method. This establishes the critical radius for the nuclear charge in the presence of highly enriched uranium or plutonium-239. As the critical radius increases, a chain reaction is maintained to generate the necessary energy for an atomic bomb. The neutron balance in the nuclear charge of this atomic bomb is used to determine the critical mass and critical size. LN is the neutron loss rate, ϑ typical velocity of a neutron produced by a nuclear fission, λ_f neutron mean free path between fission events, PN is the neutron loss rate, n is the number density of fissile nuclei, N is the free neutron number density of the sphere, v In each fission an average number of neutrons is produced, and T The nuclear generation lifetime s .

$$PN = N((4/3)\pi R^3)((v-1)/T) \quad (3.18)$$

$$T = \lambda_f / \vartheta \quad (3.19)$$

$$\lambda_f = 1/n\sigma_f \quad (3.20)$$

$$LN = \vartheta N(4\pi R^2) \quad (3.21)$$

By developing an expression for calculating the critical radius of the sphere of nuclear charge and equating LN and PN. where a is the ratio of N in the innermost sphere to that in the outer spherical layer $(\eta = \lambda_f / \lambda_{\text{scattering}})$.

$$R_c = \frac{1}{3\eta} \frac{(2\alpha(\nu-1)+1)^{1/3}}{(2\alpha(\nu-1)+1)^{1/3}-1} \lambda_f \quad (3.22)$$

This formula overestimates R_c by factors of 1.1 and 1.5 and HEU and Pu by factors of 1.3 and 3.2 in M_c , respectively. The formula is an easy tool for experts in the field of nuclear disarmament.

Simple statistical reasoning and basic calculus are the only tools used in the second technique created to calculate the critical mass of a fissile material sphere. This method uses only basic calculus and simple statistical arguments to formulate the problem in terms of the threshold condition that the number of neutrons in the sphere does not change with time. The material must have an average neutron path length that is sufficient to produce an adequate quantity of fission neutrons to compensate for losses caused by absorption from nuclear reactions and surface leakage. The total path length, with the distance between scatterings, is the sole connection between the nuclear reaction component of the problem and the geometry and mechanics of neutron transport, thereby determining the sphere radius. The solution of the diffusion equation is not necessary to obtain a critical radius expression. Compared to recognized critical masses, the agreement is negligible. Furthermore, impure materials may be subjected to analysis, whether isotopically or otherwise, and can be used as a design guide for general neutronics estimations or for order-of-magnitude checking of Monte Carlo N-Particle (MCNP) simulations.

Parameter	²³⁵ U	²³⁸ U	²³⁹ Pu	Units
ν	2.57	2.28	3.156	cm ⁻³
σ_e	3.557	4.827	3.33	barns
σ_i	1.926	2.541	1.85	"
$\sigma_{tr} = \sigma_e + \frac{1}{2}\sigma_i$	4.52	6.10	4.25	"
σ_a	0.082	0.07	0.0172	"
σ_f	1.24	0.308	1.96	"
$\sigma_0 = \sigma_a + \sigma_f$	1.322	0.378	1.98	"
ρ	18.9	18.9	19.61	g cm ⁻³
R_c	8.37	—	4.96	cm
m_c (Eq. 11)	46.5	—	10.00	kg
m_c (MCNP6)	46.4 ± 1.7	—	10.2	kg

Table 3.2

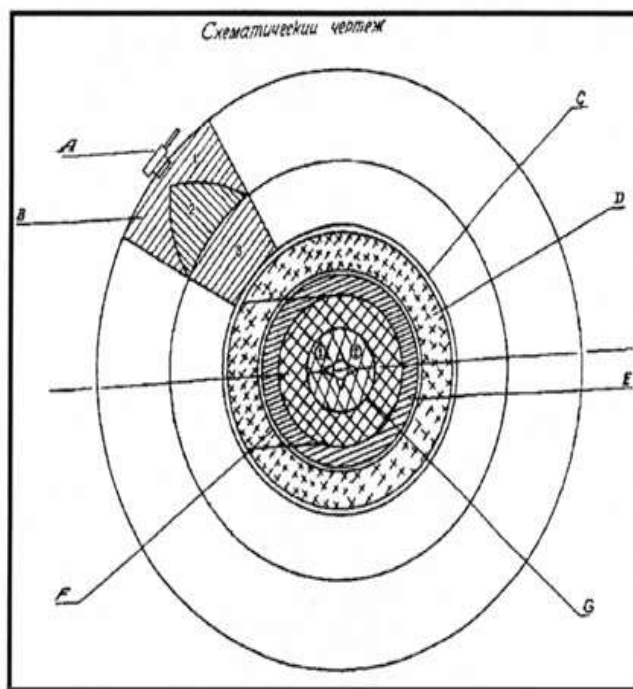
Enrichment %	$\sigma_f = .302$ m_c kg	$\sigma_f = .1$ m_c kg	Ref.[11] m_c kg
100	46.5	46.6	NV
93	51.2	52.3	53
70	75.2	85.2	87
45	139.5	204.1	185
30	250.5	669.1	367
27.02	293.6	3258	NV
19.75	502.8	x	782
14.6	3106	x	x
14.5	x	x	x

Table 3.3

$$R_C = \frac{\epsilon}{n \sigma_{tr} \sigma_0} \left[-\ln \left(1 - \frac{\sigma_0}{\sigma_f V - \sigma_0} \right) \right]^{1/2} \dots\dots\dots(3.23)$$

$\epsilon=0.89$

In the derivation of the previous formula, there are several simplifications. The initial simplification involves the assumption that the essential reaction parameters, cross sections, and the number of neutrons liberated per fission are energy-independent at approximately 1 MeV. Consequently, we can utilize values that are averaged across energy spectra. The uniformity of the neutron density within the material and the existence of a position-independent root mean square average distance—the distance a neutron must travel to leave the sphere, which is dictated by the sphere radius and scattering (diffusive) processes—are further approximations. According to the neutron conservation law, the critical radius is computed, and the mean free path of a neutron concept is used to derive this formula at the critical radius. A constant number of neutrons that is neither rising nor falling is the critical state for any nuclear reaction involving fissile materials. Since the neutron number density stays constant, it is clear that the rate of neutron production in a sphere of fissile materials is equal to the rate of neutron loss. The term “critical mass” refers to this sphere of fissile material. A nuclear chain reaction in a mass of fissile material is considered to be in a critical state when it is self-sustaining but not expanding, meaning that neither the temperature nor the power nor the number of neutrons increases or decreases. The accuracy of a critical mass is contingent upon the effective neutron multiplication factor k , which is the average number of neutrons released per fission event that continue to cause another fission event rather than being absorbed or leaving the material. The probability of neutrons evading a nucleus is the probability that they will not collide with it. The likelihood of the particles escaping the mass without colliding is greatly decreased because they bounce. Elastic scattering is therefore related to escape probability. The probability of escape is an important physical property in fission reaction.



