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# Simultaneity at Earth, in the Planetary System, and in the Universe

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## Abstract
The International Astronomical Union (IAU) realized, that General Relativity Theory (GRT) with its postulates about coordinate systems is insufficient for space flight, [1,2]. And the IAU formulated the problem of finding a useful and Adequate Coordinate System (ACS) in astronomy. While a general method for the construction of an ACS, and a terrestrial test of the ACS, have been provided in a previous paper, analytic derivations of the ACS are derived here [3]. This provides a more founded and deeper understanding of the ACS, of its uniquely determined velocity, and of its origin. In GRT, in general, clocks are not synchronized and the concept of simultaneity cannot be applied. With help of the ACS, a synchronization procedure is derived step-by-step for all clocks at Earth, in the planetary system, for all clocks on celestial bodies stabilized by self-gravity, and for all clocks everywhere in the universe. With it, the concept of simultaneity can be applied everywhere in the universe. Based on that derived and founded ACS, tests, predictions and applications are derived for Terrestrial Time, Lunar time, time at the planets and pulsar time signals. It will be interesting and exciting to see the measurement results in the future.

**Keywords:** Relativity, Adequate Coordinate System, Frame, Observation, Clock, Sychronization, Simultaneity, Space Navigation, Earth, Planets, Motion

### Introduction
In nature, in space navigation, in geodesy, in the planetary system, and in the universe, the phenomena of time dilation and simultaneity are essential, see [4]. For instance, a gravitational potential difference between two points A and B causes a gravitational time dilation, [5,6]. Similarly, a speed v can cause a kinematic time dilation, see [7]. General Relativity Theory (GRT) provides an equation that describes both types of time dilation in a coherent manner with help of a very founded, robust and valuable scalar product $ds^2$ in four-dimensional spacetime, [6-9]. Though GRT provides the key equations for time dilation, GRT does not provide sufficient information about the Adequate Coordinate Systems (ACS), in which these equations predict an observation correctly. This ACS problem and corresponding solutions will be outlined below. Moreover, in nature, in space navigation, in geodesy, in the planetary system, and in the universe, the concept of simultaneity cannot be applied in general. Therefore, the simultaneity problem occurs, to find systems, in which simultaneity is applicable.

In this paper, solutions of the simultaneity problem will be derived and corresponding tests and applications will be developed. Moreover, especially founded solutions of the ACS problem will be derived below.

First, elaborations concerning the ACS problem are presented, recommended coordinate systems, [2], and a derived solution, [3]: The International Astronomical Union (IAU) realized, that the Equation of Time Dilation (ETD) of GRT is insufficient for space navigation [2,3]. In particular, in order to use the ETD in space navigation, an ACS is needed in addition, 2688 [2]. That ACS should be in accordance with the ETD of GRT, 2687 [2]. Correspondingly, the IAU formulated the problem of finding a useful and Adequate Coordinate System [2]. Alternatively, the ACS is named adequate frame, see [10]. Generally, two items can provide a prediction in physics: a physical equation and a coordinate system, in which this equation holds. Solutions of the ACS problem of GRT are presented in the next paragraph:

The IAU recommended two ACS: Firstly, the IAU proposed a Geocentric Celestial Reference System (GCRS) for the case of space navigation in the vicinity of Earth. This corresponds to the empirical finding, that an Earth Centered Inertial (ECI) system describes correctly the clocks onboard a spacecraft in the vicinity of Earth, see [7]. Secondly, the IAU recommended a Barycentric Celestial Reference System (BCRS), for the case of space navigation in our planetary system. The BCRS is nearly heliocentric. This BCRS is in accordance with the empirical finding that space missions in the planetary system are adequately described by a heliocentric or barycentric coordinate system, see e. g., [2,11,12]. Moreover, for each point P in the universe and for each instant of time $t_I$ , the ACS for GRT has been derived with help of a measurement procedure in a founded manner, see [3]. Thereby, the ACS has also been founded with help of the gravitational field. Hereby, an observation at Earth's ground additionally confirmed this ACS, see [3].

