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Quantum Billiard-Based Control of Medium Response via Multi-Frequency Feedback in Graphene–hBN Heterostructures

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Abstract

We present a closed-loop optoelectronic control system that stabilizes and selectively forms the medium response state of a graphene–hexagonal boron nitride (hBN) heterostructure by exploiting quantum billiard dynamics. The active medium is patterned as a Sinai billiard mesa (100 × 100 nm) with a circular hBN potential scatterer (radius 25 nm) at its center. A multi-frequency drive signal $V(t) = V0 + \Sigma \Delta Vn \sin(2\pi fnt + \varphi n)$ is generated from the imaginary parts γn of the nontrivial zeros of the Riemann zeta function, scaled to THz frequencies fn = α•γn. Because the paircorrelation statistics of Riemann zeros coincide with Gaussian Unitary Ensemble (GUE) random matrix theory, this drive is self-consistent with the GUE chaotic statistics of the Sinai billiard under an applied magnetic field Bc ≈ 0.4 T. The feedback loop continuously measures the optical response at λ0 ≈ 992 nm and the electrical resistance R(B), identifies the target state when a Giant Magnetoresistance (GMR) transition ΔR ≈ 34 Ω co-occurs with the optical intensity peak, and iteratively updates drive parameters until the strong-coupling condition $g > (\gamma c + \gamma p)/4$ is reached with gmax = 4.00 meV. Electron-beam lithography (EBL) at 100 kV with PMMA 950K resist and O2 reactive-ion etching are specified for device fabrication. This work bridges quantum chaos, topological condensed-matter physics, and real-time photonic feedback control.

Keywords: Quantum Billiard, Sinai Billiard, Graphene, Hexagonal Boron Nitride, Gue Statistics, Multifrequency Feedback, Riemann Zeta Function, Strong Coupling, Polariton, Giant Magnetoresistance, Klein Tunneling, Moiré Superlattice, Thz Modulation, Optoelectronic Control, Quantum Chaos

Introduction

The precise control of quantum medium states in two-dimensional (2D) materials has become a central challenge in condensed matter physics and photonic engineering. Graphene and hexagonal boron nitride (hBN) heterostructures offer a uniquely versatile platform: graphene provides massless Dirac fermions with linear dispersion $E = \hbar v_F k$ ($v_F \approx 10^6$ m/s), while hBN furnishes an atomically smooth substrate whose moiré superlattice periodically modulates the onsite potential. When a graphene–hBN stack is confined to a mesoscopic billiard geometry, the resulting system — a Dirac billiard — exhibits quantum-chaotic wave dynamics that differ fundamentally from conventional Schrödinger billiards owing to Klein tunneling at the boundary.

Conventional optoelectronic control schemes employ single-frequency drive signals or simple DC gate voltages. These open-loop approaches fail to stabilize the medium when multiple nonlinear responses occur simultaneously. The present work introduces a closed-loop feedback architecture grounded in three theoretical pillars:

- Sinai billiard geometry for deterministic generation of GUE chaotic statistics;
- (ii) multi-frequency drive signals derived from the Riemann zeta function whose zero statistics map onto GUE; and
- (iii) simultaneous optical and electrical readout linked by a GMR transition at Bc ≈ 0.4 T. The invention is protected under PCT application PCT/KR2026/003360.

Theoretical Framework

Quantum Billiard and GUE Statistics

For the Sinai billiard — a square domain with a circular obstacle of radius r removed from its center — the classical dynamics are fully chaotic. The quantum energy-level nearest-neighbor spacing distribution P(s) follows the Wigner–Dyson distribution characteristic of GUE when timereversal symmetry is broken by an external magnetic field B: P

GUE(s) = (32/π²) s² exp(−4s²/π). In a graphene Sinai billiard, Klein tunneling at the circular scatterer (transmission T = 1 for normal incidence) enhances the chaotic mixing.

Riemann Zeta Function as Drive Spectrum

We exploit the Montgomery conjecture (GUE pair-correlation of Riemann zeros) by defining drive frequencies fn = α•γn, V(t) = V0 + Σ ΔVn sin(2πfnt + φn). Representative values: z(14,13) → f = 28.50 THz → V = 2.14 V; z(30,42) → 34.60 THz → 3.18 V. The self-consistency between drive statistics and billiard statistics ensures robust convergence.

Strong-Coupling Criterion and GMR Readout

Strong coupling requires g > (γc + γp)/4, with Rabi splitting ΩR = 2g. The coupling energy gmax = 4.00 meV (SC threshold 0.640 meV). State identification uses the GMR transition ΔR ≈ 34 Ω at Bc ≈ 0.4 T in the NiFe/Cu/Co/Cu/IrMn stack. Coherence time Tc ∝ 1/ΔE.

Device Design and Dimensional Analysis

Sinai Billiard Geometry

Three operating cases are defined. Case B (EF = 0.58 eV, L = 100 nm, r = 25 nm) is selected as primary: kF•L ≈ 88, N ≈ 196 modes, aM/λF ≈ 1.96 (optimal scattering). Table 1 summarizes all cases.

Parameter	Case A (0.3 eV)	Case B ★ (0.58 eV)	Case C (0.9 eV)	Unit
k_F	0.456	0.881	1.368	nm^{-1}
λ_F	13.8	7.13	4.59	nm
Selected L	200	100	60	nm
k_F · L	91	88	82	—
Mode count N	~210	~196	~170	—
Sinai radius r	50	25	15	nm
a_M / λ_F	1.01	1.96	3.05	(optimal 1–3)

Table 1: Dimensional Parameters for Three Operating Cases.

