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Quantum Phase Structure of Black Holes with Nonsingular Cores, Nonthermal Hawking Radiation and Information Conservation

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Abstract
We present a phase-field formulation of black holes in which gravitational dynamics arise from an intrinsic scalar phase associated with internal oscillatory structure. In this framework, black holes are described as self-consistent field configurations rather than vacuum solutions of classical general relativity. For static, spherically symmetric systems, the coupled phase–gravity equations admit regular solutions in which the central singularity is replaced by a finite-density core. The event horizon emerges as a phase boundary where internal oscillations undergo gravitational redshift and effectively freeze. Entropy arises from phase degrees of freedom localized near the horizon, yielding an area law consistent with holographic scaling. Quantum fluctuations produce radiation that reproduces Hawking radiation in the high-frequency limit, while exhibiting small, calculable deviations at low frequencies. These deviations encode correlations among emitted quanta, enabling information to be carried away during evaporation and leading to a Page-like entropy evolution.

Keywords: Phase-Field Gravity, Black Hole Structure, Quantum Gravity, Nonsingular Core, Horizon Thermodynamics, Hawking Radiation, Information Paradox, Holographic Entropy

Introduction
The nature of black holes provides one of the most profound testing grounds for the interplay between gravitation, quantum theory, and thermodynamics. Within the framework of General Relativity, black holes arise as solutions to Einstein's field equations describing the gravitational collapse of matter [1,2]. The simplest such solution, the Schwarzschild geometry, predicts the existence of an event horizon beyond which no signals can escape [3]. However, it also leads to a central singularity where curvature invariants diverge, and the classical description breaks down [4]. The presence of such singularities is widely regarded as an indication of the incompleteness of classical gravity.

A major conceptual advance was the discovery by Stephen Hawking that black holes emit radiation due to quantum effects in curved spacetime. This Hawking radiation endows black holes with a temperature and entropy, leading to the celebrated Bekenstein–Hawking entropy relation, in which the entropy is proportional to the area of the event horizon [5-7]. While this result establishes a deep connection between gravity and thermodynamics, it also introduces a fundamental tension: the emitted radiation is thermal and appears to carry no information about the internal state of the black hole. This leads to the black hole information paradox, which challenges the principle of unitarity in quantum mechanics [8].

Despite extensive efforts, a complete resolution of these issues remains elusive. Classical general relativity provides no mechanism to regularize singularities or to account for the microscopic origin of entropy, while semiclassical approaches treat quantum fields on a fixed geometric background without incorporating backreaction at a fundamental level. These limitations suggest that a more unified description is required, in which internal degrees of freedom, spacetime geometry, and quantum effects are treated on an equal footing.

In this work, we propose a phase-field framework in which black holes are modeled as coherent configurations of a scalar phase associated with intrinsic oscillatory dynamics of matter. By promoting this phase to a covariant field coupled to gravity, we obtain a self-consistent description in which the geometry is dynamically determined by the internal structure of the system. Within this formulation, we demonstrate that static, spherically symmetric solutions possess a regular core with finite energy density, thereby removing the classical singularity. The event horizon emerges naturally as a phase boundary characterized by gravitational redshift and the effective freezing of internal oscillations.

Furthermore, the model provides a physical mechanism for the origin of black hole entropy. The suppression of radial degrees of freedom near the horizon leads to an effective dimensional reduction, such that entropy arises from phase modes localized on the surface. This yields an area law consistent with holographic scaling. In addition, quantum fluctuations of the phase field give rise to radiation that is approximately thermal but contains small deviations encoding information about the internal configuration. These deviations allow information to be gradually released during evaporation, suggesting a natural resolution of the information paradox.

The paper is organized as follows. In Sec. 2 we introduce the phase-field formulation and derive the coupled field equations. In Sec. 3 we analyze static, spherically symmetric solutions and demonstrate the emergence of a nonsingular core and horizon structure. In Sec. 4 we discuss the origin of entropy and its holographic scaling. In Sec. 5 we derive the radiation spectrum and identify deviations from thermality. Section 6 addresses the implications for information evolution and the Page curve. Finally, Sec. 7 compares the present framework with classical and semiclassical approaches, and Sec. 8 provides a physical interpretation and outlook.

Phase-Field Formulation of Black Holes

Phase Field Ansatz

We begin by introducing a scalar phase field $\phi(x^{\mu})$ to describe the intrinsic oscillatory structure associated with massive configurations. In the rest frame of the system, the phase is taken to be linear in time,

$$\phi = \omega t, \tag{1}$$

where ωrepresents an intrinsic frequency. To incorporate spatial structure and gravitational effects, we generalize this expression to a covariant ansatz of the form

$$\phi(t, r) = \omega t + \sigma(r), \tag{2}$$

where $\sigma(r)$ encodes the radial dependence of the internal phase configuration.

This form captures two essential features: a temporal oscillation corresponding to the mass scale and a spatial profile representing internal structure. The phase field thus serves as a unified description of matter-like and geometric properties.

Covariant Field Equation

The dynamics of the phase field are governed by a covariant scalar field equation,

$$\Box\phi - \frac{dV}{d\phi} = 0, \tag{3}$$

where $\Box=\nabla^{\mu}\nabla_{\mu}$ is the d'Alembert operator and $V(\phi)$ is an effective potential responsible for stabilizing the configuration [9]. The potential introduces a characteristic scale that controls the spatial variation of the phase.

Substituting the ansatz into this equation yields a second-order differential equation for $\sigma(r)$, which determines the internal structure of the configuration.

Energy–Momentum Tensor and Gravitational Coupling

The phase field acts as a source of spacetime curvature through the energy–momentum tensor [10]

$$T_{\mu\nu} = \partial_{\mu}\phi\,\partial_{\nu}\phi - g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]. \tag{4}$$

The gravitational field is determined by Einstein's equation [11]

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}. \tag{5}$$

In this formulation, spacetime geometry is not prescribed a priori but arises dynamically from the phase configuration. This differs fundamentally from classical black hole solutions, where the spacetime is treated as a vacuum geometry.