In the present paper, based on key experiments, [7,13] and on the hypothetic deductive method, [14-16], the ACS is related to the range of validity of the ETD [7,13-16]. Moreover, for each celestial body that is stabilized by self-gravity, the ACS is derived from the above mentioned scalar product $ds^2$. On that exact and robust foundation, in section (6), useful and valuable special cases and linear approximations are derived. In section (7), the following is shown: The observer's motion relative to the ACS causes the kinematic time dilation (nonzero or zero), as described by the ETD. Consequently, the ACS moves together (i. e. the relative velocity is zero) with that part of spacetime, which causes the kinematic time difference (nonzero or zero). By itself, that part describes a causal relation. A physical understanding of that part is beyond the scope of the present paper, that part is elaborated in [10,17-19]. Furthermore, the elements of the ACS are analyzed, and the ACS is identified as the *natural coordinate system of space and time*, which has been looked for in [1,18]. On that basis, and based on the above mentioned scalar product $ds^2$, for each point $P$ in the universe and for each instant of time $t_I$ since the Big Bang, the existence, uniqueness, and a measurement procedure of the ACS are derived in section (8). In a solution of the simultaneity problem is derived. Tests and applications of that solution are derived in sections. The results are discussed.

In the present paper, all results are derived for the case of classical physics. More general results have been derived in a founded theory that provides gravity, relativity and quantum physics (as special cases), see [10,18-20].

## Equation of Time Dilation, ETD

In this section, a general derivation of the Equation of Time Dilation (ETD) is presented.

Basically, that ETD is based on the scalar product that Einstein proposed in his Eq. 3 in § 8 [1].

$$ds^2 := \sum_{i=0}^{i=3} \sum_{j=0}^{j=3} g_{ij} dx_i dx_j \tag{1}$$

At each point in spacetime, this scalar product can be presented in a local orthogonal coordinate system. Then the scalar product is as follows:

$$ds^2 := \sum_{i=0}^{i=3} g_{ii} dx_i^2 = g_{00} c^2 dt^2 + \underbrace{\sum_{i=1}^{i=3} g_{ii} dx_i^2}_{-d\vec{r}^2} = dt^2 c^2 \left( g_{00} - \frac{1}{c^2} \cdot \underbrace{\frac{d\vec{r}^2}{dt^2}}_{\vec{v}^2} \right) \tag{2}$$

Hereby, the minus sign is introduced in order to describe the time evolution of a light flash starting at $dt = 0$ at the origin with $0 = ds^2$, so that $0 = c^2dt^2\ d\vec{r}^2$.
The increment ds is chosen for the own system, so that the spatial changes $d\vec{r}$ are zero, and only $ds = c \cdot d\tau$ remains, whereby $d\tau$ marks dt of the own system, and the root provides the following ETD:

$$d\tau = dt \cdot \sqrt{g_{00} - \frac{\vec{v}^2}{c^2}}. \tag{3}$$

Next, the range of validity of that ETD is investigated.

**Figure 1: A Foucault Pendulum in the Pantheon in Paris**

## Range of Validity of the ETD

Einstein proposed the postulate of covariance, which states that each coordinate system would be equivalent for the description of nature: 'The only possibility is that all coordinate systems are in principle equivalent for the description of nature [1]. It is clear that a physics that obeys this postulate does also fulfill the postulate of relativity.' This postulate suggests that the range of validity of the ETD would include all coordinate systems in an equivalent manner.

However, Foucault investigated the following research question: WHAT IS THE ROTATIONAL MOTION (NONZERO OR ZERO) OF A PENDULUM'S PLANE OF OSCILLATION AT EARTH'S NORTH POLE IN COMPARISON TO EARTH? He showed that Earth rotates with the angular velocity $\frac{360^o}{24\ \mathrm{h}}$ relative to a pendulum's plane of oscillation at Earth's north pole, see Figure. (1) [13].

This observation has an important implication for the ACS: In a coordinate system that describes nature, i. e. in an ACS, the Galileo principle of inertia holds [21]. As a consequence, in an ACS, a pendulum's plane of oscillation at Earth's north pole remains at rest, as no torsional moment acts upon the pendulum. Therefore, the ACS has zero rotational motion relative to the pendulum's plane of oscillation at Earth's north pole, and Earth rotates with the angular velocity $\frac{360^o}{24\ \mathrm{h}}$ relative to the ACS.

Another example is provided by the Spaceslab D1 mission, see and Figure. (2) [7]. In that mission, among other navigation experiments, the following research question has been effectively investigated: WHAT IS THE TRANSLATIONAL MOTION (NONZERO OR ZERO) OF THE COORDINATE SYSTEM, IN WHICH THE ETD DESCRIBES THE CLOCK'S TIME OR THE CLOCK'S OSCILLATION FOR A PAIR OF CLOCKS AT THE SPACELAB AND AT EARTH'S GROUND?