Implosion sphere drawing purportedly made by Klaus Fuchs found in KGB files after the end of the Cold War. This drawing is not to scale. A) Detonator, B) Explosive lens and Inner Charge, C) Felt Layer, D) Aluminum Pusher, E) Uranium Tamper, F) Boron Plastic Shell, G) Plutonium with Urchin. (Author)

Figure 3.14

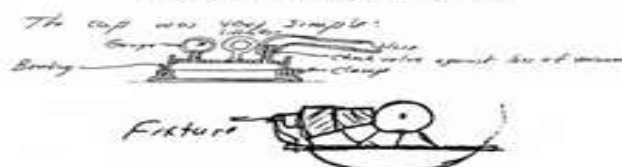
The criticality of fissile materials depends on several factors, such as shape, temperature, a temper to preserve shape, and use as a neutron reflector. On the other hand, altering the shape to a less perfect sphere will result in a subcritical state and a decrease in reactivity. A neutron reflector placed around a spherical critical mass lowers the mass required for criticality even more. Using a neutron reflector to raise the probability of a fission event also boosts the reaction's overall fuel efficiency. The reaction necessitates a lower critical mass as fuel efficiency increases. Heavy metals like beryllium, graphite, and tungsten carbide are frequently used as reflectors; the kind of material used is determined by the particular bomb design and the neutron velocity. High “reflectivity” and low neutron absorption are desirable properties for the material used in a neutron reflector. Cross sections of scattering and absorption can be used to characterize these.

Nuclide	Total cross section [barn]	Absorption cross section [barn]
Hydrogen	82.03667	0.33268
Deuterium	7.64333	0.00052
Beryllium	7.63222	0.00769
Graphite	5.55933	0.00354
Oxygen	4.23267	0.00019
Chromium	3.49222	3.05889
Iron	11.62101	2.56333
Nickel	18.53333	4.49162
Lead	11.11878	0.17122

Table 3.4



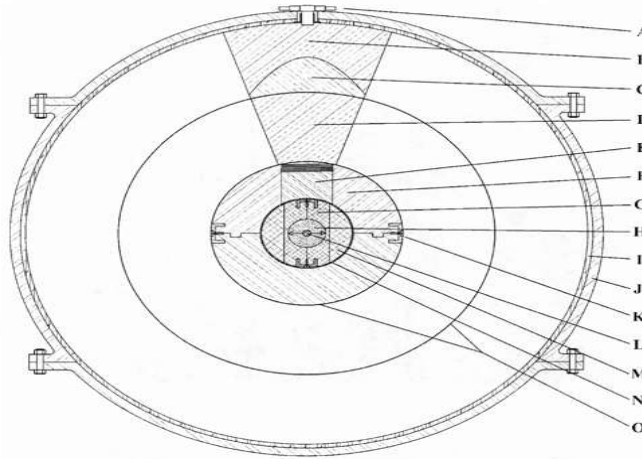
Same Y-1222 implosion design with six of the 12 large hexagonal Dural external sphere plates hoisted in place over the HE blocks. The 1/2" cork lining is visible on the inner surface of the Dural sphere along with the tape used to hold the felt covering in place over the 24.5" aluminum pusher-sphere. Since this was an early test unit, no detonator holes were drilled into the Dural sphere. The same 72-block non-leak design with the 24.3" pit was also used in the later Y-1291 model, which ultimately utilized the Y-1561 Dural case. (LANL/Courtesy of James Kuratka)



These exclusive drawings by Manhattan Project scientist Richard Bice shows both the vacuum cap used to move the HE blocks along with one of the fixtures, attached to the lower polar cap, that helped secure the HE blocks during the later Y-1561 sphere assembly process. (Courtesy of Steve Bice)