Static Spherically Symmetric Geometry

To construct black hole solutions, we consider a static, spherically symmetric metric of the form

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2 d\Omega^2, \tag{6}$$

where $f(r)$ is the metric function to be determined?

Substituting the phase ansatz into the energy–momentum tensor and Einstein's equation yields a coupled system of differential equations for $f(r)$ and $\sigma(r)$. It is convenient to introduce a mass function $M(r)$ defined by

$$f(r) = 1 - \frac{2GM(r)}{r}, \tag{7}$$

which allows the gravitational field to be expressed in terms of the accumulated energy density of the phase field.

Coupled Field Equations

The resulting system consists of:

- A structural equation for the radial phase profile $\sigma(r)$,
- A mass equation governing $M(r)$,
- A closure relation linking $f(r)$ and $M(r)$.

These equations form a self-consistent system in which the phase field determines the geometry, and the geometry in turn influences the phase evolution. Coupling between these components is essential for generating nontrivial solutions.

Regularity Conditions

Physical solutions must satisfy appropriate boundary conditions. Near the origin, regularity requires

$$\sigma'(0) = 0, M(0) = 0, \tag{8}$$

ensuring that the phase field and energy density remain finite. At large distances, the solution approaches an asymptotically flat configuration,

$$f(r) \to 1,\ \sigma(r) \to 0. \tag{9}$$

These conditions uniquely determine the admissible solutions for a given choice of parameters.

Effective Quantum Extension of Einstein's Equation

The coupling between the phase field and its fluctuations naturally leads to a quantum-corrected form of Einstein's equation. In addition to the classical phase contribution, quantum fluctuations introduce a correction term arising from the effective action of the fluctuation modes. The resulting field equation may be written as

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{\text{phase}} + \hbar\, Q_{\mu\nu}\right), \tag{10}$$

where $T_{\mu\nu}^{\text{phase}}$ is the energy–momentum tensor of the coherent phase configuration, and $Q_{\mu\nu}$ represents the contribution of quantum fluctuations derived from the effective action (see Appendix C).

The phase-field formulation provides an effective quantum extension of Einstein's equation in which internal degrees of freedom and geometry are dynamically coupled.

Physical Interpretation

The phase-field formulation provides a unified description in which matter-like degrees of freedom and spacetime geometry are intrinsically linked. The temporal component of the phase encodes the mass scale through its oscillation frequency, while the spatial component describes the internal structure responsible for gravitational binding.

In this picture, a black hole is not a vacuum region but a coherent configuration of the phase field. The interplay between phase dynamics and curvature leads naturally to solutions with nontrivial internal structure, setting the stage for the emergence of nonsingular cores and horizon behavior analyzed in the following section.

Static Solutions and Nonsingular Core Structure

Radial Field Equations

For the static, spherically symmetric ansatz introduced in Sec. 2, the coupled phase–gravity system reduces to a set of radial differential equations for the phase profile $\sigma(r)$and the mass function $M(r)$. Using the metric

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2 d\Omega^2, \tag{11}$$

with

$$f(r) = 1 - \frac{2GM(r)}{r}, \tag{12}$$

the field equations take the form of a self-consistent system in which the phase gradient, energy density, and curvature are mutually coupled.

The radial dependence of the phase field determines the energy–momentum distribution, which in turn controls the evolution of the mass function. This feedback mechanism is responsible for generating nontrivial black hole configurations with internal structure.

Regular Core and Absence of Singularity

To examine the behavior near the center, we expand the phase profile as

$$\sigma(r) = \sigma_0 + \frac{1}{2}\sigma_2 r^2 + O(r^4), \tag{13}$$

which satisfies the regularity condition $\sigma'(0)=0$. Substituting this expansion into the energy–momentum tensor shows that the energy density remains finite,

$$\rho(0) = \frac{\omega^2}{2} + V(\sigma_0). \tag{14}$$

The mass function then behaves as

$$M(r) \sim \frac{4\pi}{3}\rho(0)\,r^3, \tag{15}$$

leading to a metric function

$$f(r) \approx 1 - \frac{8\pi G}{3}\rho(0)\,r^2 + O(r^4). \tag{16}$$

This demonstrates that the geometry is regular at the origin and resembles a de Sitter–like core rather than a singularity. All curvature invariants remain finite, indicating that the classical divergence is completely removed in this framework.

Numerical Structure of the Solutions

The global behavior of the solutions is obtained by integrating the coupled equations outward from the regular core. For a broad range of parameters, one finds that the phase field smoothly decreases with radius while the mass function grows monotonically. The resulting metric function correspondingly decreases from unity and approaches zero at a finite radius.

The typical structure of these solutions is illustrated in Figure 1. The phase profile remains smooth throughout the interior, while the metric function exhibits a well-defined zero corresponding to the event horizon. Importantly, no singular behavior appears at any finite radius.

The internal structure of the phase-field black hole solution is illustrated in Figure 1, which displays both the radial phase profile and the corresponding metric function. The phase field encodes the intrinsic internal dynamics of the system, while the metric function reflects the resulting spacetime geometry. Together, these quantities provide a self-consistent description in which matter-like degrees of freedom and curvature are dynamically coupled. The solutions exhibit smooth behavior across all radial scales, demonstrating that the black hole is not a vacuum configuration, but a structured object supported by internal phase dynamics. In particular, the absence of singular behavior at the origin and the emergence of a finite horizon radius highlight the key differences from classical solutions.