## The Time Difference $\delta t$ of the Two Clocks is as Follows

The ETD describes the time difference $\delta t$ of the time $d\tau_p$ measured by the pilot onboard the Spaceshuttle, minus the time $dt_{gr}$ measured at Earth's ground, as follows:

$$\delta t_{theo} = d\tau_p - dt_{gr} = dt_{gr} \cdot \left( \sqrt{g_{00} - \frac{\vec{v}^2}{c^2}} - 1 \right). \tag{4}$$

In this ETD, the pilot has a translational motion in Earth's coordinate system.

Thereby, the velocity $\vec{v}$ relative to Earth was as follows:
The Spacelab was at a circular orbit, $h$ = 383 km above the ground, with the radius $r_{\mathrm{orbit}} = r_e + h$ = 6761 137 m, and with the speed

$$v_{orbit} = \sqrt{\frac{GM_\oplus}{r_{orbit}}} = 7678.193\ \frac{\mathrm{m}}{\mathrm{s}}. \tag{5}$$

Hereby, $GM_\oplus$ = 3.986004418. $10^{-14}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg}}$ and Earth's radius at the equator is $r$ = 6378 137 m. As $v_{\mathrm{orbit}}$ is very large compared to the circular velocity of a clock on Earth's ground (caused by Earth's rotation), Earth's rotation can be omitted in an appropriate approximation. For a full analysis, see [10].

Moreover, the metric was as follows
In general, in the Schwarzschild metric, at a radial coordinate r, the tensor element for time is $g_{00} = 1 - R_S/r$ [22]. Thereby, the Schwarzschild radius is $R_S = \frac{2GM}{c^2}$ This is applied to the case of the orbit as follows.

$$g_{00,orbit} = 1 - \frac{R_S}{r_{orbit}} = 1 + \frac{2\Phi_{orbit}}{c^2} \text{ and } g_{00} - 1 = \frac{2\Delta\Phi}{c^2} = 7.877\,9199 \cdot 10^{-11}. \tag{6}$$

Thereby, $R_S = \frac{2GM_\oplus}{c^2}$. Hereby, the orbit's potential is $\Phi_{orbit} = \frac{-GM_\oplus}{r_{orbit}}$, the ground's potential is $\Phi_{ground} = \frac{-GM_\oplus}{r_e}$, and the potential difference is $\Delta\Phi = \Phi_{orbit} - \Phi_{ground} = 3.540\,1606 \cdot 10^6\ \frac{\mathrm{J}}{\mathrm{kg}}$.

Inserting $g_{00}$ -1 in Eq. (6) and $|\vec{v}|$ in Eq. (5) into the ETD in Eq. (4) yields the following time difference:

$$\delta t_{theo} = -1.597\,\mu s. \tag{7}$$

The measurement showed that at each orbital time dt, the pilot's time dτ is

$$\delta t_{obs} = -1.6\,\mu s, \text{ with } dt = 5533 \text{ s} \tag{8}$$

shorter than the corresponding time $dt$ at the ground. This observation is in accordance with the hypothesis that the ETD holds in Earth's coordinate system.

According to the postulate of covariance, Earth's coordinate system and the pilot's coordinate system can be interchanged. This hypothesis is tested next:

- After that interchange of coordinate systems, the squared velocity $v_{orbit}^2$ remains unchanged.
- After that interchange of coordinate systems, the potential difference has the opposite sign, $\Delta\Phi = \Phi_{ground} - \Phi_{orbit} = -3.540\,1606 \cdot 10^6\ \frac{\mathrm{J}}{\mathrm{kg}}$.
- Consequently, $g_{00} - 1$ has the opposite sign $g_{00} - 1 = -7.877\,9199 \cdot 10^{-11}$.
- Therefore, the interchange of the coordinate systems would imply the postulated or hypothetic time difference $\delta t_{hypo}$

$$dt_{gr} = d\tau_p \cdot \sqrt{g_{00} - \frac{\vec{v}^2}{c^2}} \text{ or } d\tau_p = \frac{dt_{gr}}{\sqrt{g_{00} - \frac{\vec{v}^2}{c^2}}} \text{ and}$$

$$\delta t_{hypo} = d\tau_p - dt_{gr} = dt_{gr} \cdot \left( \frac{1}{\sqrt{g_{00} - \frac{\vec{v}^2}{c^2}}} - 1 \right) = 2.033\ \mu\text{s}. \tag{9}$$

This time difference is based on the hypothetic or postulated covariance. This hypothesis is falsified by the observation in Eq. (8).