Figure 3.16

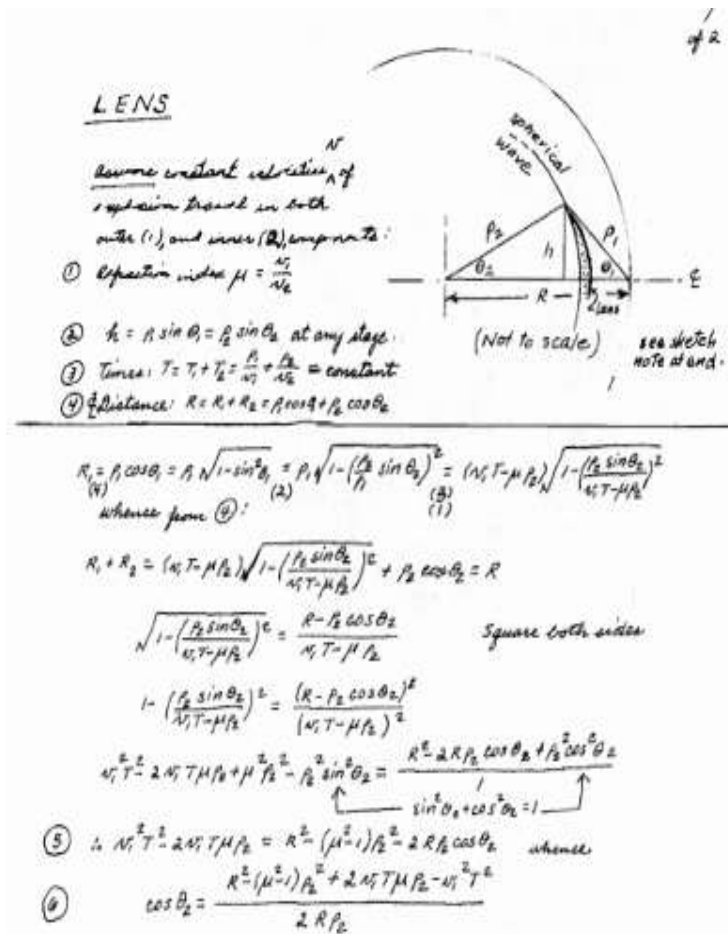
The scattering cross section, denoted as σ_s , denotes the probability of neutron scattering, whereas the absorption cross section, denoted as σ_a , denotes the probability of neutron absorption. A probability that can be represented as the sum of the scattering and absorption cross sections is the total cross section. The scattering cross section is defined. Neutron scattering is clearly more likely to occur when the scattering cross section is higher. To reduce the critical mass and delay the expansion of the reacting material by utilizing its inertia, the fissile material is enclosed by a dense layer of material. This prolonged delay in thermal expansion maintains the supercritical state of the fissioning fissile mass. The same layer often serves as a neutron reflector and a tamper. Critical mass is inversely proportional to density: as density increases, critical mass decreases. Varying the pressure or tension or altering the crystal structure can alter the density of a material at a constant temperature. Subcriticality will occur when an ideal mass is permitted to expand. In contrast, the same mass will undergo supercritical expansion when compressed. The impact on critical mass is further complicated by temperature effects and the manner in which the material expands or contracts as the temperature increases. Additionally, a change in temperature may affect density. The absorption and fission cross-sections increase as the relative neutron velocity decreases, which is influenced by temperature effects on fission reactions. As the bomb charge temperature rises, the probability of fission/absorption falls because neutrons with a given energy appear more quickly.



Cross-section drawing of the Y-1561 implosion sphere showing component placement. For simplicity, only one set of the 32 lenses, inner charges, and detonators is depicted. Numbers in () indicate quantity of identical components. Drawing is shown to scale. (Author)

- A) 1773 EBW detonator inserted into brass chimney sleeve (32)
- B) Comp B component of outer polygonal lens (32)
- C) Cone-shaped Baratol component of outer polygonal lens (32)
- D) Comp B inner polygonal charge (32)
- E) Removable aluminum pusher trap-door plug screwed into upper pusher hemisphere
- F) 18.5" diameter Aluminum pusher hemispheres (2)
- G) 5" diameter Tuballoy two-piece tamper plug
- H) 3.62" diameter Pu-239 hemispheres with 2.75" I.D. jet ring
- I) 0.5" thick Cork lining
- J) 7-piece Y-1561 Duralumin sphere
- K) Aluminum cup holding pusher hemispheres together (4)
- L) 0.8" diameter Polonium-Beryllium Urchin initiator
- M) 8.75" diameter Tuballoy tamper sphere
- N) 9.0" diameter Boron plastic shell
- O) Felt padding layer under lenses and inner charges

Figure 3.15



in which T can be found from initial $\frac{R}{v_1}$ dimensions, and the line curve plotted for selected values of P_2 . These points can be converted, if desired, to x, y coordinates as usual by $x = P_2 \cos \theta_2$ and $y = P_2 \sin \theta_2$.

Figure 3.16

If, instead, it is preferred to calculate ρ_2 as a function of θ_2 , (5) yields

$$(7) \quad (\mu^2 - 1)\rho_2^2 + 2(R \cos \theta_2 - \mu T)\rho_2 + \mu^2 T^2 - R^2 = 0$$

which can be solved by quadratic formula for ρ_2 .

Also from (5) or (7), by differentiating:

$$(8) \quad \frac{d\rho_2}{d\theta_2} = \frac{R \sin \theta_2 \rho_2}{(\mu^2 - 1)\rho_2 + R \cos \theta_2 - \mu T}$$

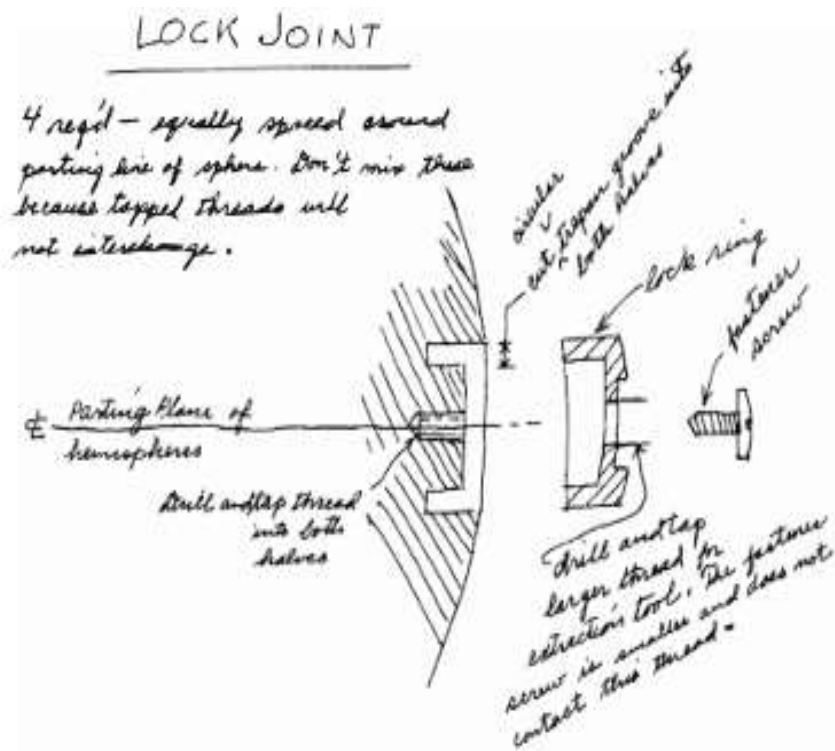
Note: $\frac{d\rho_2}{d\theta_2} \rightarrow 0$ as $\theta_2 \rightarrow 0$, so the curve begins at the $\hat{e}_z$

with a central circular arc of radius ρ_2 .