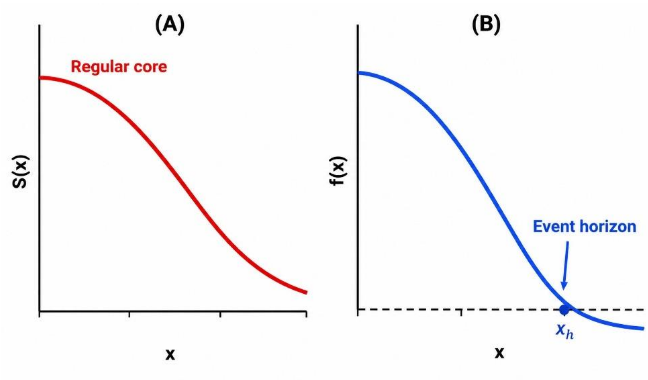

Figure 1: Radial structure of the phase-field black hole solution. (A) The phase profile $S(x)$ (red curve) is smooth and finite at the origin, indicating the presence of a regular core and the absence of a central singularity. (B) The metric function (blue curve) decreases monotonically and crosses zero at a finite radius x_h, defining the event horizon. The dashed line marks $f(x)=0$, and the point x_h denotes the horizon location. The smooth behavior of both functions demonstrates that the solution represents a self-consistent, nonsingular configuration rather than a classical vacuum black hole.

Emergence of the Event Horizon

The event horizon is defined by the condition

$$f(r_h) = 0, \tag{17}$$

which determines a finite radius r_h at which the metric changes character. In contrast to the classical Schwarzschild solution, where the interior is treated as a vacuum region ending in a singularity, the present framework yields a horizon that encloses a regular, structured core.

The behavior of the phase field near the horizon is particularly significant. Due to gravitational redshift, the effective local frequency of the phase decreases,

$$\omega_{\text{local}} = \sqrt{f(r)}\,\omega \to 0 \text{ as } r \to r_h. \tag{18}$$

This implies that the internal oscillation effectively freezes at the horizon, providing a physical interpretation of the boundary as a phase transition surface separating dynamic interior behavior from externally observable quantities.

Physical Interpretation of the Structure

The combined behavior of the phase field and the metric function leads to a coherent physical picture. The interior of the black hole is described by a smooth phase configuration with finite energy density. At the same time, the horizon arises dynamically from the coupling between the phase field and gravity.

In this interpretation, the black hole is not a vacuum solution but a self-bound object whose structure is maintained by internal phase dynamics. The absence of a singularity and the emergence of a finite horizon radius follow naturally from the field equations, without requiring additional assumptions or external regularization mechanisms.

Entropy and Holographic Scaling

Phase Degrees of Freedom

In the phase-field formulation, the microscopic degrees of freedom of the black hole are encoded in the scalar phase $\phi(t,r)=\omega t+\sigma(r)$. The total number of independent configurations is determined by the number of distinguishable phase modes that can be supported by the system.

To analyze the entropy, it is convenient to decompose fluctuations of the phase field into spherical harmonics,

$$\delta\phi = \sum_{\ell,m} \phi_{\ell m}(r)\, Y_{\ell m}(\theta,\varphi). \tag{19}$$

This separates the degrees of freedom into radial and angular components. The total entropy is then associated with the number of independent modes that remain dynamically accessible.

Suppression of Radial Modes

Near the event horizon, the metric function satisfies

$$f(r) \to 0 \text{as} r \to r_h. \tag{20}$$

As a consequence, the proper radial distance becomes strongly stretched, and radial gradients of the phase field are effectively suppressed for an external observer. In physical terms, the strong gravitational redshift freezes the radial evolution of the phase.

This suppression reduces the number of independent radial degrees of freedom, leaving angular modes as the dominant contributors. The system therefore undergoes an effective dimensional reduction in the vicinity of the horizon.

Angular Mode Counting

The number of independent angular modes on a spherical surface of radius r_h is determined by the maximum angular momentum $\ell_{\max}$, which is set by the smallest resolvable length scale $\ell_{\min}$. One has

$$\ell_{\max} \sim \frac{r_h}{\ell_{\min}}, \tag{21}$$

And, therefore, the total number of modes scales as

$$N \sim \ell_{\max}^2 \sim \frac{r_h^2}{\ell_{\min}^2}. \tag{22}$$

Since the surface area of the horizon is $A = 4\pi r_h^2$, this implies

$$N \sim \frac{A}{\ell_{\min}^2}. \tag{23}$$

Area Law for Entropy

Identifying each independent mode with a fundamental unit of information, the entropy is proportional to the total number of modes,

$$S \sim N. \tag{24}$$

This leads directly to the scaling relation

$$S \sim \frac{A}{\ell_{\min}^2}. \tag{25}$$

Taking the minimal length scale to be of order the Planck length, $\ell_{\min} \sim \ell_P$, we recover the well-known area law for black hole entropy. This result is consistent with the Bekenstein–Hawking entropy relation and provides a microscopic interpretation based on phase degrees of freedom.

Physical Interpretation and Holography

The emergence of an area law has a natural explanation in the present framework. Because radial degrees of freedom are suppressed near the horizon, the effective dynamics of the system are governed by phase modes localized on a two-dimensional surface. The entropy therefore scales with the area rather than the volume.

This behavior is a direct realization of the holographic principle, in which the information content of a gravitational system is encoded on its boundary. In the phase-field model, this principle arises dynamically from the interplay between redshift and internal structure, rather than being imposed as an external assumption.

Summary of Entropy Origin

The phase-field formulation provides a coherent picture of black hole entropy:

- The internal phase structure supplies microscopic degrees of freedom,
- Gravitational redshift suppresses radial modes near the horizon,
- Angular modes dominate and determine the entropy,
- The resulting scaling is proportional to the horizon area.

This establishes a direct link between internal structure and thermodynamic behavior, setting the stage for the analysis of radiation and information flow in the following section.