**Interpretation**
Firstly, this falsification is interpreted in the sense of the hypothetic deductive method, see [14-16].

Secondly, the postulated covariance, which means the hypothetic interchangeability of coordinate systems, is interpreted as a hypothetic maximal range of validity of the ETD.

Thirdly, the postulated covariance includes a hypothetic uniformity of spacetime, which would underly a uniform rate of clock's. Such a hypothetic uniformity represents a typical idealization in physics, see [23]. In general, an idealizing physical theory describes nature in some incorrect or falsified manner, see [24].

Fourthly, the pendulum is interpreted as a key experiment which clarified Earth's rotation relative to the ACS. Similarly, the Spacelab D1 mission (and possibly similar missions) is interpreted as a key experiment: It clarified that in Earth's vicinity, the translational motion of Earth and of the ACS are the same [7,13].

Consequently, the observation in Eq. (8) shows that the range of validity of the ETD is restricted to particular Adequate Coordinate Systems, ACS. These are derived next.

## Celestial Body With Self-Gravity
In many cases, an ACS occurs in the vicinity of a single celestial body, which is stabilized by its own gravity, i. e. by self-gravity. Therefore, in this section, such celestial bodies are presented in more detail. Some celestial bodies, such as

planets, exhibit the property of self-gravity, see 131 [25]. That means that such a celestial body has sufficient gravity to hold itself together. As a consequence, a slow mass in the vicinity of such a celestial body falls onto that celestial body. In this Manner, such a celestial body is stabilized by self-gravity. For instance, in the solar system, planets and dwarf planets are celestial bodies with self-gravity, see [25]. Another example is the Moon, see Fig. (3). Similarly, many moons of other planets stabilize by self-gravity, see [25].

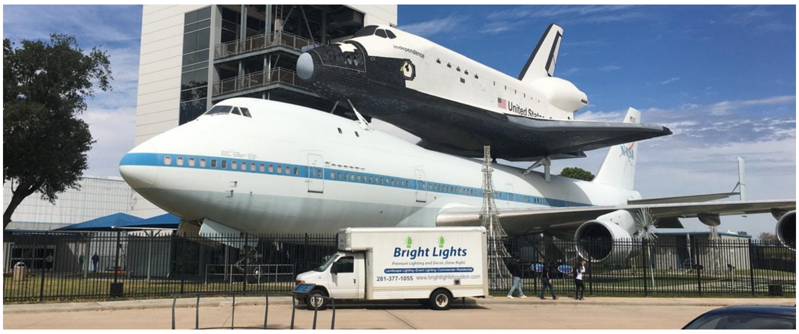


**Figure 2: Spaceshuttle: With it, Starker et al. (1985) Showed that the Pilot's Time is Shorter than the Corresponding Time At the Ground. Therefore, the Pilot's Coordinate System and the Ground's Coordinate System Are Not Interchangeable in An Equivalent Manner [7]**

In this paper, celestial bodies that stabilize by self-gravity are marked by $CB$ or $CB_j$, with a subscript. In this study, only single celestial bodies with self-gravity are considered.

**Figure 3: When a Large Crater at the Moon Formed, Material was Ejected, Fell Back To the Moon and Formed Mountains That Are Directed in A Radial Outward Manner. For Instance, This is Seen Clearly At the Copernicus Crater (Large in the Mare Area), at the Kepler Crater (Smaller in the Mare Area), and at the Tycho Crater (At the Right of the Mare Area)**

### Celestial Body With Self-Gravity Implies the ETD and an ACS

In the following, as usual, all investigated time intervals describe the same pair of events. In the vicinity of a celestial body with self-gravity $CB_j$ with a mass $M_j$, the metric can be applied [22].

**Figure 4: Celestial Body Stabilized By Crystallization: A Meteorite Consisting of Crystallized Iron (Here in a Frame) is Stabilized by Electromagnetic Forces and by Electrons in the Conduction Band With Their Common Wave Function.**

Corresponding to Fig (4), some asteroids are single stones that are mainly stabilized by condensation, crystallization, or chemical binding. Such asteroids are examples of celestial bodies that are not stabilized by self-gravity.