Sketch Note: As the explosion spreads out along the lens surface, the wave in the inner slow component is spherical all the way back to the $\hat{e}_z$. When it reaches the casting boundary, the entire wave is spherically formed. It will then continue to shrink spherically all the way to the center of the sphere, no matter what the rest of the space to the center is filled with. The only part of the casting (in principle) that needs the slow component is the dotted volume between this sphere and the lens surface. The rest of the casting inward can be H.E., or any other homogeneous material.

These exclusive equations by Manhattan Project engineer Leonard Della-Moretta show the principles involved in calculating the proper lens curve for the Baratol portion of the explosive lens used in the implosion device. Although not the actual equations used to create the lenses, these do illustrate the general principles involved. (Author)

Figure 3.17

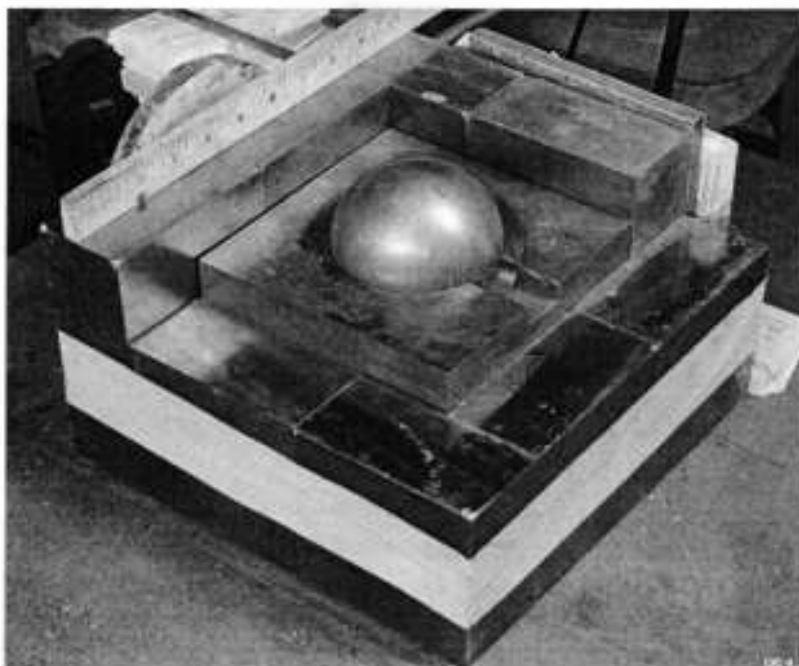


This exclusive sketch from Manhattan Project engineer Leonard Della-Moretta shows how the lock joint cups worked on the aluminum pusher hemispheres. The extraction tool mentioned was used in case the lock ring had to be removed so the pusher could be disassembled. The same principle was applied to the lock joint cups (trepanned rings) used to hold the two halves of the tamper plug together. (Author)



This is artist Jim Sanborn's interpretation of what the nickel and gold plated polonium-beryllium Urchin initiator might have looked like based on information found in the KGB files in the early 1990's. Los Alamos sources have told the author the interior was ribbed similar to what is shown here. (Author)

Figure 3.18

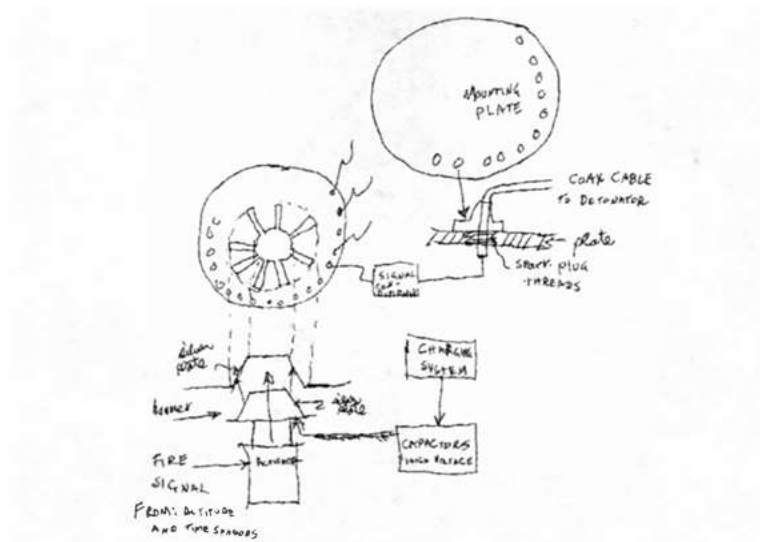


Daghlian accident reconstruction assembly showing Pu core surrounded by WC blocks. (LANL)

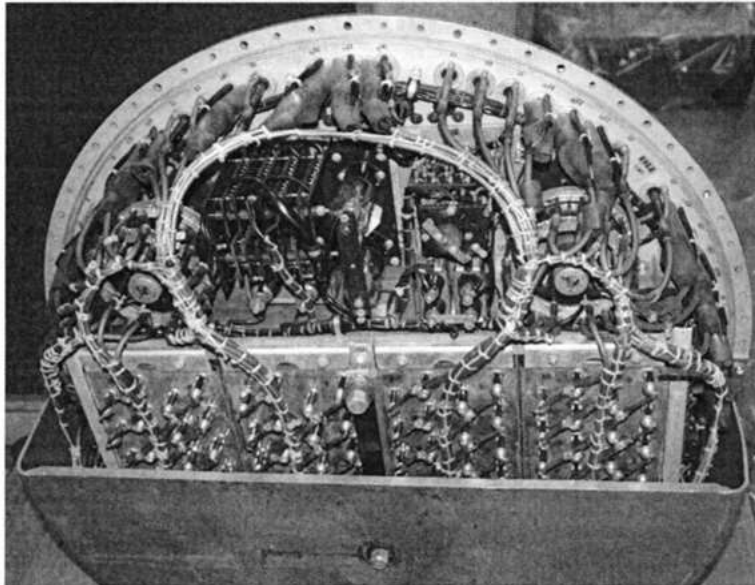


Slotin accident reconstruction assembly showing beryllium hemispheres. Note use of the screwdriver to hold the beryllium hemispheres apart and the two Pu hemispheres sitting atop the lead blocks. (LANL)

