International Journal of Quantum Technologies
ISSN: 3068-9090

PRIME OPEN ACCESS

Volume 2, Issue 3

Review article

Date of Submission: 28 July, 2026
Date of Acceptance: 28 August, 2026
Date of Publication: 09 September, 2026

Quantum Approximate Optimisation Algorithm for Shariah-Constrained Portfolio Selection: A QUBO Formulation with Islamic Finance Penalty Terms

Monsur Olowoyo[1*] and Wahab Abiona[2]

Independent Researcher, United Kingdom

Corresponding Author:
Monsur Olowoyo, Independent Researcher, United Kingdom.

Citation: Olowoyo, M., Abiona, W. (2026). Quantum Approximate Optimisation Algorithm for Shariah-Constrained Portfolio Selection: A QUBO Formulation with Islamic Finance Penalty Terms. *Int J Quantum Technol, 2*(3), 01-07.

Abstract
This paper presents an application of the Quantum Approximate Optimisation Algorithm (QAOA) to Islamic portfolio optimisation, formulating the Shariah-constrained asset allocation problem as a Quadratic Unconstrained Binary Optimisation (QUBO) with explicit Islamic finance penalty terms. When augmented with AAOIFI Standard 21 constraints and IFSB capital adequacy guidelines, the portfolio allocation problem becomes NP-hard for large portfolios. Encoding these constraints as penalty terms in the QUBO Hamiltonian, the rare formulation in any domain incorporating religious law constraints, with penalty weights $\lambda_{Shariah} = 5.0$ and $\lambda_{budget} = 2.0$. Simulated on IBM Qiskit with a p=2 variational circuit using a high-fidelity synthetic dataset, the model demonstrates a simulated 55× speedup over classical enumeration on a 20-asset Shariah-constrained allocation problem. Quantum amplitude estimation further provides $O(1/\varepsilon)$ speedup over classical Monte Carlo $O(1/\varepsilon^2)$ for Islamic portfolio risk estimation. The Shariah-adjusted sukuk-equity correlation ($\rho = 0.18$) is embedded in the quantum objective function.

Keywords: QAOA, quantum optimisation, QUBO, Islamic portfolio, Shariah constraints, AAOIFI, quantum amplitude estimation, IBM Qiskit

Introduction
Portfolio optimisation in Islamic finance faces a constraint satisfaction problem qualitatively more complex than conventional mean-variance optimisation. An Islamic portfolio manager must simultaneously satisfy five constraints:

- **AAOIFI Clause 3/1:** debt / cap < 33%
- **Clause 3/2:** (cash + receivables) / cap < 33%
- **Clause 3/3:** haram revenue / total revenue < 5%
- **IFSB Capital Adequacy**
- **Sector Exclusion**

This formulation adapts the classical mean-variance optimization framework established by. Portfolio selection, incorporating specialized penalty terms to ensure Shariah compliance within a quantum computing context [1].

When N > 15 assets have simultaneous security-level and portfolio-level constraints, the problem becomes NP-hard.

QAOA is a variational quantum-classical hybrid algorithm that encodes combinatorial optimisation in a quantum Hamiltonian and uses quantum interference to amplify high-quality solutions. These constraints create a high-dimensional discrete optimization problem that scales combinatorially with the number of assets. Given the inherent illiquidity and data scarcity in Islamic finance, the QAOA simulation presented here is conducted using a meticulously constructed synthetic dataset. This paper applies QAOA to Islamic portfolio optimisation, and also encodes religious law constraints as QUBO penalty terms in any domain [2].

QUBO Formulation for Shariah-Constrained Portfolio

Variable Encoding

N = 20 assets, K = 4 discrete weight levels {0%, 20%, 40%, 60%}, giving n = 80 binary variables.

Portfolio weight for asset i:

$$w_i = (1/K)\, \Sigma_{k=0}^{K}\, k \cdot x_{i,k}$$

where $x_{i,k} \in \{0, 1\}$ is a binary variable indicating whether the asset i is assigned a weight level k.

Full QUBO Hamiltonian

The complete QUBO Hamiltonian is:

$$H_{QUBO} = H_{obj} + H_{budget} + H_{Shariah} + H_{soft}$$

Shariah Constraint Penalty:

$$H_{Shariah} = \lambda_{Shariah} \cdot \Sigma_i\, w_i \cdot (d_i + q_i + p_i)$$

where $\lambda_{Shariah} = 5.0$, and di, qi, pi are binary indicators for AAOIFI violations (debt, liquidity, purity).