In this paper, the reference systems or coordinates systems are named as in the original paper [8]: One system is regarded at rest, it is named `rest system`, see § 3 and § 4, in particular pages 903 and 904 [8]. For it, the subscript $\rho$ (Greek r) is used. In the rest system, another system is regarded as moving. It is named `moving system`, see 3 and 4, in particular pages 903 and 904 [8]. For it, the subscript $\mu$ (Greek m) is used. Alternatively, the moving system can be marked by a pilot (subscript $p$), who is moving in the rest system.

Here and in the following, a first subscript marks an object or an object's system (regarded as moving with nonzero or zero velocity), and a second underlined subscript marks the used coordinate system or reference system, in which the object moves. In this section, the rest system is a mass $M_j$ Centered Inertial ($M_j$CI) system, and local orthogonal coordinates are used.
Consequently,

$$d\vec{r}^{\,2}_{\mu,\underline{\rho}} = \sum_{i=1}^{i=3} g_{ii} dx^{2}_{i,\mu,\underline{\rho}} \quad \text{and} \tag{10}$$

$$\vec{v}^{\,2}_{\mu,\underline{\rho}} = \frac{d\vec{r}^{\,2}_{\mu,\underline{\rho}}}{dt^2} \tag{11}$$

In the $M_j$CI system, in spherical polar coordinates, the diagonal tensor element for time is as follows

$$g_{00}(r_{\mu,\underline{\rho}}) = 1 - \frac{R_S}{r_{\mu,\underline{\rho}}} \tag{12}$$

Therefore, in this $M_j$CI rest system, the ETD including the pilot's time in the moving system $d\tau_\mu$ have been derived in section (2), see Eq. (3):

$$d\tau_\mu = dt_\rho(r_{\mu,\underline{\rho}}) \cdot \sqrt{g_{00}(r_{\mu,\underline{\rho}}) - \frac{\vec{v}^{2}_{\mu,\underline{\rho}}}{c^2}}. \tag{13}$$

As a consequence, in this $M_j$CI $\rho$ system, the ETD holds as a result of the derivation in section (2). Consequently, this $M_j$CI $\rho$ system is a derived ACS. Therefore, the ACS exists, as a result of a derivation:

$$\vec{v}_{\mu,\underline{ACS}} = \vec{v}_{\mu,\underline{\rho}}. \tag{14}$$

Thereby, in this $M_j$-centered $\rho$ system, the ETD, the tensor element $g_{00}$ and ACS - velocity relative to the pilot or moving system, $\vec{v}_{\rho,\underline{p}} = \vec{v}_{\rho,\underline{\mu}}$, are derived, and determined in a unique manner:

$$\vec{v}_{ACS,\underline{p}} = \vec{v}_{\rho,\underline{p}} = \vec{v}_{\rho,\underline{\mu}} = -\vec{v}_{\mu,\underline{\rho}} = -\vec{v}_{p,\underline{\rho}}. \tag{15}$$

Consequently, the ACS - velocity is uniquely determined, in a derived manner.

Moreover, the pilot can measure this velocity, for instance with help of the Doppler effect, see [6,26]. Consequently, the ACS - velocity is uniquely determined, in a derived and in a measurable manner.

As a result of the derivation, the ACS is essentially caused by the celestial body's mass $M_j$. Thereby, the transmission of the ACS - information from $M_j$ to the $\mu$ system, or to the pilot, can be described by the curvature of spacetime (described by $g_{ij}$) or by the gravitational field $\vec{G}^*$, both generated by the celestial body's mass $M_j$. For more details and implications see [10,18,19].