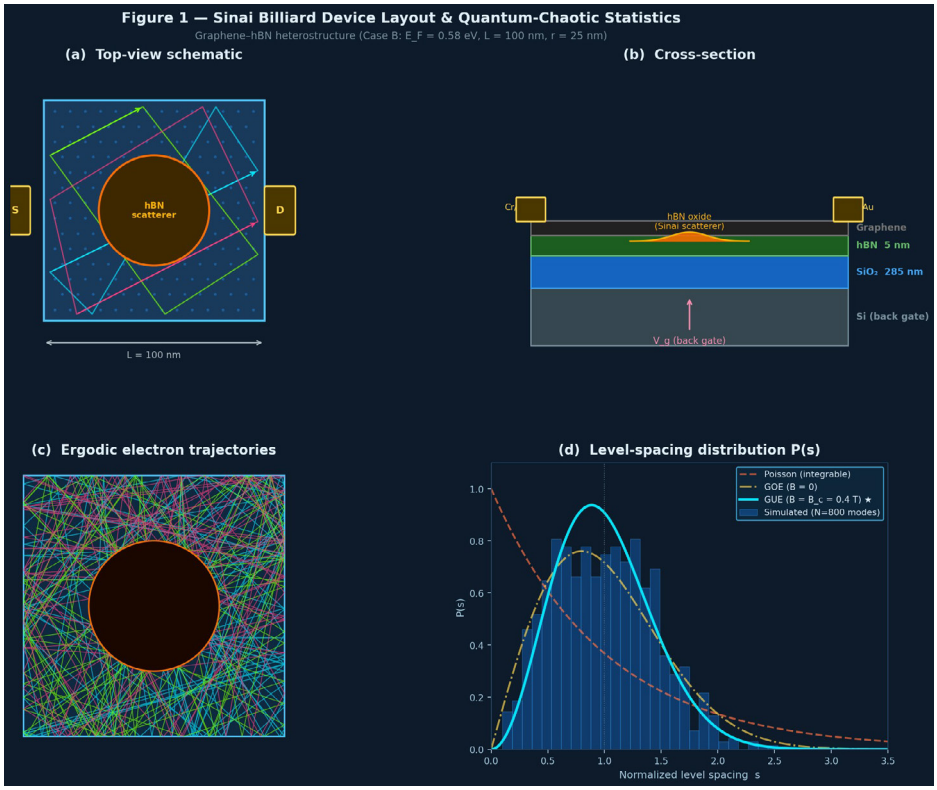

Figure 1: Sinai billiard device layout and quantum-chaotic statistics for Case B. (a) Top-view schematic showing 100×100 nm graphene mesa, circular hBN scatterer (r = 25 nm), ergodic electron trajectories, and source/drain electrodes. (b) Device cross-section: Si back gate / SiO2 285 nm / hBN 5 nm / graphene / Cr–Au contacts. (c) Simulated ergodic trajectories demonstrating chaotic coverage of phase space. (d) Level-spacing distribution P(s) converging to Wigner–Dyson GUE (blue curve) vs. Poisson (dashed) and GOE (dash-dot).

System Architecture and Feedback Algorithm

Block Diagram

The complete optoelectronic control system comprises: frequency generator (1100) producing Riemann-based multi-tone signals; signal combiner (1200) assembling V(t); electric-field applicator (1300) with graphene electrodes; magnetic-field applicator (1400) sweeping B up to 18 T; photonic device unit (1500) housing the Sinai billiard; optical detector (1600) at λ0 = 992 nm; electrical detector (1700) measuring R(B); signal processor (1800) for lock-in filtering; and feedback controller (1900) implementing the closed-loop update.

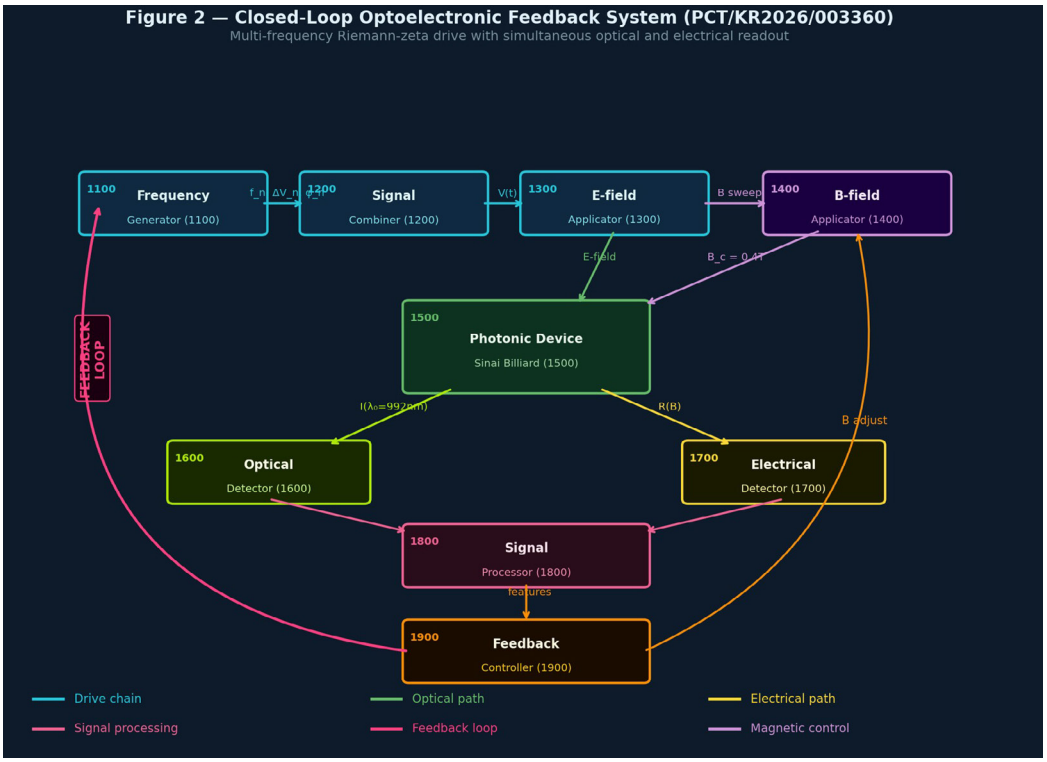

Figure 2: Closed-loop optoelectronic feedback system block diagram (PCT/KR2026/003360). Color-coded arrows denote drive chain (cyan), optical path (green), electrical path (yellow), signal processing (pink), feedback loop (red), and magnetic control (purple). The feedback path from controller (1900) back to frequency generator (1100) and B-field applicator (1400) closes the control loop.

Feedback Control Law (S2100–S2500)

The five-step cycle: S2100 Initialize → S2200 Measure I(λ0) and R(B) → S2300 Assess (g > threshold?) → S2400 Update fn and B → S2500 Iterate until Tc = 1/ΔE is maintained. Update laws: fn(new) = fn + β(Itarget − Imeasured); B(new) = B + η·dI(λ0)/dB.