Figure 3.19



This exclusive sketch of part of the interior layout of the X-unit by Firing Team member Larry DeCuir shows component placement and the use of modified aircraft sparkplugs to transmit the firing signal from the capacitors to the EBW detonators. Sparkplugs were used because the insulation could withstand the high voltage firing signal and the o-ring at the base provided the requisite pressure seal. (Author)



This exclusive photo shows a cutaway of the X-unit interior. The banks of capacitors can be seen at the bottom with two (of four) circular spark-gap switches just above them with their leads going to the aircraft sparkplugs spread around the rim that, in turn, were connected to each of the 32 EBW detonators. (Glen McDuff)

Figure 3.20



This very rare photo shows Raemer Schreiber steadying the 5" diameter Pu-filled uranium "capsule" as it is about to be loaded into the F-31 *Fat Man* Dural sphere inside Tinian Assembly Bldg. #3 in 1945. The top plate on this F-31 sphere is the same style as the one used on the Trinity sphere shown on the previous page. This top had a metal funnel built into it to protect the surrounding explosive blocks while it was being loaded into the 8.75" diameter Uranium tamper sphere down in the very center of that sphere. Schreiber was present a few weeks earlier in the McDonald Ranch House to observe the assembly of that same style capsule for the Trinity device. In an extensive 1996 interview, he gave the author a complete description of exactly how that capsule was assembled. (Author)

Figure 3.21

The criticality state for fissionable material is defined as the state in which the neutron production rate equals the neutron loss rate. A multiplication factor is the ratio of the number of neutrons in one generation to the number of neutrons in the previous generation. A generation is essentially the duration of a neutron. The symbol for the multiplication factor is k effective, or k_{eff} . A steady-state chain reaction of nuclear fissioning is maintained by a critical system, with $k_{eff} = 1$. Designing atomic bombs and managing the waste from nuclear explosions require an understanding of k_{eff} . More accurately, criticality calculations for fission reactions for fissionable materials have found a significant application in the design of atomic bombs. The preferred technique for examining the criticality state of atomic bombs and nuclear explosions is Monte Carlo methods. MCNP from Los Alamos National Laboratory, KENO from Oak Ridge National Laboratory, and MONK from AEA Technology are the three production Monte Carlo computer codes that are most frequently used in the United States. VIM is implemented extensively at Argonne National Laboratory. A novel approach, the Fission Matrix, has been integrated into the Acceleration Method of MCNP. Fission source convergence for large or weakly coupled problems using Monte Carlo criticality calculations. It computes a low-order fundamental eigenvector to speed up fission source convergence by counting the spatially discretized fission kernel (the linear operator stochastically applied during power iteration). Utilizing a sparse structure, the fission matrix is efficiently stored in MCNP, necessitating minimal additional effort for tallying. Furthermore, higher eigenvalues and eigenmodes from the fission matrix are computed, which are not possible with traditional Monte Carlo. The main steps for Monte Carlo K calculation are step one: begin by estimating the initial fission source and eigenvalue. Second step: Preserve fission sites for the subsequent iteration; simulate neutron histories. Third step: Determine the eigenvalue by comparing the size of the new source to that of the old source. Fourth step: Repeat steps 13 until convergence is achieved. Fifth step:

Proceed with iteration and calculate the quantities of interest. The fission matrix is primarily associated with Monte Carlo criticality calculations. Nevertheless, it is also employed in the calculation of deterministic criticality. The probability that a fission source neutron born in region j of the system will contribute to the subsequent birth of a fission source neutron in region i is expressed by the $(i;j)$ th element of the fission matrix. Calculating the fission matrix from a series of deterministic calculations may be a somewhat tedious endeavor. Moreover, the fission matrix can be estimated in a Monte Carlo calculation by monitoring the Monte Carlo simulated neutrons. A fission matrix is composed of $(i;j)$ elements, each of which is the sum of the number of neutrons produced in region i as a result of neutrons starting in region j and the number of neutrons starting in region j . Ratios (and probabilities) that comprise the fission matrix elements may be more precise than the Monte Carlo fission source. The values of the numerators and denominators in the fission matrix elements associated with region j will be less than their converged values. These errors are more indicative of the converged solution and tend to cancel out in the fission matrix elements. Therefore, the eigenvector of the fission matrix converges at a faster rate than the Monte Carlo fission source. The spatial discretization of the fission matrix eigenvector means that it does not exactly converge to the spatially continuous Monte Carlo fission source. The eigenvector demonstrates a second-order spatial truncation error. The neutron transport equation is a neutron balance equation that is applicable to both fixed-source and criticality calculations. In the Los Alamos National Laboratory, Monte Carlo methods have been developed to calculate the fundamental mode eigenfunction and k_{eff} of critical systems for little boy atomic bomb and fat man atomic bomb. The majority of Monte Carlo codes utilize the standard power method to resolve k -eigenvalue issues. Each (outer) iteration cycle in this method corresponds to a single fission generation in the simulation. To estimate a new k_{eff} and source distribution, single-generation random walks are conducted for a "batch" of neutrons, with a fission neutron source distribution and an estimate of k_{eff} . Until the source distribution and k_{eff} have converged, iterations will continue. The third method of calculating criticality involves the application of the mean free path concept to neutron travel with time-dependent diffusion theory.

$$\left(\frac{\text{reactions per}}{\text{second}} \right) = \left(\frac{\text{incident neutron}}{\text{flux per second}} \right) \left(\frac{\text{fraction of surface area}}{\text{occupied by cross - section}} \right) \quad (3.23)$$

Diffusion theory generates the subsequent differential equation for the t time rate of change of N the neutron number density in a spherical bomb core,

$$\frac{\partial N}{\partial t} = \frac{v_{neut}}{\lambda_f} (\nu - 1)N + \frac{\lambda_t v_{neut}}{3} (\nabla^2 N) \quad (2.24)$$

v_{neut} is the average neutron velocity.
 ν is number of neutrons generation per fission event.
 λ_f is neutron mean free path for fission event.
 λ_t is total mean free path of neutron.