### Usual Special Cases and Linear Approximations

In the previous section, the ETD has been derived together with its ACS. For the case of classical physics and in the vicinity of a mass $M$ with self-gravity, this pair ETD with ACS constitutes a complete theory of time dilation. On that basis, in this section, usual special cases and linear approximations are derived:
In Eq. (13), the time $dt_\rho$ is subtracted, in order to derive the *time difference*:

$$\delta t = d\tau_\mu - dt_\rho = dt_\rho \cdot \left( \sqrt{g_{00} - \frac{\vec{v}^2_{\mu,\rho}}{c^2}} - 1 \right). \tag{16}$$

The above equation is divided by $dt_\rho$ in order to obtain the fractional *time difference*:

$$\delta t_{frac} = \frac{\delta t}{dt_\rho} = \sqrt{g_{00} - \frac{\vec{v}^2_{\mu,\rho}}{c^2}} - 1. \tag{17}$$

In the case of zero velocity, $\vec{v}_{\mu,\rho} = 0$, the *gravitational fractional time difference* is derived:

$$\delta t_{grav,frac} = \sqrt{g_{00}} - 1. \tag{18}$$

In many cases, the tensor element of time is very small compared to one, $1 - g_{00} \ll 1$ For that case, the *first order linear approximation of the gravitational fractional time difference* is derived:

$$\delta t_{grav,frac} = \sqrt{1 - (1 - g_{00})} - 1 \doteq \frac{g_{00} - 1}{2}. \tag{19}$$

For instance, in the Schwarzschild (1916) metric, at a radial coordinate $R$ from a central mass $M$, the tensor element is $g_{00} = 1 - R_S/R$. Hereby, $R_S = \frac{2GM}{c^2}$ is the Schwarzschild radius. Consequently,

$$\delta t_{grav,frac} \doteq \frac{-R_S}{2R} = \frac{\Phi}{c^2}. \tag{20}$$

Hereby, $\Phi = \frac{-GM}{R}$ is the potential.

In the case of zero gravitational fractional time difference, the *kinematic fractional time difference* remains, and $0 = \sqrt{g_{00}} - 1$, see Eq. (18). Consequently, Eq. (17) implies

$$\delta t_{kin,frac} = \sqrt{1 - \frac{\vec{v}^2_{\mu,\rho}}{c^2}} - 1. \tag{21}$$

Therefore, the *first order linear approximation of the kinematic time difference*, with respect to $\frac{\vec{v}^2_{\mu,\rho}}{c^2} \ll 1$, is as follows:

$$\delta t_{kin,frac} \doteq -\frac{\vec{v}^2_{\mu,\rho}}{2c^2}. \tag{22}$$

Of course, at first order, the kinematic time difference and the gravitational time difference are

$$\delta t_{kin} \doteq -dt_\rho \cdot \frac{\vec{v}^2_{\mu,\rho}}{2c^2} \quad \text{and} \quad \delta t_{grav} \doteq dt_\rho \cdot \frac{g_{00} - 1}{2}. \tag{23}$$

### Natural Coordinate System of Space and Time

More generally, the ACS will be derived for an arbitrary point P in the universe. For it, the manner in which the ACS describes nature is analyzed in a more detail:
The ETD shows that the pilot's velocity $\vec{v}_{p,\rho}$ relative to the rest system, which is the ACS in the ETD in Eq. (13), is the cause of any velocity - based time difference $\delta t_{kin}$. That $\delta t_{kin}$ is called kinematic time difference. For instance, the kinematic time difference $\delta t_{kin}$ of a pilot (or clock or object) is zero, when the ACS (the rest system) and the pilot move together.

Therefore, in the ACS, the motion is zero, which causes a kinematic time difference. As a consequence, the ACS moves together with that part of spacetime, that causes the kinematic time difference. Remind that the Einstein idea of the equivalence of all coordinate systems is an idealization, so that this idea cannot prohibit the identification of the part of spacetime, that causes the kinematic time difference, see section (3) [1]. The useful part of that idea in is that a 'coordinate system for description of nature' is searched [1]. Here, we derived that such a coordinate system moves together with that part of spacetime, that causes the kinematic time difference. Similarly, in meteorology, a natural coordinate system is used that moves together with atmospheric motion, see [27]. Based on the fact that the ACS moves together with that part of spacetime, that causes the kinematic time difference, the ACS is also called *natural coordinate system of space and time (NCSST)*, see [18]. Of course, additionally, the NCSST does not rotate relative to a Foucault pendulum [13].

A Natural Coordinate System for Space and Time (NCSST) includes the following items:

- A three-dimensional coordinate system in space with a time coordinate, forming four-dimensional spacetime.
- A defined rotational motion (nonzero or zero) of the coordinate system.
- A defined translational motion (nonzero or zero) of the coordinate system.
- Time evolution

**Thereby, the Following Properties Occur**

- the above first item implies the invariance of the scalar product $ds^2$ in four dimensional spacetime and the ETD, see section (2).
- The rotational motion (nonzero or zero) of an NCSST is defined according to the pendulum [13]. Therefore, a gyroscope that moves together with a local NCSST shows zero angular velocity $\vec{\omega}$.
- The translational motion (nonzero or zero) of an NCSST is derived in the following section.