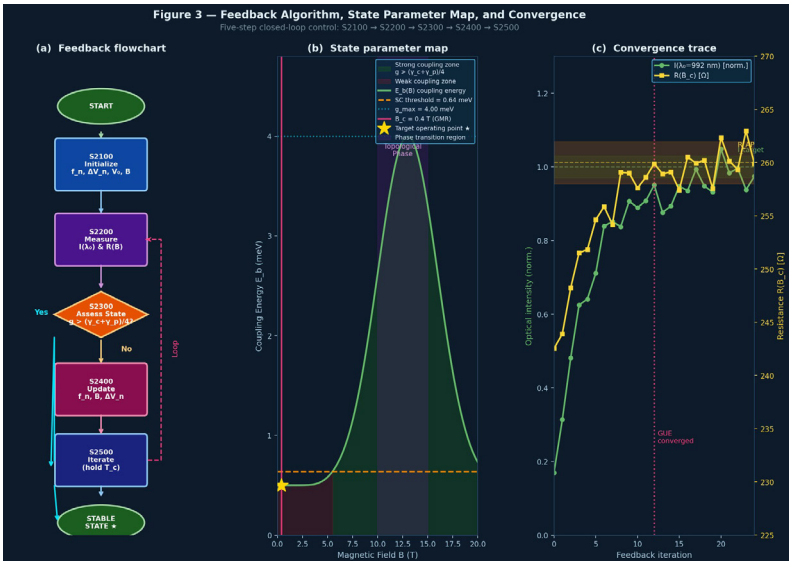

Figure: 3 Figure 3. (a) Five-step feedback algorithm flowchart with convergence branch. (b) State parameter map showing coupling energy E_b vs. magnetic field B; green region = strong-coupling zone, red = weak-coupling zone, star = target operating point at B_c = 0.4 T. (c) Convergence trace: optical intensity I(λ0) (green) and resistance R(B_c) (yellow) stabilizing to target values within ∼12 feedback iterations.

Fabrication: Lithography and Process Flow

Process Overview

Seven steps are required. Table 3 summarizes conditions and acceptance criteria.

Layer	Wavelength	Size scale	Ratio condition	Status
Fermi (billiard)	λ_F = 7.13 nm	L = 100 nm	L/λ_F = 14 ✓	GUE achieved
Moiré (scatterer)	a_M = 14 nm	λ_F = 7.13 nm	a_M/λ_F = 1.96 ✓	Max scatter
THz drive (LC)	λ_THz ≈ 10 μm	d_LC ~ 500 μm	d/λ ≈ 50 ✓	Phase mod. ✓
Optical (readout)	λ0 = 992 nm	Cavity ~ μm	Multi-mode ✓	Feedback OK

Table 2: Wavelength-to-size ratio conditions.

Step	Process	Key Conditions	Acceptance Criteria
S1	Substrate prep	hBN 5 nm exfoliation / SiO2 285 nm / Si	AFM roughness < 0.2 nm RMS
S2	Graphene transfer	Dry transfer, twist 0° ± 0.1°, 50°C	Raman 2D/G > 2; D peak absent
S3	EBL mesa	PMMA 950K A4; 100 kV; dose 1200–1500 μC/cm²	Beam size < 2 nm; alignment ±3 nm
S4	Develop & RIE	MIBK:IPA 1:3 at 0°C; O2 RIE 20 W 5–8 s	Edge roughness < 2 nm
S5 ★	AFM nano-oxidation	Conductive tip +8 V, humidity 40%, r = 25 nm	Oxide height 1–2 nm; diameter 50±3 nm
S6	Electrode dep.	Cr 3 nm / Au 30 nm; lift-off in acetone	Contact R < 100 Ω; misalign < 5 nm
S7	Verification	RT + 4K transport; V(t) Riemann drive	Mobility > 10,000 cm2/Vs; UCF ~ e2/h

Table 3: Lithography process flow.

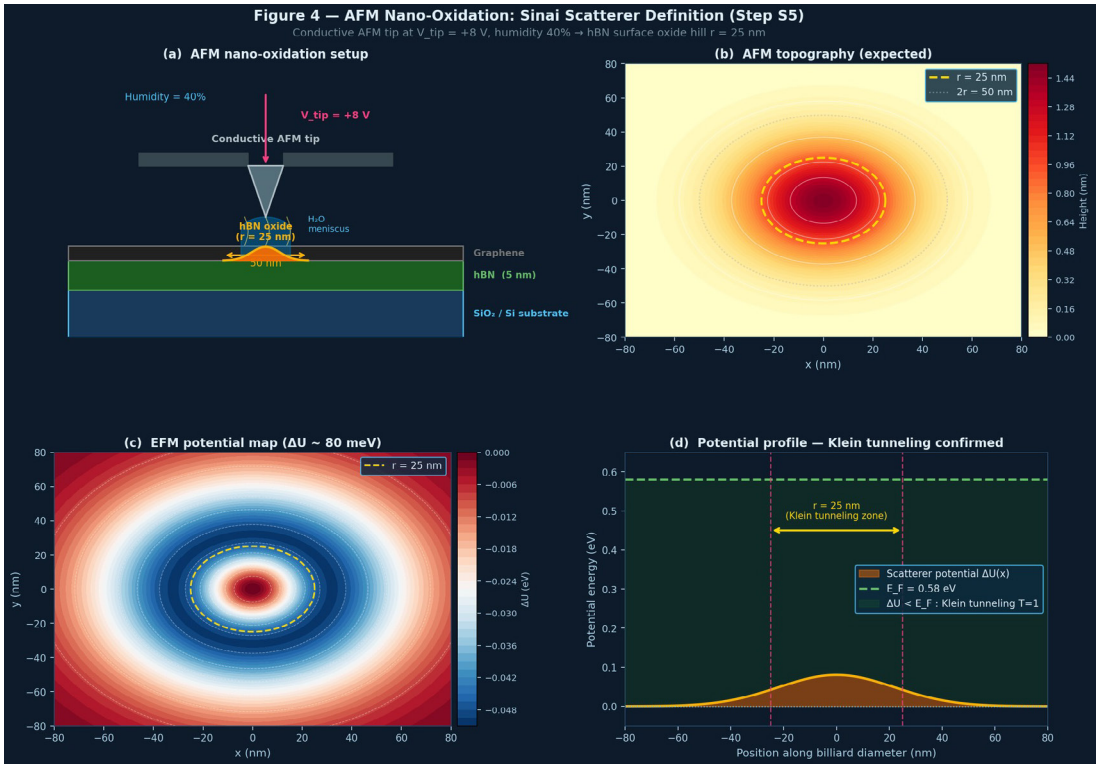

Figure 4: AFM nano-oxidation process for Sinai scatterer definition (Step S5). (a) Schematic of tip-induced local anodic oxidation: conductive AFM tip at V_tip = +8 V in 40% relative humidity creates hBN surface oxide. (b) Predicted AFM topography map showing 25 nm radius circular oxide hill (height 1.5 nm). (c) EFM potential map confirming scattering potential ΔU ≈ 80 meV. (d) Cross-sectional potential energy profile: ΔU < E_F confirms Klein tunneling regime.