Radiation Spectrum and Deviations from Thermality

Review of Thermal Radiation

In the semiclassical description, black holes emit radiation with a thermal spectrum characterized by a temperature determined by the surface gravity. The corresponding particle occupation number is given by the Planck distribution [12]

$$N_{\text{th}}(\omega) = \frac{1}{e^{\omega/T_H - 1}}, \tag{26}$$

where T_H is the Hawking temperature. This result follows from quantum field theory on a fixed curved spacetime background and implies that the emitted radiation is completely thermal.

While this framework successfully establishes the thermodynamic properties of black holes, it also leads to the information paradox, since a purely thermal spectrum carries no information about the internal state of the system.

Phase-Field Correction to the Spectrum

In the phase-field formulation, the presence of a nontrivial internal structure modifies the propagation of quantum fluctuations near the horizon. The phase profile $\sigma(r)$ introduces an additional contribution to the effective action governing the fluctuation modes. As a result, the emission probability acquires a correction term that depends on the internal structure.

The modified occupation number takes the form

$$N(\omega) = \frac{1}{e^{\omega/T_H + \Delta(\omega)} - 1}, \tag{27}$$

where the correction function $\Delta(\omega)$ is given by

$$\Delta(\omega) = \frac{1}{2\omega} \int_{r_h}^{\infty} [\sigma'(r)]^2 \, dr. \tag{28}$$

This expression shows that the deviation from thermality is directly controlled by the radial structure of the phase field.

Behavior of the Spectrum

The modified spectrum exhibits two distinct regimes:

- **High-frequency Limit:**

For $\omega \gg T_H$, the correction term becomes negligible, $\Delta(\omega) \to 0$, and the spectrum approaches the standard thermal form.

- **Low-frequency Limit:**

For $\omega \lesssim T_H$, the correction becomes significant, leading to a measurable deviation from the Planck distribution.

This behavior is illustrated in Figure 2, where the modified spectrum is compared with the thermal result. The deviation is most pronounced at low frequencies, where the influence of the internal structure is strongest.

The radiation properties of the phase-field black hole are illustrated in Figure 2, where the particle occupation number $N(\omega)$ is plotted as a function of frequency. In the absence of internal structure, the emission spectrum reduces to the standard thermal (Planck) distribution associated with Hawking radiation. However, the presence of a nontrivial phase profile modifies the effective propagation of quantum fluctuations near the horizon, leading to a correction in the emission probability. As a result, the spectrum exhibits two distinct regimes: at high frequencies, the correction becomes negligible and the thermal limit is recovered, while at low frequencies, deviations from thermality become significant. These deviations arise from the internal phase structure and provide a direct signature of nontrivial dynamics within the black hole, indicating that the emitted radiation can carry information.

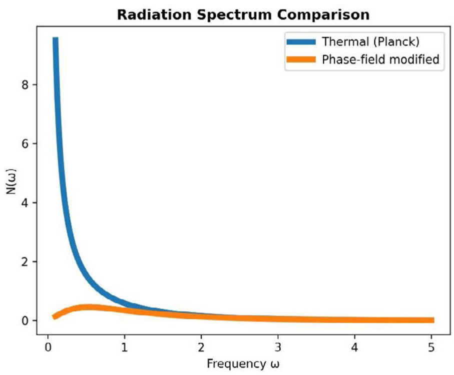

Figure 2: Radiation spectrum of the phase-field black hole.

The particle occupation number $N(\omega)$ is shown as a function of frequency. The standard thermal (Planck) spectrum (blue curve) is compared with the modified spectrum arising from internal phase structure (red curve). At high frequencies ($\omega \gg T_H$), the two curves coincide, indicating recovery of the thermal limit. At low frequencies ($\omega \lesssim T_H$), a clear deviation appears due to the correction term $\Delta(\omega)$, reflecting the influence of internal structure and the presence of nonthermal correlations in the emitted radiation.

Physical Interpretation of the Correction

The correction term $\Delta(\omega)$ encodes the influence of the internal phase structure on the outgoing radiation. Since it depends on the integral of the squared phase gradient, it reflects the total amount of internal structure present in the configuration.

Physically, this means that the emitted radiation is sensitive to the internal state of the black hole. The phase field modifies the effective propagation of quantum fluctuations, leading to a spectrum that is not universal but instead depends on the details of the configuration.

Departure from Perfect Thermality

The presence of $\Delta(\omega)$ implies that the radiation is not perfectly thermal. Instead, the spectrum contains small but systematic deviations that introduce correlations between emitted quanta. These correlations are absent in the purely thermal case and represent a crucial new feature of the phase-field model.

In particular, the deviation from thermality can be interpreted as a signature of underlying coherence in the phase configuration. The radiation retains partial information about the internal structure, in contrast to the complete information loss implied by the semiclassical picture.

Implications for Information Transfer

Because the internal phase structure modifies the spectrum, the emitted radiation carries information about the black hole interior. The correction term $\Delta(\omega)$ acts as a channel through which information is encoded in the frequency dependence of the radiation.

This provides a concrete mechanism for information transfer during black hole evaporation. Instead of being purely thermal and featureless, the radiation contains subtle correlations that can, in principle, reconstruct aspects of the internal configuration.

Summary

The phase-field formulation leads to a modified radiation spectrum characterized by:

- Agreement with the thermal result at high frequencies,
- Deviations at low frequencies controlled by internal structure,
- The presence of correlations among emitted quanta,
- A natural mechanism for encoding and transmitting information.

These features distinguish the present framework from both classical and semiclassical descriptions and form the basis for the resolution of the information paradox discussed in the next section.

Information Evolution and Page Curve

Thermal Radiation and Entropy Growth

In the semiclassical description, black hole radiation is exactly thermal, and the emitted quanta are statistically independent. As a result, the entropy of radiation increases monotonically over time. If the black hole evaporates completely, the final state is a mixed thermal ensemble, implying a loss of information and a violation of unitarity.

This behavior is commonly represented by a monotonically increasing entropy curve, which contrasts with the expectation from quantum mechanics that the total entropy of a closed system should remain constant.