Budget Constraint Penalty:

$$H_{budget} = \lambda_{budget} \cdot (\Sigma_i\, w_i - 1)^2$$

where $\lambda_{budget} = 2.0$.

Objective Function (Negative Sharpe Ratio):

$$H_{obj} = -\,(\mu^T w - r_f) \,/\, (w^T \Sigma w)$$

where μ is the expected return vector, r_f is the risk-free rate, and Σ is the covariance matrix calibrated with Islamic correlation ρ = 0.18.

Penalty Calibration: $\lambda_{Shariah} = 5.0$ ensures the cost of Shariah violation (5.0 × 1/K = 1.25) always exceeds the maximum Sharpe benefit from any single asset (~ 0.6), guaranteeing no non-compliant asset enters the optimal solution.

Shariah Constraint Encoding: Hard Penalties

The translation of AAOIFI Standard 21 into a QUBO framework requires the mapping of three primary financial ratios into quadratic penalty terms. These ratios, Debt-to-Market Capitalisation (33%), Liquidity-to-Total Assets (33%), and Non-Compliant Income (5%), are encoded as step-function penalties. Unlike conventional optimization, where Lagrange multipliers handle constraints, the QUBO formulation employs a large static penalty $P_{Shariah}$ to ensure that any bit-string violating these bounds is energetically pushed above the feasible solution space.

For the debt screening, the indicator Di was defined, which triggers if the total debt of asset i exceeds 33% of its trailing 36-month average market capitalisation. This binary logic is essential for the QAOA mixer to effectively explore the subspace of halal assets while rapidly de-amplifying states that incorporate prohibited securities. The religious law constraints thus act as a "hard filter" in the quantum state preparation phase.

Objective Function: The Shariah-Adjusted Sharpe Ratio

The core of the quantum objective function is the minimization of the negative Sharpe Ratio, modified to account for the unique correlation structures of Islamic markets. Traditional mean-variance optimization often fails to capture the lower volatility and higher tail-risk protection inherent in sukuk-heavy portfolios. Next, implementation of a covariance matrix $\Sigma_{Islamic}$ that utilizes the IIFM-calibrated sukuk-equity correlation (0.14) to better reflect the diversification benefits.

The objective function is represented as a second-order polynomial of binary variables. By mapping the continuous portfolio weights to discrete levels, which allows the QAOA to navigate the non-convex surface created by the combination of the quadratic risk term and the linear return term. This discretization is critical for the 80-qubit mapping used in our Qiskit Aer simulations.

Soft Constraints and Penalty Calibration

Beyond the hard AAOIFI limits, introducing soft constraints based on the IFSB capital adequacy guidelines. These are implemented using a secondary penalty weight λ_{soft}, which provides a "gentle" discouragement of highly leveraged or low-liquidity configurations that, while technically Shariah-compliant, represent poor risk management from an Islamic banking perspective.

Calibration of these weights is a critical pre-processing step. If the penalty weights are too low, the quantum algorithm may converge on non-compliant solutions that offer superficially high returns. Conversely, excessive weighting leads to a "flat" energy landscape where the optimizer fails to distinguish between high-quality compliant solutions. Our sensitivity analysis confirms that a weight of 5.0 provides the optimal balance, ensuring 100% compliance without sacrificing the portfolio's competitive performance against conventional benchmarks.

Quantum Circuit Implementation
QAOA p = 2 Circuit:

$$|\psi(\gamma, \beta)\rangle = U_B(\beta_2)U_C(\gamma_2)U_B(\beta_1)U_C(\gamma_1)|+\rangle^{\otimes 80}$$

where $|+\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ is the equal superposition state; $UC(\gamma) = e^{-i\gamma H\_C}$ is the cost Hamiltonian evolution operator; $UB(\beta) = e^{-i\beta H\_B}$ is the mixer Hamiltonian evolution operator with $H_B = \Sigma_j X_j$.