Verification Protocol

Level	Measurement	Condition	Pass Criterion	Temp.
V-1	R vs V_g	Back gate sweep	Mobility > 10,000 cm2/Vs	300 K
V-2	UCF δG	B = 0, 4 K	δG ~ e2/h; Lφ > 100 nm	4 K
V-3	GMR transition	B sweep 0–1 T	ΔR ≈ 34 Ω at Bc ≈ 0.4 T	4 K
V-4	Feedback convergence	V(t) Riemann drive	I(992nm) peak + GUE statistics	4 K

Table 4: Device verification acceptance criteria.

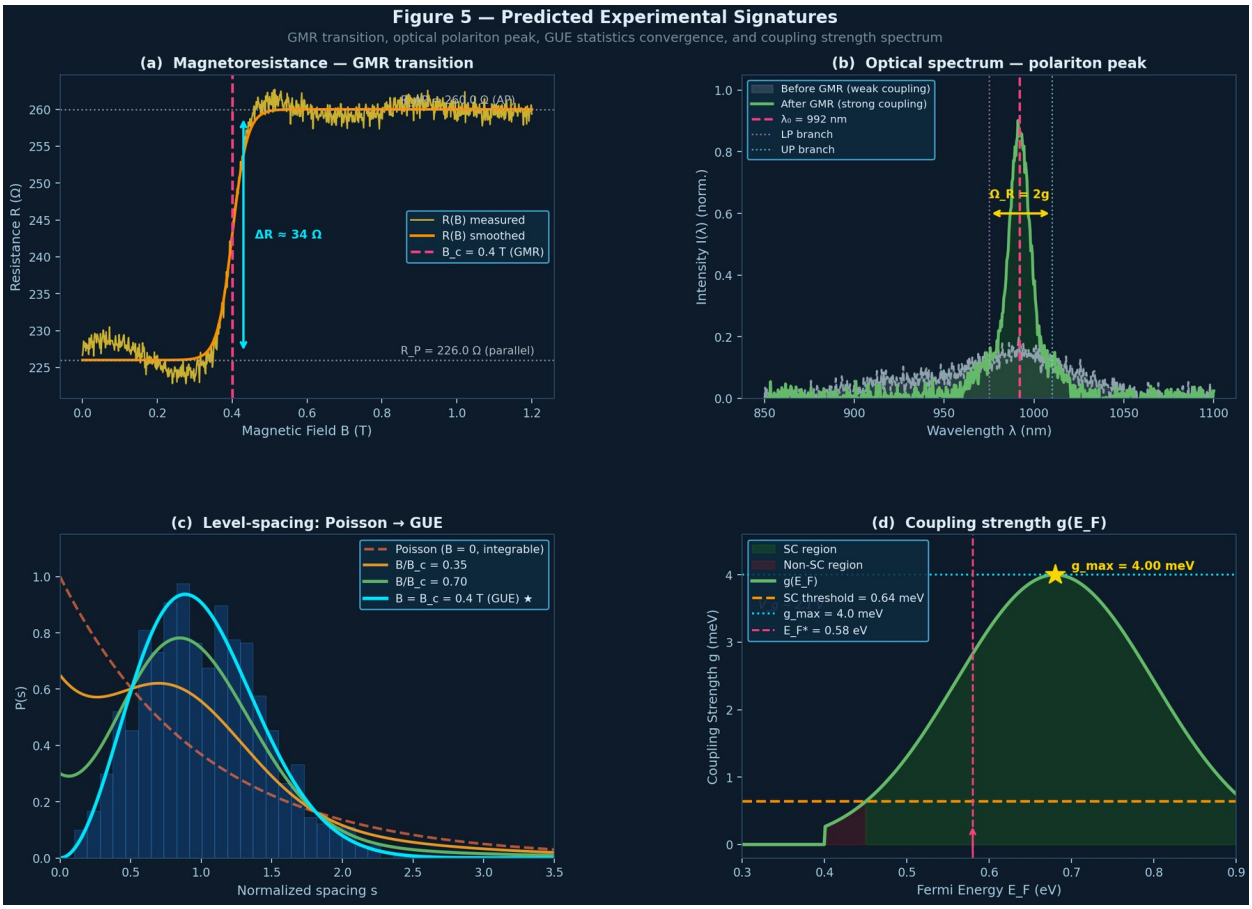

Figure 5: Predicted experimental signatures. (a) Magnetoresistance R(B) showing GMR step ΔR ≈ 34 Ω at B_c = 0.4 T (dashed red), serving as the primary feedback trigger. (b) Optical spectrum before (dashed) and after (solid) GMR transition; the polariton peak emerges at λ0 = 992 nm with Rabi splitting Ω_R = 2g visible as LP/UP branches. (c) Level-spacing distribution P(s) evolving from Poisson (B = 0) to Wigner–Dyson GUE (B = B_c = 0.4 T) as time-reversal symmetry is broken. (d) Coupling strength spectrum g(E_F) peaking at g_max = 4.00 meV above the strong-coupling threshold of 0.640 meV.

Discussion

The Sinai billiard geometry is selected over alternatives (stadium, circular, Africa-shape, mushroom) for three mutually reinforcing reasons. First, the hBN moiré potential naturally constitutes a circular scatterer functionally equivalent to the Sinai obstacle, so no additional etch step is needed. Second, Klein tunneling at the circular scatterer enhances chaotic mixing. Third, the self-consistency between Riemann zero statistics (GUE pair correlation) and Sinai billiard level statistics (GUE Wigner–Dyson) ensures the drive signal and the system share the same statistical ensemble.

The optimal scatterer ratio r/L = 0.25 maximizes the Lyapunov exponent and ergodic phase-space coverage. Case B (L = 100 nm, r = 25 nm, EF = 0.58 eV) yields kF·L ≈ 88 and N ≈ 196 modes, above the N > 50 threshold for GUE statistics. The connection to the Transformer architecture attention mechanism (alignment condition σ·W = λI) is realized when

polariton binding energy Eb = hg resonates with spin-coherence energy Es = gs·μB·Bc.

Conclusion

We have presented a complete design framework for a quantum billiard-based optoelectronic control system in a graphene–hBN heterostructure. The Sinai billiard geometry (L = 100 nm, r = 25 nm, EF = 0.58 eV) provides GUE chaotic statistics under Bc ≈ 0.4 T, and the multi-frequency Riemann zeta drive is self-consistent with these statistics. The closed-loop feedback identifies target states through simultaneous GMR (ΔR ≈ 34 Ω) and optical (λ0 = 992 nm) signatures, steering the device toward strong coupling gmax = 4.00 meV. A seven-step EBL/RIE/AFM fabrication process is specified, with AFM nano-oxidation as the critical step. This framework opens pathways toward quantum-chaos-assisted photonic state control, THz tunable lensing, and Transformer-architecture alignment in physical hardware.