The neutron number density must increase to criticality in both space and time to produce a chain fission reaction. The full dimension for the bomb core and tamper design is the solution to this differential equation. The equation is resolved by utilizing a separation variable function.

$$N(t, r) = N_t(t) N_r(r) \quad (2.25)$$

$$\frac{1}{N_t} \left(\frac{\partial N_t}{\partial t} \right) = \left(\frac{\nu - 1}{\tau} \right) + \frac{D}{N_r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial N_r}{\partial r} \right) \right] \quad (2.26)$$

$$D = \frac{\lambda_t v_{neut}}{3}, \tau = \frac{\lambda_f}{v_{neut}} \quad (2.27)$$

$$N_t(t) = N_0 e^{(\alpha/\tau)t} \quad (2.28)$$

$$d = \sqrt{\frac{\lambda_f \lambda_t}{3(-\alpha + \nu - 1)}}, \quad x = \frac{r}{d} \quad (2.29)$$

$$N_r = \frac{\sin x}{x} \quad (2.30)$$

To reduce the loss of neutron flux in the bomb charge, a tamper is employed. The device is constructed using reflective materials to enhance the rate of fission reactions, decrease the charge critical mass within the bomb charge, and retrieve escaped neutrons. This accomplishes two objectives: (1) it decreases the critical mass, and (2) it delays the core's inevitable expansion, thereby providing additional time for fissions to occur until the core density reaches a level at which criticality is no longer maintained.

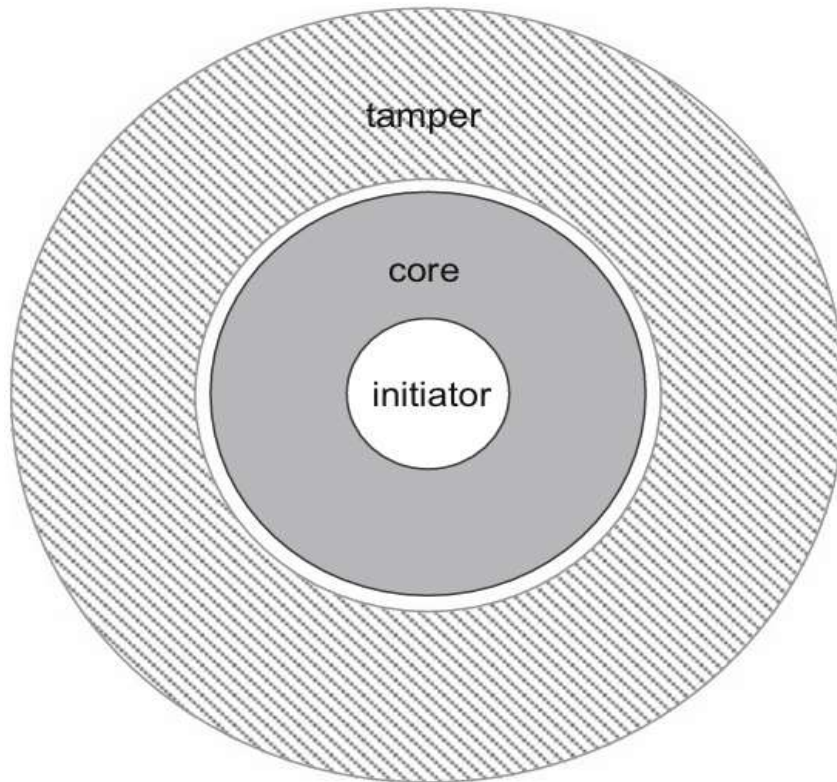
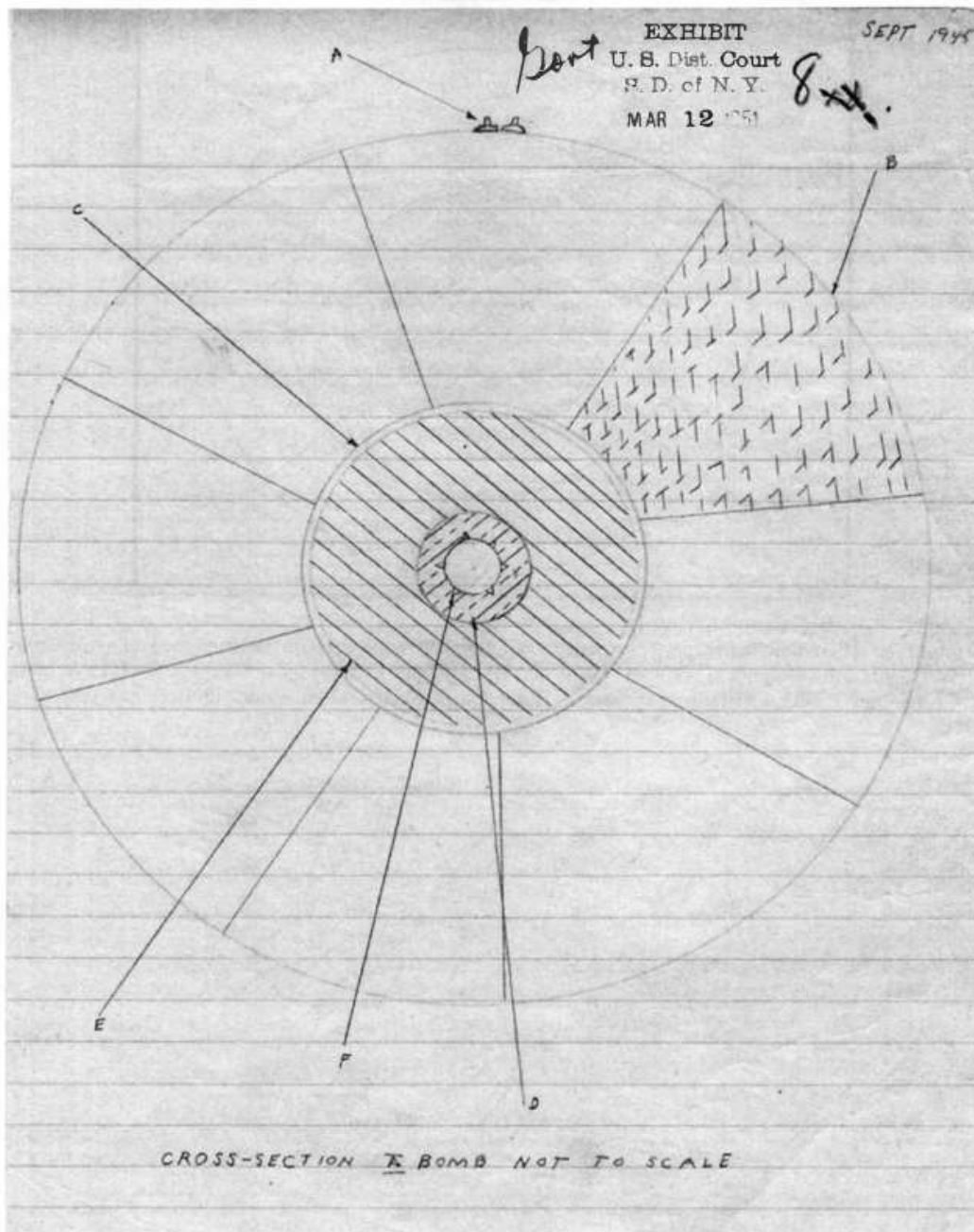