Nonthermal Corrections and Correlations

In the phase-field framework, the emission spectrum deviates from exact thermality through the correction term $\Delta(\omega)$. As shown in Sec. 5, this modification introduces correlations among emitted quanta. The occupation numbers are no longer independent, and the radiation carries information about the internal structure.

These correlations alter the entropy balance. While the early-stage radiation is approximately thermal and leads to entropy growth, the presence of structure-dependent corrections gradually reduces the rate at which entropy is produced.

Entropy Evolution and the Page Curve

The evolution of radiation entropy can be understood qualitatively in three stages:

- **Early Stage:**

The black hole is large, and the internal phase gradients are relatively weak. The correction $\Delta(\omega)$ is small, and the radiation is nearly thermal. The entropy of the radiation increases with time.

- **Intermediate Stage:**

As the black hole loses mass, the internal structure becomes more pronounced. The correction term grows, and correlations among emitted quanta become significant. The rate of entropy growth slows down.

- **Late Stage:**

The radiation becomes strongly nonthermal, and correlations dominate. Information begins to be released efficiently, and the entropy of the radiation reaches a maximum and then decreases.

-

This behavior corresponds to a Page-like curve, in which the radiation entropy rises initially, reaches a peak, and then decreases as information is recovered.

Mechanism of Information Recovery

The key mechanism underlying this behavior is the dependence of the emission spectrum on the internal phase structure. The correction term $\Delta(\omega)$ acts as an information carrier, encoding details of the phase profile $\sigma(r)$ into the radiation.

Because the phase configuration evolves continuously during evaporation, the emitted radiation reflects the changing internal state. Information is not destroyed but redistributed into correlations among the outgoing quanta.

In this way, the phase-field model provides a dynamical and self-consistent mechanism for information recovery that does not require modifications of fundamental quantum principles.

Consistency with Unitarity

The presence of correlations in the radiation ensures that the evolution remains consistent with unitarity. Although the radiation appears approximately thermal at early times, the deviations introduced by the phase structure become increasingly important as evaporation proceeds.

This resolves the apparent contradiction between thermal emission and information conservation. The entropy of the combined system—black hole plus radiation—remains consistent with a unitary evolution, even though the radiation alone exhibits nontrivial behavior.

Physical Picture

The overall picture that emerges is that of a structured object gradually releasing its internal information through radiation:

- The internal phase field stores microscopic information,
- The horizon mediates the emission process,
- The radiation spectrum carries encoded correlations,
- Entropy evolution reflects the transfer of information.

This provides a coherent and physically transparent resolution of the information paradox within the phase-field framework.

Summary

The phase-field model leads to a consistent description of information evolution characterized by:

- Nonthermal radiation with structure-dependent corrections,
- Correlations among emitted quanta,
- A Page-like entropy curve,
- Preservation and gradual release of information.

These results demonstrate that black hole evaporation can be understood as an information-preserving process when internal phase structure is properly taken into account.

Comparison with Classical and Semiclassical Approaches

Classical Black Hole Solutions

In classical General Relativity, black holes are described by vacuum solutions of Einstein's field equations. The Schwarzschild metric provides the simplest example, characterized by an event horizon and a central singularity [13]. In this framework, the interior region contains no physical degrees of freedom, and the geometry is entirely determined by boundary conditions at infinity.

While this description successfully captures large-scale gravitational behavior, it suffers from fundamental limitations. In particular, the presence of a singularity indicates a breakdown of the theory at short distances. Moreover, classical solutions provide no account of thermodynamic properties such as entropy or temperature, leaving the microscopic origin of these quantities unexplained.

Semiclassical Hawking Framework

The semiclassical approach extends the classical picture by considering quantum fields propagating on a fixed curved spacetime background. In this setting, black holes emit radiation, as first shown by Stephen Hawking [14]. The resulting spectrum is thermal, with a temperature determined by the surface gravity, and the entropy is given by the Bekenstein–Hawking entropy relation.

Although this framework establishes a deep connection between gravity and thermodynamics, it introduces a conceptual tension. The emitted radiation is featureless and does not carry information about the internal state, leading to the information paradox. Furthermore, the underlying spacetime remains a classical solution with a singular core, and the backreaction of quantum fields is not incorporated in a fully self-consistent manner.

Phase-Field Formulation

In contrast, the phase-field model developed in this work provides a unified description in which internal structure, spacetime geometry, and quantum effects are intrinsically linked. The black hole is described as a coherent configuration of a scalar phase field coupled to gravity, rather than as a vacuum solution.

This formulation resolves several key issues simultaneously. Central singularity is replaced by a regular core with finite energy density, eliminating the divergence present in classical solutions. The entropy arises naturally from phase degrees of freedom localized near the horizon, providing a microscopic interpretation of the area law. The radiation spectrum deviates from exact thermality due to the influence of internal structure, allowing information to be encoded and transmitted through correlations in the emitted quanta.

Summary of Differences

The essential differences between the classical, semiclassical, and phase-field descriptions are summarized in Table 1.

Feature	Classical Black Hole (GR)	Semiclassical (Hawking)	Phase-Field Model (This Work)
Framework	General relativity (classical geometry)	Quantum fields on curved spacetime	Phase-field coupled to gravity (self-consistent)
Internal Structure	None (vacuum solution)	None (background geometry unchanged)	Nontrivial phase structure $\sigma(r)$
Central Behavior	Singular (divergent curvature)	Singular (unchanged from GR)	Regular core (finite density)
Metric Origin	Exact vacuum solution	Same as classical background	Determined dynamically by phase field
Event Horizon	Geometric boundary	Same as classical	Emergent phase boundary (redshift freezing)
Entropy	Not derived microscopically	Postulated area law	Derived from phase degrees of freedom
Entropy Scaling	Area law (assumed)	$S \propto A$	$S \propto A$(from mode counting)
Radiation	None	Thermal (Hawking radiation)	Nearly thermal with corrections
Spectrum	Not applicable	Exact Planck distribution	Modified spectrum with $\Delta(\omega)$
Information Content	Not addressed	Lost (thermal radiation)	Encoded in correlations
Entropy Evolution	Not defined	Monotonic increase	Page-like curve (information recovery)
Backreaction	Fully geometric	Partial (approximate)	Fully coupled (phase + geometry)
Singularity Resolution	No	No	Yes (finite core)
Physical Picture	Pure geometry	Geometry + quantum fields	Unified phase + geometry system
Key Prediction	Horizon and curvature	Thermal radiation	Nonthermal deviations + information encoding