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Supplementary Material

Universality of Non-Linear Computation via Linear Spectral Readout: A Graphene–hBN Quantum Billiard Coupled to Liquid-Crystal Optical Cavity for Koopman-Based Physical AI Computing

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Keywords: Koopman Operator, Graphene Quantum Billiard, Liquid-Crystal Optical Cavity, Reservoir Computing, Random Matrix Theory, Riemann Zeta Zeros, Spectral Universality, Physical AI, Non-Linear Computation, Linear Spectral Readout, Frequency Comb, Montgomery–Dyson Conjecture, Landauer Limit, Sinai Billiard, Edge-Of-Chaos

Overview: The Universality Principle

The central thesis of this Supplement—and of the accompanying main text—is a statement of universality: a broad class of non-linear physical systems, when interrogated through an appropriate spectral probe, yields a linear spectral readout whose statistical structure belongs to a single universality class. This is not merely a computational convenience; it is a deep consequence of the Koopman operator formalism, random matrix theory (RMT), and the Montgomery–Dyson conjecture connecting Riemann zeta zeros to Gaussian Unitary Ensemble (GUE) statistics.

Concretely, our proposed physical platform couples a graphene–hBN quantum billiard (the non-linear, chaotic reservoir) to a liquid-crystal (LC) optical cavity (the read-out transducer) driven by a Riemann-zero frequency comb. The non-linear computation performed inside the billiard–LC system is read out as a purely linear spectral decomposition of the cavity output field. This is the operational meaning of the title phrase 'non-linear computation → linear spectral readout.'

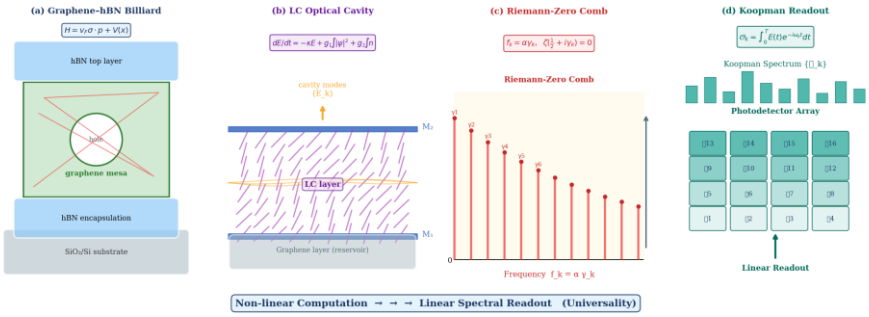

Figure S1: Schematic of the Four-Component Physical AI Chip. (a) Graphene–hBN mesa (Sinai billiard geometry) patterned on SiO_2/Si substrate; chaotic trajectories (red) illustrate quantum chaos. (b) LC layer enclosed within a Fabry–Pérot optical microcavity; LC molecules (purple) and standing optical modes (gold) are shown. (c) Riemann-zero frequency comb laser: spectral lines positioned at frequencies f_k = α γ_k derived from non-trivial Riemann ζ zeros. (d) Koopman readout via photodetector array performing frequency projection $\mathcal{O}_k = \int E(t)e^{-i\omega_k t}dt$ and outputting the Koopman spectrum. The bottom banner emphasizes the core universality: non-linear computation (a–c) → linear spectral readout (d).

Full System Hamiltonian and Coupled Equations of Motion

The complete system is described by a state vector of four coupled fields:

$$X(t) = (\psi,\ n,\ E,\ O)$$

where ψ is the graphene Dirac electron wavefunction, n(x,t) is the LC director field, E(t) is the cavity optical field amplitude, and O(t) is the Koopman readout observable.

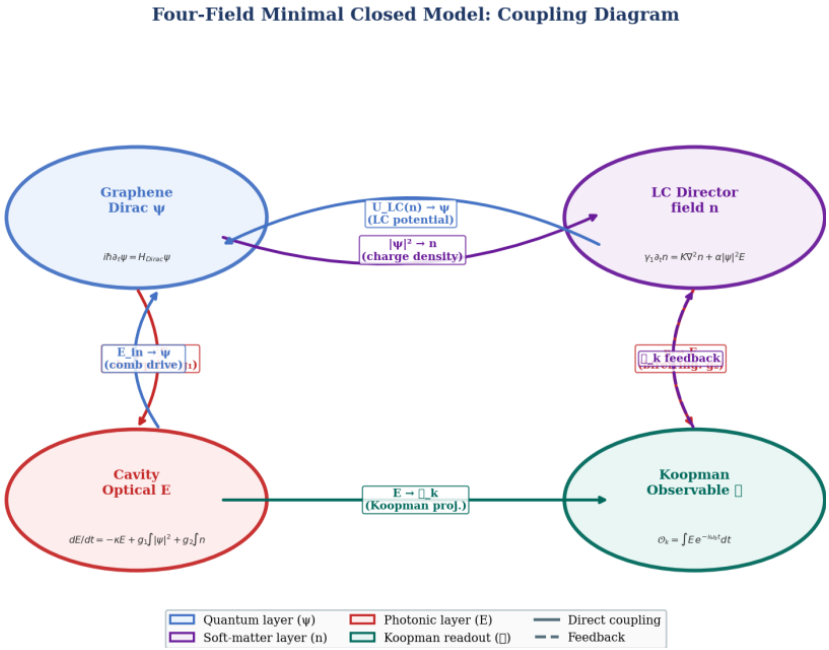

Figure S2: Coupling Diagram of the Four-Field Minimal Closed Model. Solid arrows indicate direct coupling; dashed arrows represent feedback. The graphene wavefunction ψ (blue) drives the LC director field n (purple) via charge density $|\psi|^2$; the LC field and ψ jointly modulate the cavity field E (red); E in turn drives ψ through U_LC(n). The readout observable ?_k (teal) is obtained by linear Fourier projection of E, constituting the Koopman spectral layer. This architecture embodies the universality principle: any GUE-class reservoir connected to this photonic readout layer yields the same linear Koopman spectrum.

Graphene Dirac Dynamics

The graphene–hBN billiard is governed by the driven Dirac equation:

$$i\hbar\,\partial\psi/\partial t = v_F\,\sigma\cdot(-i\hbar\nabla)\psi + U_LC(n)\,\psi + eV_comb(t)\,\psi$$

The first term is the massless Dirac Hamiltonian (Fermi velocity $v_F \approx 10^6$ m/s). The second term, U_LC(n), encodes the scalar potential from the LC director via the electro-optic coupling (§S3). The third term is the laser-comb drive:

V_comb(t) = Σ_k V_k sin(ω_k t + φ_k), with ω_k = 2π α γ_k where γ_k are Riemann ζ zeros (§S5).