Figure 3.22

The charge of the fissile material in nuclear weapons is encased in a layer of dense, neutron-reflective material, which serves as a distinctive instrument known as a tamper. Its principal objectives are to delay the expansion of the reacting material through inertia and reduce critical mass, thereby facilitating a more efficient explosion. An additional increase in yield may be achieved by employing a fissionable tamper.



Implosion sphere cross-section used in 1951 Rosenberg spy case. (National Archives/Courtesy of Alex Wellerstein)

Figure 3.23



The 72-block Y-1222 implosion design showing blotter paper-faced solid Comp B non-lens HE blocks and 24.5" diameter felt-covered aluminum pusher sphere. Note the use of both regular and irregular pentagonal shaped charges in this design. No detonator holes were used in this test unit. (Courtesy of James Lyons, Sr.)

Figure 3.24

Conclusion

Starting with first-principles calculations, this paper will provide a thorough overview of the procedures required to design different types of nuclear weapons, such as nuclear fission and nuclear fusion bombs. Detailed logarithmic instructions for nuclear fusion and fission reactions that are based on calculations for mass and energy balance. This covers nuclear weapons calculation techniques for neutron flux, nuclear reaction rate, and microscopic and macroscopic cross sections of nuclear reactions. Client requests dictate this logarithmic approach, which outlines the requirements for nuclear weapon yield and explosion power. Any successful bomb design needs to react quickly, in less than a second. In conclusion, this paper offers a comprehensive examination of the various nuclear weapons' designs. Following the development of atomic fission bombs, intensification atomic bombs, hydrogen bombs, and cold fusion bombs, the process has advanced. The primary goal of this paper is to provide a thorough understanding of the procedures involved in the development of atomic bombs and to make it more accessible to a broader audience of students and professionals who aspire to be involved in international security, arms control, or nuclear non-proliferation.

Acknowledgements

I am grateful for the unlimited support and assistance provided by the United Nations Office for Disarmament Affairs (ODA), IAEA International Atomic Energy Agency, ANS, DOE, JAEA (Japan Atomic Energy Association), and NARI (Taiwan National Atomic Research Institute) in all of my previous research endeavors.

Competing Interests

Authors have declared that no competing interests exist

References

1. Anderson, H. L., Fermi, E., & Hanstein, H. B. (1939). Production of neutrons in uranium bombarded by neutrons. *Physical review*, 55(8), 797.
2. Redder, A., Becker, H. W., Lorenz-Wirzba, H., Rolfs, C., Schmalbrock, P., & Trautvetter, H. P. (1982). The $^{15}\text{N}(\text{p}, \alpha)^{12}\text{C}$ reaction at stellar energies. *Zeitschrift für Physik A Atoms and Nuclei*, 305(4), 325-333.
3. Bernstein, J. (2002). Heisenberg and the critical mass. *American Journal of Physics*, 70(9), 911-916.