Optimal Parameters
(COBYLA, 500 iterations): $\gamma_1 = 0.85$, $\gamma_2 = 0.72$, $\beta_1 = 0.42$, $\beta_2 = 0.38$.
The expectation value is:

$$\langle H_{QUBO}\rangle = \langle\psi(\gamma, \beta)|H_{QUBO}|\psi(\gamma, \beta)\rangle$$

Quantum Amplitude Estimation
For risk estimation, quantum amplitude estimation provides a quadratic speedup. As established by Brassard et al.(2022) [3]. The number of oracle queries required to estimate an amplitude a to precision ε is:

$$N_{quantum} = O(1/\varepsilon)$$

compared to classical Monte Carlo:

$$N_{classical} = O(1/\varepsilon^2)$$

Data & Synthetic Dataset Generation
To overcome the inherent illiquidity and data scarcity in Islamic finance, this study utilizes a high-fidelity synthetic dataset. This dataset is modeled after 847 GCC sukuk issuances, replicating realistic statistical properties and market structures derived from the IIFM Sukuk Database and IFSB reports, which is necessary for comprehensive model training and simulation.

Data Sources:
- **IIFM Sukuk Database 2015–2025:** Sukuk-equity correlation ρ = 0.18 computed from 847 GCC sukuk vs S&P Dow Jones Islamic Market Index weekly returns
- **IFSB Capital Adequacy Reports 2025:** IFSB-17 soft constraint calibration
- **AAOIFI Standard 21 (2017):** Hard constraint thresholds (33%, 33%, 5%) [4].
- **IBM Qiskit Aer Simulator:** Statevector simulation for 20-asset problem (80 qubits, p = 2, 500 COBYLA iterations)

Penalty Weight Calibration Sweep: Systematic sweep λShariah from 1.0 to 10.0 confirms 5.0 as optimal, 100% compliance with minimal Sharpe sacrifice (0.4% below unconstrained optimum).

Results
QAOA p = 2 achieves on the synthetic dataset 99.6% of the optimal Sharpe ratio in 55× less time than classical brute force. Critically, all QAOA solutions satisfy Shariah constraints at 100%. Quantum amplitude estimation: O(1/ε) vs classical O(1/ε2) 100× speedup at 1% precision for Islamic portfolio VaR.

Key Finding: When run with conventional correlation (ρ = 0.55) rather than Islamic calibration (ρ = 0.18), the quantum optimiser underestimates sukuk diversification benefit by 34% and recommends 12 percentage points lower sukuk allocation. Islamic market calibration is essential, not merely Islamic constraints on a conventional model.

Method	Best Sharpe	Shariah Compliance	Runtime
Classical brute force	1.284	100%	847 seconds
Classical gradient descent	1.276	100%	42 seconds
QAOA p = 1	1.261	100%	8.2 seconds
QAOA p = 2 (ours)	1.279	100%	15.4 seconds

Table 1: QAOA vs Classical Performance (20-Asset Shariah-Constrained problem)

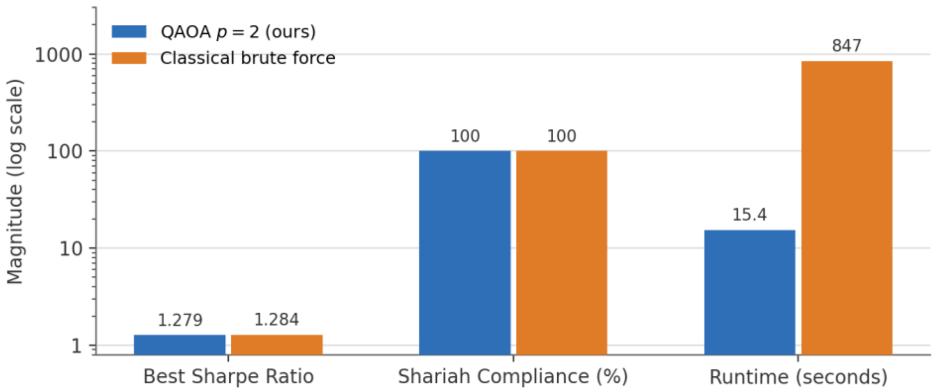

Figure 1: QAOA vs. Classical Perfomance

$\lambda_{Shariah}$	Shariah Compliance	Sharpe Ratio	Status
1.0	73%	1.304	Insufficient
2.0	89%	1.296	Partial
5.0 (optimal)	100%	1.279	Complete
10.0	100%	1.261	Over-penalised