## Translation Motion of A Local NCSST

For an arbitrary point $P$ in the universe and an arbitrary instant of time $t_I$ since the Big Bang, the NCSST is investigated:

As a consequence of the invariance of the scalar product $ds^2$ in four dimensional spacetime, the ETD holds at ($t_I$, $P$ ), see item (1) in section (7).

For the purpose of an arbitrary reference system, an *unspecified coordinate system* UCS is introduced. For instance, a UCS can be a celestial body. In general, the UCS is introduced in addition to the pilot's system and to the rest system in section (5).

The velocity $\vec{v}_{ACS,\underline{UCS}}$,UCS of the ACS relative to a UCS will be derived:

The NCSST, in which the ETD holds, is an ACS, see section (7). Consequently, the rest system in

Eq. (13) is an ACS:

$$d\tau_p = dt_{ACS} \cdot \sqrt{g_{00} - \frac{\vec{v}^{\,2}_{p,\underline{ACS}}}{c^2}}. \tag{24}$$

The velocity relative to a coordinate system can be expressed in terms of the difference of the respective velocities, all relative to a UCS:

$$\vec{v}_{p,\underline{ACS}} = \vec{v}_{p,\underline{UCS}} - \vec{v}_{ACS,\underline{UCS}} \tag{25}$$

This identity is applied to the above ETD in Eq. (24):

$$d\tau_p = dt_{ACS} \cdot \sqrt{g_{00} - \frac{(\vec{v}_{p,\underline{UCS}} - \vec{v}_{ACS,\underline{UCS}})^2}{c^2}}. \tag{26}$$

In many textbooks, [6,28], the minimum of the pilot's time $d\tau_p$ is analyzed [6,28]. That minimum $d\tau_{p,min}$ occurs, when the velocity $\vec{v}_{p,\underline{ACS}}$ in Eq. (24) is equal to c. Then the minimum $d\tau_{p,min}$ = 0 occurs. It means that no time elapses at a propagating light signal.

Here, for each $g_{00}$ > 0, the maximum of the pilot's time $d\tau_p$ is analyzed: It is marked by $d\tau_{p,max}$. This maximum is achieved, iff the difference of velocities in Eq. (26) is zero:

$$\vec{v}_{ACS,\underline{UCS}} = \vec{v}_{p,\underline{UCS}}, \quad \text{at } d\tau_p = d\tau_{p,max}. \tag{27}$$

Therefore, the velocity $\vec{v}_{ACS,\underline{UCS}}$ of the ACS is derived. Consequently, the maximum exists, and it is determined in a unique manner by the velocity $\vec{v}_{p,\underline{UCS}}$ of the pilot at the maximum of the pilot's time $d\tau_{p,max}$, see Eq. (27).

Altogether, at ($t_I$ ; $P$), the ACS - velocity $\vec{v}_{ACS,\underline{UCS}} = \vec{v}_{p,\underline{UCS}}$. Moreover, as the maximum is analyzed for ($t_I$ ; $P$), the derived maximum holds locally for ($t_I$ ; $P$), so that the derived and meaningful origin of the ACS is at ($t_I$ ; $P$).

As a consequence, when the pilot is at the origin and at rest in the ACS, the pilot has no gravitation difference to the origin of the ACS, so that the pilot has zero gravitational time difference relative to the ACS, so that $g_{00}$ = 1 at the origin of the ACS:

$$g_{00} = 1 \quad \text{at the ACS origin.} \tag{28}$$

When the pilot is in the vicinity of the ACS - origin, then the pilot has the following potential difference relative to the ACS:

$$\Delta\Phi_{p,\underline{ACS}} = \Phi_p - \Phi_{ACS} \tag{29}$$

This potential difference is equal to the gravitational energy $E_{grav,ACS\ to\ p}$, which is needed to move a pilot with a mass $m_p$ from the ACS-origin to the pilot's position, divided by $m_p$:

$$\Delta\Phi_{p,\underline{ACS}} = \frac{E_{grav,ACS\ to\ p}}{m_p}. \tag{30}$$

Therefore, the existence of that potential energy is based on the principle of energy conservation, which can be applied to the motion of the pilot from the ACS origin to the pilot's position.