4. Bohr, N. (1939). Resonance in uranium and thorium disintegrations and the phenomenon of nuclear fission. *Physical Review*, 55(4), 418.
5. Bohr, N., & Wheeler, J. A. (1939). The mechanism of nuclear fission. *Physical Review*, 56(5), 426.
6. Brown, A. (1997). *The neutron and the bomb: A biography of Sir James Chadwick*. Oxford University Press, Oxford.
7. Chadwick, J. (1932). Possible existence of a neutron. *Nature*, 129(3252), 312-312.
8. Chadwick, J. (1932). The existence of a neutron. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 136(830), 692-708.
9. D. Hafemeister, *Physics of Societal Issues: Calculations On National, Security, Environment, And Energy* (Springer, New York, 2007).
10. MacMillan, D. B. (1973). Monte Carlo confidence limits for iterated-source calculations. *Nuclear Science and Engineering*, 50(1), 73-75.
11. Glaser, A. (2006). On the proliferation potential of uranium fuel for research reactors at various enrichment levels. *Science & Global Security*, 14(1), 1-24.
12. Kreyszig, E. (1978). *Introductory functional analysis with applications*. John Wiley & Sons.
13. Derrin, E. (1990). Estimate of the critical mass of a fissionable isotope. *American Journal of Physics*, 58(4), 363-364.
14. Feather, N. (1939). The time involved in the process of nuclear fission. *Nature*, 143(3623), 597-598.
15. Fermi, E.: *Nuclear Physics*, P. 165. University of Chicago Press, Chicago, IL (1950; 1974 PAPERBACK REPRINT).
16. C.A. Bertulani, P. Danielewicz, *Intro. to Nucl. Reactions*. IOP Graduate Student Series in Physics (IOP, Bristol, 2004).
17. G.R. Satchler, *Intro. Nucl. Reactions*, 2nd edn. (McMillan, London, 1990). Chap. 3.7 ff.
18. Morrison, G. W., Mihalcz, J. T., & Irving, D. C. (1966). REACT and CONVRG Fortran Subroutines for Determining Source Convergence for the O5R Monte Carlo Neutron Transport Code (No. ORNL-TM-1325). Oak Ridge National Lab., Tenn..
19. Koch, L. J., & Paxton, H. C. (1959). Fast reactors. *Annual Review of Nuclear Science*, 9(1), 437-472.
20. Hahn, O., & Strassmann, F. (1939). On the detection and behavior of the alkaline earth metals formed during the irradiation of uranium with neutrons. *Naturwissenschaften*, 27 (1), 11-15.
21. Paetz gen Schieck, H. (2012). *Nuclear physics with polarized particles*.
22. H. Lorenz-Wirzba, P. Schmalbrock, H.P. Trautvetter, M. Wiescher, C. Rolfs, *Nucl. Phys. A* 313, 346 (1979).
23. Hyde, E. K. (1964). *The nuclear properties of the heavy elements: Detailed radioactivity properties (Vol. 2)*. Prentice-Hall.
24. Kalos, M. H., Nakache, F. R., & Celnik, J. (1968). *Computing Methods in Reactor Physics*. Greenspan H, Kelber CN, Okrent K, Eds.
25. Kadotani, H., Hariyama, Y., Shiota, M., & Takada, T. (1991, January). Acceleration of fission distribution convergence using eigen-vectors from matrix K calculations in the KENO code. In *ICNC'91: international conference on nuclear criticality safety* (pp. 1-1).
26. Kaper, H. G., Lindeman, A. J., & Leaf, G. K. (1974). Benchmark values for the slab and sphere criticality problem in one-group neutron transport theory. *Nuclear science and engineering*, 54(1), 94-99.
27. J.M. Blatt, V.F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).
28. Halton, J. H. (1994). Sequential Monte Carlo Techniques for the Solution of Linear Systems," *Journal of Scientific Computing*, 9, 2, June 1994, pages 213-257.
29. J.M. Hammersley and D.C. Handscomb, *Monte Carlo Methods*, John Wiley & Sons, New York (1964); reprinted with minor corrections by Barnes & Noble, Inc., New York (1965).
30. Carter, L. L., & Cashwell, E. D. (1975). *Particle Transport Simulation with the Monte Carlo Method*, TID-26607, ERDA Critical Review Series, Technical Information Center, U.S. Energy Research and Development Administration, Oak Ridge, TN.
31. Logan, J. (1996). The critical mass. *American Scientist*, 84(3), 263-277.
32. Lovins, A. B. (1980). Nuclear weapons and power-reactor plutonium. *Nature*, 283(5750), 817-823.
33. Kalos, M. H., & Whitlock, P. A. (1986). *Monte Carlo Methods, Volume I: Basics*, John Wiley & Sons, New York.
34. Myers, W. D., & Swiatecki, W. J. (1966). Nuclear masses and deformations. *Nuclear Physics*, 81(1), 1-60.
35. Peierls, R. (1939, November). Critical conditions in neutron multiplication. In *Mathematical Proceedings of the Cambridge Philosophical Society* (Vol. 35, No. 4, pp. 610-615). Cambridge University Press.
36. P. Marmier, E. Sheldon, *Physics of Nuclei and Particles*, Vol. I (Academic Press, New York, 1970). Chap. 11.2 ff.
37. Reed, B. C. (2007). Arthur Compton's 1941 report on explosive fission of U-235: A look at the physics. *American Journal of Physics*, 75(12), 1065-1072.
38. Reed, B.C.: Rudolf Peierls' 1939 analysis of critical conditions in neutron multiplication. *Phys.Soc.* 37(4), 10-11 (2008).
39. Reed, B. C. (2009). A brief primer on tamped fission-bomb cores. *American Journal of Physics*, 77(8), 730-733.
40. Reed, B. C. (2010). Student-level numerical simulation of conditions inside an exploding fission-bomb core. *Nat. Sci.* 2(3), 139-144.
41. Reed, B. C., & Mihailidis, D. (2011). *the Physics of the Manhattan Project*. Berlin: Springer.
42. Rutherford, E., & Soddy, F. (1902). XLI. The cause and nature of radioactivity.—Part I. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 4(21), 370-396.
43. Richard Edward Swaja, *Application of Stochastic Optimal Estimation Principles to Monte Carlo Eigenfunction*

- Convergence, Ph.D. dissertation, Carnegie-Mellon University, Pittsburgh, Pennsylvania, October, 1972, recorded as 1973 in the reprint by University Micro Ims International, Ann Arbor, MI (1993).
44. Rutherford, E., & Soddy, F. (1903). LX. Radioactive change. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 5(29), 576-591.
 45. Rutherford, E., & Geiger, H. (1908). An electrical method of counting the number of α -particles from radio-active substances. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 81(546), 141-161.
 46. Rutherford, E., & Royds, T. (1909). XXI. The nature of the α particle from radioactive substances. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 17(98), 281-286.
 47. Todd J. Urbatsch, R. Arthur Forster, Richard E. Prael, and Richard J. Beckman, \Understanding the Three-Combined keff Condence Intervals in MCNP," Proceedings of the International Conference on Nuclear Criticality Safety (ICNC), September 17-22, 1995, Albuquerque, New Mexico (1995). See also \Estimation and Interpretation of keff Condence Intervals in MCNP," Nuclear Technology, 111, 169-182, August (1995).
 48. Gol'Din, V. Y. (1964). A quasi-diffusion method of solving the kinetic equation. USSR Computational Mathematics and Mathematical Physics, 4(6), 136-149.
 49. Goad, W., & Johnston, R. (1959). A monte carlo method for criticality problems. Nuclear Science and Engineering, 5(6), 371-375.
 50. Wilfred Kaplan, Advanced Calculus, Third Edition, Addison-Wesley Publishing Company, Reading, Massachusetts (1984).
 51. Walters, W. F., & Wareing, T. A. (1996). An accurate, strictly-positive, nonlinear characteristic scheme for the discrete-ordinate equations. Transport Theory and Statistical Physics, 25(2), 197-215.