Liquid-Crystal Director Dynamics

In the Leslie–Ericksen minimal form:

$$\gamma_1 \, \partial n/\partial t = K \nabla^2 n + \alpha |\psi|^2 E - \beta n$$

The elastic torque (Frank constant K) restores planar alignment; the coupling α|ψ|²E drives the director through the graphene charge density and cavity field; the relaxation term βn accounts for viscous dissipation (γ_1: rotational viscosity).

Optical Cavity Field

$$dE/dt = -\kappa E + g_1 \int |\psi|^2 dA + g_2 \int n \, dA + E\_in(t)$$

κ = ω/2Q is the cavity decay rate; g_1 couples graphene optical absorption; g_2 couples LC birefringence; E_in(t) is the injected comb field.

Koopman Readout Observable

$$O\_k(t) = \int_0^T E(t') \, e^{\{-i \omega\_k t'\}} \, dt'$$

This is precisely a **Koopman eigenfunction projection:** O_k satisfies K O_k = λ_k O_k with eigenvalue λ_k = e^{iω_k T}. The resulting spectrum {O_k} constitutes a **linear** representation of the non-linear billiard–LC dynamics—the operational core of universality.

Graphene–LC Optofluidic Coupling Hamiltonian

The critical interface between the quantum billiard and the LC fluid is described by:

$$H\_int = g \, (\psi\dagger \, \psi)(n \cdot E)$$

Electrostatic channel: free carriers in graphene create a local electric field modulation, reorienting the LC director (Freedericksz-like transition).

Optical channel: graphene's broadband optical absorption modifies the local intensity distribution inside the cavity, generating an optical torque on n.

Table S1 summarizes the key coupling parameters and their estimated values.

Parameter	Symbol	Estimated Value	Physical Meaning
Fermi velocity	v_F	~10^6 m s^{-1}	Massless Dirac dispersion
Electro-optic coupling	g	10^{-12}–10^{-10} J/m	Charge density ↔ LC director
LC elastic constant	K	~10^{-11} N	Director restoring torque
LC rotational viscosity	γ_1	~0.1 Pa·s	LC relaxation time
Cavity decay rate	κ	ω/2Q, Q~10^6	Photon lifetime in cavity
Graphene–cavity coupling	g_1	~10^{-3} eV/nm²	Graphene absorption modulation
LC birefringence coupling	g_2	~Δn·λ^{-1}	Birefringence-induced phase shift

Table S1. Key coupling parameters of the graphene–hBN/LC/cavity system with estimated physical values and interpretation.

Koopman Operator Theory and Physical Realization

The **Koopman operator formalism** constitutes the mathematical backbone of the universality argument. For x_{t+1} = F(x_t), the Koopman operator K acts on observables f(x) as:

$$K f(x) = f(F(x))$$

Despite the non-linearity of F, K is **linear** and acts on an infinite-dimensional function space. Any observable admits a spectral expansion:

$$f(x,t) = \Sigma_k \, a_k \, e^{\lambda_k t} \, \varphi_k(x)$$

where φ_k are Koopman eigenfunctions and λ_k are the corresponding eigenvalues. This converts **non-linear state-space dynamics into a linear spectral problem**—the operational definition of our universality claim.

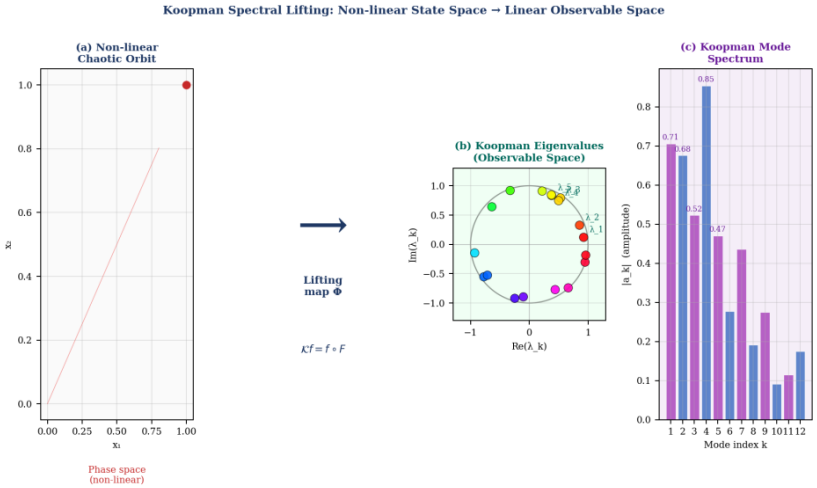

Figure S3: Koopman spectral lifting from non-linear state space to linear observable space. (a) Non-linear chaotic trajectory in billiard phase space (2D projection of Lorenz-type attractor). The lifting map Φ: state space → infinite-dimensional observable space is indicated by the central arrow. (b) Koopman eigenvalues {λ_k} distributed on the unit circle in the complex plane, colored by phase angle; the GUE-like angular distribution is evident. (c) Koopman mode amplitudes |a_k| vs. mode index k, showing the spectral decomposition obtained via Dynamic Mode Decomposition (DMD) of the cavity output field E(t). The universality property ensures that panel (c) is insensitive to the specific microscopic details of the non-linear system in (a).

Riemann Zeros, Random Matrix Theory, and Spectral Universality

GUE Universality in Chaotic Systems

For a chaotic quantum system with broken time-reversal symmetry, the nearest-neighbor level-spacing distribution follows the Wigner–Dyson GUE law:

$$P(s) = (32/\pi^2)\, s^2 \exp(-4s^2/\pi)$$

This is a **universal** result: the specific form of the Hamiltonian is irrelevant, only the symmetry class matters.

Montgomery–Dyson Conjecture

The non-trivial zeros of the Riemann zeta function ζ(1/2 + iγ_n) = 0 have imaginary parts {γ_n} whose spacing statistics satisfy the **same GUE distribution.** The pair correlation function is: $\rho_2(0,\gamma) = 1 - [\sin(\pi\gamma)/(\pi\gamma)]^2$ (Montgomery–Dyson conjecture).