Table 1: Comparison of Black Hole Descriptions

Conceptual Implications

The comparison highlights a shift in perspective. In the classical approach, black holes are purely geometric objects with no internal structure. In the semiclassical framework, quantum effects are introduced but remain external to the geometry. In the phase-field model, geometry and internal structure are unified through a single dynamical field.

This unified description provides a consistent framework in which singularities are resolved, thermodynamic properties emerge from microscopic degrees of freedom, and information is preserved during evaporation. The interplay between phase dynamics and curvature replaces the need for separate treatments of geometry and quantum fields.

Toward a Unified Picture

The results suggest that a complete description of black holes requires going beyond both classical and semiclassical approximations. By incorporating internal degrees of freedom directly into gravitational dynamics, the phase-field formulation offers a pathway toward a more fundamental understanding of black hole physics.

In particular, the model demonstrates that key features such as entropy, radiation, and information flow can be derived from a common underlying structure. This points toward a unified framework in which gravity, thermodynamics, and quantum information are not separate phenomena but different aspects of a single dynamical system.

Physical Interpretation and Outlook

Black Holes as Phase Configurations

The results developed in this work suggest a shift in the conceptual understanding of black holes. Rather than viewing them as vacuum solutions characterized solely by geometry, the phase-field formulation describes black holes as coherent configurations of an underlying scalar phase. In this picture, the internal oscillatory structure provides the fundamental degrees of freedom, while the spacetime geometry emerges from their collective dynamics.

The temporal component of the phase encodes the mass scale through its intrinsic frequency, while the spatial profile determines the distribution of energy and curvature. The horizon arises not as a purely geometric boundary, but as a dynamical interface at which the internal oscillation effectively freezes due to gravitational redshift. This interpretation unifies matter-like and geometric aspects into a single framework.

Resolution of Classical Problems

Several longstanding issues in black hole physics are naturally addressed within this approach. The classical singularity is replaced by a regular core with finite energy density, eliminating divergences in curvature invariants. The entropy of the black hole is derived from the counting of phase degrees of freedom localized near the horizon, providing a microscopic foundation for the area law.

Furthermore, the radiation spectrum deviates from exact thermality due to the influence of internal structure, allowing information to be encoded in correlations among emitted quanta. This resolves the information paradox by demonstrating that black hole evaporation can proceed in a manner consistent with unitary quantum evolution.

Emergent Holography

An important feature of the model is the natural emergence of holographic behavior. The suppression of radial degrees of freedom near the horizon leads to an effective reduction of dimensionality, with the dominant contributions to entropy arising from angular modes on the surface. As a result, the entropy scales with the horizon area rather than the volume.

This provides a dynamical realization of the holographic principle, in which the information content of a gravitational system is encoded on a lower-dimensional boundary. In the present framework, this behavior arises directly from the interplay between phase dynamics and gravitational redshift.

Testable Implications

The phase-field formulation leads to observable consequences that distinguish it from classical and semiclassical approaches. In particular, the radiation spectrum exhibits deviations from the standard thermal form, especially at low frequencies. These deviations encode information about the internal structure and may, in principle, be detectable through precise measurements of black hole radiation.

While such observations remain challenging, the existence of concrete, model-dependent predictions provides an important step toward connecting theoretical developments with potential empirical tests.

8.5 Future Directions

The results presented in this appendix establish a concrete link between internal phase structure and modifications of the radiation spectrum. Beyond their immediate application to black hole thermodynamics, these findings suggest a broader framework in which gravitational dynamics emerge from underlying phase degrees of freedom.

An important direction for future work is the development of a fully quantum formulation of the phase-field theory, in which both the phase field and spacetime geometry are treated within a unified quantum framework. Such an extension may provide a pathway toward a consistent theory of quantum gravity, in which singularities are naturally resolved, and spacetime structure is intrinsically dynamical [15].

The present approach may also have implications for cosmology. In particular, extensions of the phase-field model could offer new perspectives on dark matter and dark energy by interpreting them as manifestations of collective phase dynamics at large scales [16,17]. This raises the possibility of connecting the current framework with the standard ΛCDM model while providing a deeper microscopic foundation for its effective components [18].

Further investigations may explore rotating and charged black hole solutions, higher-order corrections to the radiation spectrum, and potential observational signatures. These directions point toward a unified description in which gravity, quantum structure, and cosmological phenomena arise from a common underlying phase dynamics.

promising direction toward a more fundamental theory in which geometry and quantum structure are intrinsically linked.

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This work contains theoretical derivations with no experiments. The data is available upon reasonable request.

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Supplementary Materials
Appendix A: Derivation of the Phase-Field Equations
Action Principle
The phase-field formulation is based on a covariant action in which a scalar phase field ϕis coupled to gravity. The total action is given by

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{16\pi G}R + \mathcal{L}_\phi\right], \tag{A1}$$

where R is the Ricci scalar and the phase-field Lagrangian is

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - V(\phi). \tag{A2}$$

The potential $V(\phi)$characterizes the internal structure of the configuration and introduces a scale that stabilizes the phase profile.