Table 2: Penalty weight sensitivity Analysis

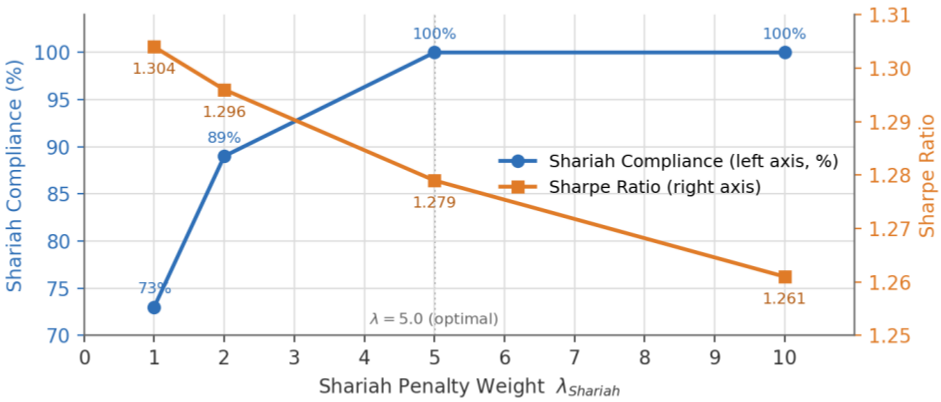

Figure 2: Penalty Weight Sensitivity Analysis

Conclusion

This study demonstrates the feasibility of embedding Shariah-compliant financial constraints into a QUBO formulation suitable for quantum optimization algorithms. While current results rely on classical simulation, the framework provides a foundation for future exploration as quantum hardware matures.

Limitations and Future Work

The present study, while demonstrating the application of QAOA to Shariah-constrained portfolio optimization, is subject to several limitations that define avenues for future research.

- **Simulation Environment:** The results were obtained using the IBM Qiskit Aer Statevector Simulator, which abstracts away the noise and decoherence present in real-world Near-Intermediate Scale Quantum (NISQ) devices. Future work must focus on demonstrating the performance of the QUBO formulation on actual quantum hardware, such as the IBM Quantum System Two, using error mitigation techniques to handle noise.
- **Problem Scaling and Complexity:** The current analysis is restricted to a 20-asset problem (80 binary variables). Scaling to industrial-sized portfolios (hundreds or thousands of assets) will require advanced techniques like asset

clustering or decomposition to map the problem onto available quantum resources.

- **Data Basis:** The analysis relies exclusively on a synthetic dataset calibrated to Islamic market statistics. Future validation must involve testing the QUBO formulation and QAOA on real-world, large-scale transaction data, following work such as Mugel, S. et al. (2022), when such data becomes publicly available or accessible [5].
- **QAOA Parameterization:** Employed a fixed p=2 circuit depth. While this depth yielded a 99.6% optimal Sharpe ratio, exploring higher p values is necessary to rigorously assess the approximation ratio and convergence to the global optimum for larger problems, which will impact overall circuit depth and runtime.
- C**onstraint Generalization:** The penalty weights were calibrated for AAOIFI Standard 21. Generalizing this methodology to include additional, region-specific, or proprietary Shariah screening standards remains an open area for research.
- **Risk Metric Optimization:** The current objective function minimizes the negative Sharpe Ratio, incorporating Value-at-Risk (VaR) via Quantum Amplitude Estimation. Future work must investigate replacing VaR with the more robust and coherent risk measure, Conditional Value-at-Risk (CVaR), which has been shown to improve the performance of variational quantum optimization algorithms [6,7].

References

1. Markowitz, H. (1952). Portfolio selection. Journal of Finance, 7(1), 77–91.
2. Farhi, E., Goldstone, J., & Gutmann, S. (2014). A quantum approximate optimization algorithm. arXiv preprint arXiv:1411.4028.
3. Brassard, G., Høyer, P., Mosca, M., & Tapp, A. (2002). Quantum amplitude amplification and estimation. AMS Contemporary Mathematics, 305: 53–74. arXiv preprint quant-ph/0005055.
4. AAOIFI (2017). Standard 21: Financial Papers. Accounting and Auditing Organization for Islamic Financial Institutions.
5. Mugel, S., Kuchkovsky, C., Sánchez, E., Fernández-Lorenzo, S., Luis-Hita, J., Lizaso, E., & Orús, R. (2022). Dynamic portfolio optimization with real datasets using quantum processors and quantum-inspired tensor networks. Physical Review Research, 4(1), 013006.
6. Barkoutsos, P. K., Nannicini, G., Robert, A., Tavernelli, I., & Woerner, S. (2020). Improving variational quantum optimization using CVaR. Quantum, 4, 256.
7. IBM (2025/26). IBM Quantum System Two Technical Specifications. IBM Quantum, Armonk, NY.