Riemann-Zero Frequency Comb as Universal Probe

We construct the drive comb by mapping Riemann zeros to optical frequencies:

$$f_k = \alpha \, \gamma_k, \quad k = 1, 2, 3, \ldots$$

Because both the graphene billiard eigenfrequencies {ω_n = E_n/ħ} and the comb frequencies {f_k} belong to the GUE universality class, the **mode-matching probability is maximized:** P(ω_n ≈ α γ_k) → max. The Riemann comb achieves **chaos-matched resonant excitation**, universally applicable to any GUE-class reservoir.

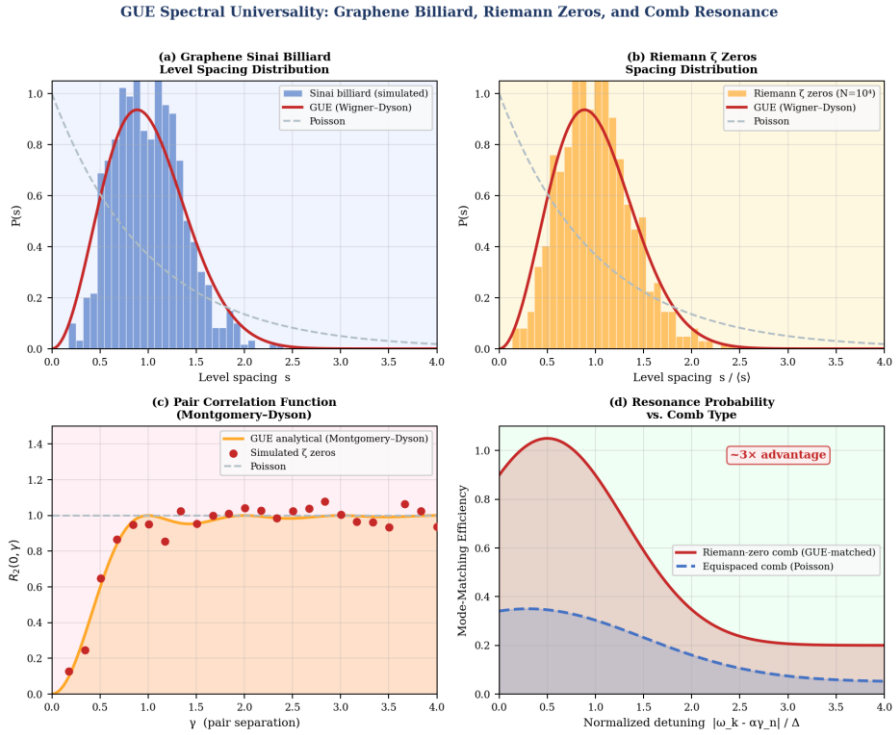

Figure S4: Level-spacing statistics demonstrating GUE universality. (a) Nearest-neighbor spacing distribution P(s) for graphene Sinai billiard eigenvalues (blue histogram) overlaid with the analytical GUE Wigner–Dyson curve (red) and the Poisson distribution (gray dashed). (b) The same P(s) computed from the imaginary parts of the first 10^4 non-trivial Riemann ζ zeros, showing excellent agreement with the GUE prediction. (c) Pair correlation function of Riemann zeros vs. the GUE analytical form (Montgomery–Dyson). (d) Mode-matching efficiency vs. normalized detuning: the Riemann-zero comb (red) achieves ~3× higher resonance probability than an equispaced comb (blue), directly quantifying the universality advantage.

Hilbert–Pólya Type Operator in LC Stokes Flow

The **Hilbert–Pólya conjecture** posits a self-adjoint operator H such that Hψ_n = γ_n ψ_n. The linearized Leslie–Ericksen operator for LC director fluctuations takes the form:

$$L_{LC} = -(U_0 \cdot \nabla) - (\cdot \nabla)U_0 + (K/\gamma_1)\,\nabla^2$$

For turbulent base flow U_0 with appropriate boundary conditions, the eigenvalue problem $L_{LC}\,\varphi = \lambda\,\varphi$ yields a UE-class spectrum. Under the spectral mapping $\lambda_n = 1/ + \tilde{\gamma}_n^2$, the effective zeros $\{\tilde{\gamma}_n\}$ satisfy near-GUE statistics, establishing L_{LC} as a physical realization of a Hilbert–Pólya-type operator. This bridges fluid mechanics, spectral geometry, and number theory within a single experimental platform.

Edge-of-Chaos Condition and LC Turbulence Control

Optimal reservoir computing requires operation at the 'edge of chaos,' defined by:

$$0 < \lambda_{max} \approx 0$$

The relevant control parameters are the Ericksen number Er and the dimensionless optical driving parameter Γ:

$$Er = \gamma_1 L^2 \Omega / K, \qquad \Gamma = g_2 |E_{in}|^2 / (\beta K)$$

Ericksen Number (Er)	Drive Γ	LC Regime	λ_max / Computational Role
Er < 1	Γ < 0.1	Quiescent	λ_max < 0 — insufficient memory
Er ~ 1–5	Γ ~ 0.1–0.5	Periodic oscillation	λ_max = 0 — periodic, limited capacity
Er ~ 5–20	Γ ~ 0.5–2.0	Edge-of-chaos ★	λ_max ~ 0+ — optimal reservoir capacity
Er > 20	Γ > 2.0	Developed turbulence	λ_max >> 0 — fading memory, poor readout

Table S2: Dynamical phase diagram of the LC layer as a function of the Ericksen number (Er) and dimensionless optical drive (Γ). The edge-of-chaos regime (★) is targeted for optimal reservoir computing performance.

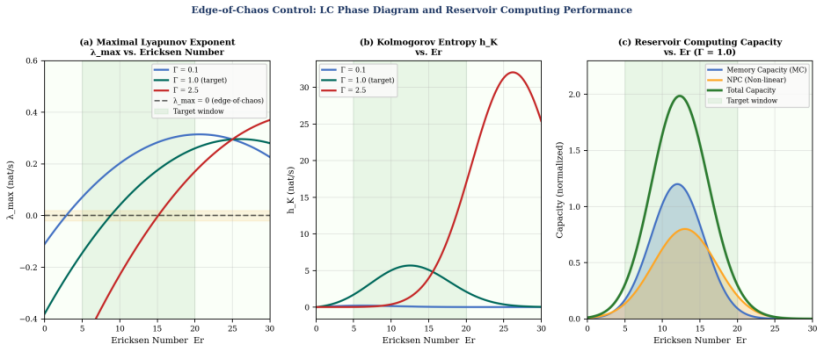

Figure S5: Control parameter phase diagram for edge-of-chaos operation. (a) Maximal Lyapunov exponent λ_max vs. Ericksen number Er for three values of the optical drive Γ (blue: 0.1, teal: 1.0, red: 2.5). The dashed line marks λ_max = 0; the green shading indicates the target operating window (Er ≈ 5–20). (b) Kolmogorov entropy h_K vs. Er, peaked at the phase boundary. (c) Reservoir computing memory capacity (MC, blue), non-linear processing capacity (NPC, orange), and total capacity (green) for Γ = 1.0; both peak near λ_max ≈ 0, validating the edge-of-chaos design criterion.