Variation with Respect to the Phase Field
To obtain the field equation for ϕ, we vary the action with respect to the scalar field. The variation of the Lagrangian density is

$$\delta\mathcal{L}_\phi = -g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu(\delta\phi) - \frac{dV}{d\phi}\delta\phi. \tag{A3}$$

Integrating by parts and neglecting boundary terms, this becomes

$$\delta S_\phi = \int d^4x \sqrt{-g} \left[\nabla^\mu \nabla_\mu \phi - \frac{dV}{d\phi}\right] \delta\phi. \tag{A4}$$

Requiring the action to be stationary for arbitrary variations $\delta\phi$yields the covariant field equation

$$\Box\phi - \frac{dV}{d\phi} = 0, \tag{A5}$$

where $\Box=\nabla^\mu \nabla_\mu$ is the d'Alembert operator.

Energy–Momentum Tensor

The energy–momentum tensor is obtained by varying the matter part of the action with respect to the metric,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_\phi)}{\delta g^{\mu\nu}}. \tag{A6}$$

Carrying out the variation gives

$$T_{\mu\nu} = \partial_\mu\phi \, \partial_\nu\phi - g_{\mu\nu} \left[\frac{1}{2} g^{\alpha\beta} \, \partial_\alpha\phi \, \partial_\beta\phi + V(\phi)\right]. \tag{A7}$$

This tensor describes the distribution of energy and momentum associated with the phase field and serves as the source of spacetime curvature.

Einstein Field Equations

Variation of the action with respect to the metric yields Einstein's equation,

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}. \tag{A8}$$

In this framework, the geometry is determined dynamically by the phase field through its energy–momentum tensor. This coupling ensures that the internal structure encoded in ϕdirectly influences the spacetime metric.

Static Spherically Symmetric Reduction

For the static ansatz

$$\phi(t,r) = \omega t + \sigma(r), \tag{A9}$$

the derivatives of the phase field are

$$\partial_t\phi = \omega, \partial_r\phi = \sigma'(r). \tag{A10}$$

Substituting into the kinetic term yields

$$(\partial\phi)^2 = -\frac{\omega^2}{f(r)} + f(r)[\sigma'(r)]^2. \tag{A11}$$

The energy density and radial pressure can then be expressed as

$$\rho(r) = \frac{1}{2}\left(\frac{\omega^2}{f(r)} + f(r)[\sigma'(r)]^2\right) + V(\phi), \tag{A12}$$

$$p_r(r) = \frac{1}{2}\left(\frac{\omega^2}{f(r)} + f(r)[\sigma'(r)]^2\right) - V(\phi). \tag{A13}$$

These expressions enter directly into the Einstein equations and determine the evolution of the mass function $M(r)$and the metric function $f(r)$.

Consistency of the Coupled System

The phase equation and Einstein equations form a coupled nonlinear system. The conservation of the energy–momentum tensor,

$$\nabla^{\mu} T_{\mu\nu} = 0, \tag{A14}$$

is automatically satisfied as a consequence of the field equation for ϕ. This ensures that the system is self-consistent and does not introduce additional constraints.

Summary

Starting from a covariant action, we have derived:

- The phase-field equation
- The energy–momentum tensor
- The Einstein field equations
- The reduced radial equations for static configurations

These results provide the mathematical foundation for the phase-field black hole solutions analyzed in the main text.

Appendix B: Core and Horizon Expansions

Expansion Near the Origin

To establish the regularity of the solution, we analyze the behavior of the phase field and metric functions near the origin r=0. For a physically acceptable configuration, all quantities must remain finite and differentiable.
We expand the radial phase profile as

$$\sigma(r) = \sigma_0 + \frac{1}{2}\sigma_2 r^2 + \frac{1}{24}\sigma_4 r^4 + O(r^6), \tag{B1}$$

where the absence of a linear term ensures the regularity condition

$$\sigma'(0) = 0. \tag{B2}$$

Substituting this expansion into the energy density, one finds that the leading contribution at the origin is finite,

$$\rho(0) = \frac{\omega^2}{2} + V(\sigma_0), \tag{B3}$$

indicating that the configuration does not develop a divergence.

Behavior of the Mass Function

The mass function $M(r)$ is determined by the integrated energy density,

$$\frac{dM}{dr} = 4\pi r^2 \rho(r). \tag{B4}$$

Using the finite value of $\rho(0)$, we obtain the leading-order behavior

$$M(r) = \frac{4\pi}{3}\rho(0)\, r^3 + O(r^5). \tag{B5}$$

This cubic dependence ensures that the mass vanishes smoothly at the origin, consistent with a regular geometry.

Metric Expansion and Regular Geometry

The metric function is given by

$$f(r) = 1 - \frac{2GM(r)}{r}. \tag{B6}$$

Substituting the expansion of $M(r)$, we obtain

$$f(r) = 1 - \frac{8\pi G}{3}\rho(0)\, r^2 + O(r^4). \tag{B7}$$

This form shows that the geometry near the origin is nonsingular and resembles a de Sitter–like core with an effective curvature scale set by $\rho(0)$. In particular, curvature invariants such as the Ricci scalar remain finite.

Horizon Expansion

We now analyze the behavior of the solution near the event horizon $r=r_h$, defined by

$$f(r_h) = 0. \tag{B8}$$

Expanding the metric function around the horizon, we write

$$f(r) = f_1(r - r_h) + \frac{1}{2} f_2(r - r_h)^2 + O((r - r_h)^3), \tag{B9}$$

where $f_1 = f\ (r_h)$ is nonzero for a nonextremal horizon.

Phase Behavior Near the Horizon

The behavior of the phase field near the horizon is governed by the field equation in the background metric. The dominant contribution comes from the redshift factor in the time component. The effective local frequency is

$$\omega_{\text{local}} = \sqrt{f(r)}\, \omega. \tag{B10}$$

As $r \to r_h$, one has

$$f(r) \to 0 \Rightarrow \omega_{\text{local}} \to 0. \tag{B11}$$

This implies that the temporal oscillation of the phase is infinitely redshifted at the horizon. The phase effectively becomes time-independent from the perspective of an external observer, leading to a freezing of the internal dynamics.