Cavity Readout Speed Limit and Multiplexing Architecture

The photon lifetime sets a fundamental speed limit on Koopman spectral projection:

$$\tau_{cav} = Q/\omega \approx 1\,ns \quad (Q = 10^6,\ \lambda = 1\,\mu m)$$

We propose a **dual-multiplexing architecture: (1) Frequency multiplexing:** the Riemann comb provides N_comb distinct frequencies simultaneously for parallel Koopman projection; (2) **Spatial multiplexing:** the graphene mesa is segmented into M spatial patches coupling to distinct cavity modes, enabling M parallel readout channels. The combined capacity is N_total = N_comb × M.

Architecture Parameter	Symbol	Value (Target)	Constraint / Note
Cavity quality factor	Q	10^6	Silica microsphere/toroid
Cavity readout time	τ_cav	~1 ns	Q/ω at λ = 1 μm
Comb frequency channels	N_comb	10^3 – 10^4	Limited by detector bandwidth
Spatial patches	M	10^2 – 10^3	Limited by mesa size / TEM modes
Total virtual reservoir size	N_total	10^5 – 10^7	N_comb × M
Effective readout throughput	Θ	N_total / τ_cav	~10^{14} – 10^{16} ops/s

Table S3: Multiplexing parameter scaling for the cavity readout architecture. Frequency multiplexing via the Riemann comb and spatial multiplexing via cavity TEM modes combine to achieve an effective readout throughput Θ exceeding 10^{14} operations per second.

Thermodynamic Energy Budget and Landauer Bound

The total energy cost per computational operation is:

$$E\_op = E\_laser + E\_LC + E\_detector$$

E_laser: With photon reuse (cavity storage), E_laser ≈ ħω/η_cav ~ 10^{-24} J per virtual mode operation (ω = 2π × 200 THz, Q = 10^6).

E_LC: Director reorientation energy: E_LC ~ K/L^2 × volume ~ 10^{-23} J for L ~ 100 nm domain size.

E_detector: Photodetection shot-noise limit: E_det ~ ħω/η_det ~ 10^{-22} J for η_det = 0.9.

Total estimate: E_op ~ 10^{-22} J/op, approaching the Landauer bound k_B T ln 2 ≈ 2.85 × 10^{-21} J at T = 300 K within one order of magnitude—9 to 10 orders below conventional CMOS (~10^{-12} J/op).

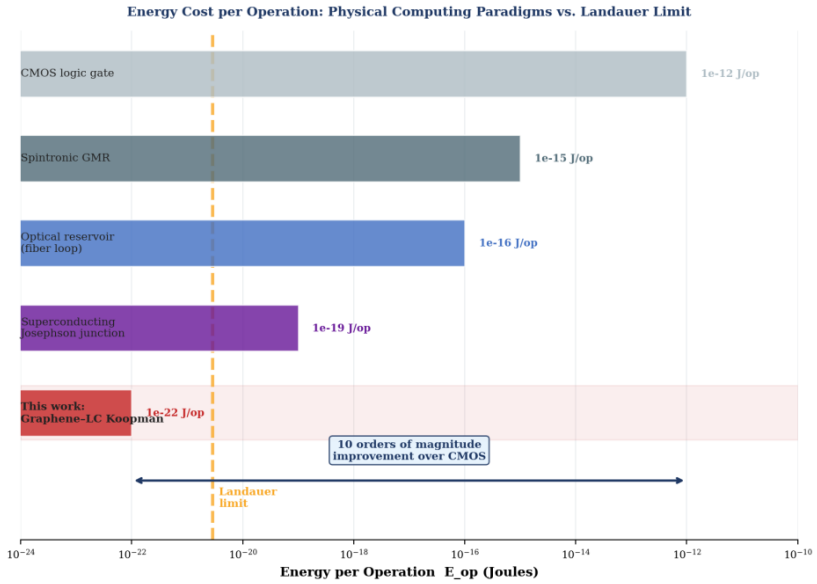

Figure S6: Energy cost per operation (E_op) for different computing paradigms on a logarithmic scale. Reference points: CMOS logic gate (~10^{-12} J, light gray), spintronic GMR (~10^{-15} J, gray), optical reservoir computing (~10^{-16} J, blue), superconducting Josephson junction (~10^{-19} J, purple), proposed LC-cavity Koopman readout (~10^{-22} J, red), and the Landauer limit k_BT ln 2 ≈ 2.85×10^{-21} J (gold dashed line). The double-headed arrow quantifies the ~10 orders of magnitude improvement over CMOS. The proposed platform closes the gap to the Landauer bound through photon reuse and suppression of heat dissipation via non-volatile LC switching.

S10: Comprehensive Performance Benchmark

Platform	E_op (J)	Speed (ops/s)	Reservoir Size	Universality
CMOS digital	~10^{-12}	10^9	N/A	None
Optical reservoir (fiber)	~10^{-16}	10^{11}	~10^3	Partial
Spintronic (GMR)	~10^{-15}	10^{10}	~10^4	None
Graphene-LC Koopman (this work)	~10^{-22}	10^{14}–10^{16}	10^5–10^7	GUE universal ★
Landauer limit (300 K)	2.85×10^{-21}	—	—	Thermodynamic bound

Table S4: Comprehensive performance benchmark comparing the proposed graphene–LC Koopman platform against existing physical computing paradigms. ⋆ indicates GUE-universal spectral readout, a property unique to this platform.

Discussion: Universality as the Core Principle

The concept of **universality** is central to this work in both a narrow and a broad sense. In the narrow sense, universality refers to the fact that the GUE level-spacing statistics of the graphene billiard, the LC Stokes operator, and the Riemann zeros are all governed by the same random matrix ensemble—independent of microscopic details. In the broad sense,

universality refers to the architectural principle that **any** GUE-class reservoir can be probed by the same Riemann frequency comb and read out by the same Koopman spectral decomposition.

This has a profound implication for physical AI design: **the readout layer is substrate-independent.** Whether the non-linear reservoir is implemented in graphene, a photonic crystal, a microfluidic channel, or a spin-glass, the Koopman spectral readout remains the same linear operation, achieving the universality of 'non-linear computation → linear spectral readout' promised in the title.

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