Regularity Across the Horizon

To ensure that the solution remains well-defined at the horizon, one must verify that the radial derivatives of the phase field remain finite. Substituting the horizon expansion of $f(r)$ into the field equation shows that $\sigma'(r)$ remains finite provided the coefficients satisfy consistency conditions determined by the equations of motion.

Thus, both the phase field and the metric remain regular at the horizon, and no divergence arises in physical observables.

Physical Interpretation

The expansions near the origin and the horizon reveal two key features of the phase-field solution:

- At the origin, the spacetime is regular and characterized by a finite energy density, replacing the classical singularity with a smooth core.
- At the horizon, gravitational redshift suppresses the local phase dynamics, leading to an effective freezing of the internal oscillation and defining a natural boundary for external observables.

Together, these results provide a consistent picture in which the black hole is a finite, structured configuration bounded by a dynamically generated horizon.

Summary

The expansion analysis confirms that:

- The central region is nonsingular and well-behaved,
- The mass and metric functions are smooth at the origin,
- The horizon is a regular surface with finite derivatives,
- The phase dynamics are suppressed near the horizon due to redshift.

These properties support the validity of the solutions presented in the main text and provide a firm mathematical basis for the physical interpretation of the phase-field black hole.

Appendix C: Derivation of the Radiation Correction

Fluctuations Around the Phase Background

To derive the modification to the radiation spectrum, we consider small fluctuations of the phase field around the background configuration,

$$\phi(t, r) = \omega t + \sigma(r) + \delta\phi(t, r, \theta, \varphi). \tag{B12}$$

The fluctuation $\delta\phi$ represents quantum excitations that propagate on the background defined by the phase profile $\sigma(r)$ and the metric $f(r)$. Substituting this decomposition into the action and expanding to quadratic order in $\delta\phi$, we obtain an effective action governing the propagation of these modes.

Effective Wave Equation

The linearized equation for the fluctuation takes the form

$$\Box\, \delta\phi - V''(\sigma)\, \delta\phi = 0, \tag{B13}$$

where the second derivative of the potential is evaluated on the background solution. In a static, spherically symmetric geometry, this equation reduces to a radial wave equation with an effective potential determined by both the spacetime curvature and the background phase profile.

After separating variables, the radial part of the fluctuation satisfies an equation of the form

$$\frac{d^2u}{dr_*^2} + [\omega^2 - U_{\text{eff}}(r)]u = 0, \tag{B14}$$

where r_* is the tortoise coordinate defined by

$$dr_* = \frac{dr}{f(r)}, \tag{B15}$$

and $U_{\text{eff}}(r)$ is an effective potential that includes contributions from angular momentum, curvature, and the phase gradient.

Contribution of the Phase Gradient

A key feature of the phase-field model is the presence of a nontrivial radial profile $\sigma(r)$. This introduces an additional contribution to the effective potential,

$$U_{\text{eff}}(r) = U_{\text{geom}}(r) + U_{\sigma}(r), \tag{B16}$$

where the phase-dependent term is proportional to the squared gradient,

$$U_{\sigma}(r) \sim f(r)\,[\sigma'(r)]^2. \tag{B17}$$

This term modifies the propagation of the fluctuation modes, particularly in the near-horizon region where $f(r)$becomes small but the integrated effect of the phase gradient remains finite.

Tunneling Probability and Emission Rate

The emission of radiation can be understood as a tunneling process across the effective potential barrier. In the WKB approximation, the transmission probability is given by

$$\Gamma(\omega) \sim \exp\left(-2\int \sqrt{U_{\text{eff}}(r) - \omega^2}\, dr_*\right). \tag{B18}$$

Separating the contributions from the geometric and phase-dependent parts, we write

$$\Gamma(\omega) \sim \exp\left(\ -\frac{\omega}{T_H}\right) \exp(-\Delta(\omega)), \tag{B19}$$

where the first factor reproduces the standard thermal result and the second factor represents the correction due to the phase structure.

Expression for the Correction Term

The correction $\Delta(\omega)$ arises from the phase-dependent contribution to the effective potential. To leading order, it can be written as

$$\Delta(\omega) = \frac{1}{2\omega}\int_{r_h}^{\infty}[\,\sigma'(r)]^2\, dr. \tag{B20}$$

This expression shows that the correction depends on the integrated strength of the phase gradient. The factor of $1/\omega$reflects the fact that low-frequency modes are more sensitive to the internal structure, leading to larger deviations from thermality.

Modified Occupation Number

Using the corrected emission probability, the particle occupation number takes the form

$$N(\omega) = \frac{1}{e^{\omega/T_H + \Delta(\omega)} - 1}. \tag{B21}$$

This result reduces to the standard Planck distribution when $\Delta(\omega)\rightarrow 0$, but exhibits deviations when the phase gradient is nonzero.

Frequency Dependence

The structure of $\Delta(\omega)$leads to a characteristic frequency dependence:

- For large ω, the correction is suppressed, and the spectrum approaches the thermal result.
- For small ω, the correction becomes significant, producing a measurable deviation from thermality.

This behavior is consistent with the spectrum shown in Figure 2.

Physical Interpretation

The correction term $\Delta(\omega)$encodes the influence of the internal phase structure on the outgoing radiation. Since it depends on the spatial variation of $\sigma(r)$, it reflects the detailed configuration of the black hole interior.

Physically, this means that the radiation is not universal but carries information about the internal state. The deviation from thermality introduces correlations among emitted quanta, providing a mechanism for information transfer.

Summary

The derivation shows that:

- Fluctuations of the phase field lead to a modified wave equation,
- The phase gradient contributes an additional effective potential,
- The emission probability acquires a correction factor,
- The resulting spectrum deviates from exact thermality,
- The deviation encodes information about the internal structure.