Consequently, the tensor element $g_{00}$ is as follows:

$$g_{00} \doteq 1 - \frac{2 \cdot \Delta\Phi_{p,\underline{ACS}}}{c^2} \tag{31}$$

Therefore, at each ($t_I$ ; $P$ ), the ETD is as follows, with $g_{00}$ in Eq. (31):

$$d\tau_p = dt_{ACS} \cdot \sqrt{g_{00} - \frac{\vec{v}_{p,\underline{ACS}}^{\,2}}{c^2}}. \tag{32}$$

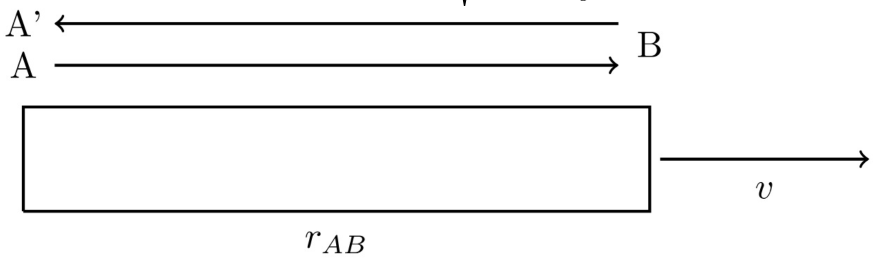


**Figure 5: Simultaneity in Relativity Theory: A Rod Has A Length $r_{AB}$ and A Clock at Each end A and B. The rod moves with a speed v in the direction from A to B, relative to a so-called system at rest, see Einstein (1905), page 892 [8]. A light signal starts at A (event A), is reflected at B (event B), and is detected at A (event A'). The forward propagation occurs in the time interval $t_B - t_A$, the backward propagation occurs in the time interval $t_A' - t_B$, see Einstein (1905), pages 892**

### Violation of Synchronization Excludes Simultaneity

In relativity theory, synchronously running clocks and simultaneity of events are investigated in one so-called `system at rest`, see [8], page 892, and in another system that moves with a speed v relative to the so-called `system at rest`, see [8].

Einstein proposed the following example, see Figure. (5): A rod has a length $r_{AB}$ and two clocks $A$ and $B$ at its ends [8]. The rod moves with a speed $v$ in the direction from $A$ to $B$, relative to a so-called system at rest. A light signal starts at $A$, is reflected at $B$, and is detected at the at the event A'. These three events, emission $A$, reflection $B$ and absorption $A'$ of the signal are studied:

In general, the two clocks are defined to run in a synchronous manner, if $t_B - t_A = t_{A'} - t_B$, see Einstein (1905), page 894:

$$t_B - t_A = t_{A'} - t_B \tag{33}$$

This definition makes sense, as the light requires the same time in the forward and backward direction. In the so-called `system at rest`, clocks detect these events at the times $t_A$, $t_B$, and $t_{A'}$. Moreover, the light signal arrives at $B$ at the time $t_B = \frac{r_{AB}}{c-v}$, and it arrives at A at the $t_{A'} = 2\ t_B$. Consequently, the observed times fulfill Eq. (33). Therefore, in the *rest system*, the clocks at $A$ and at $B$ are synchronized. Clocks at the ends of the rod, at $A$ and $B$, detect the three events at respective times $\bar{t}_A,\ \bar{t}_B$, and $\bar{t}_{A'}$. As the speed of light c is invariant, an observer in the *system at rest* observes the three events, that occur at the moving clocks, as follows:

$$\bar{t}_B - \bar{t}_A = \frac{r_{AB}}{c-v}, \quad \text{and} \tag{34}$$

$$\bar{t}_{A'} - \bar{t}_B = \frac{r_{AB}}{c+v}, \tag{35}$$

see [8], pages 896-897 (This does not imply a velocity larger $c$, as the ratio describes a part of a light travel distance, where time and length are transformed additionally) [8]. Even if these times are divided by the Lorentz factor or by any constant factor, the above time differences do not become equal. Therefore, the clocks at $A$ and at $B$ are not synchronized in the moving system.

Based on the definition of synchronized clocks in Eq. (33), the simultaneity of events in the system at rest and in the rod's system can be analyzed:

The forward and backward propagation are started simultaneously at a time zero. In the rest system, both signals reach the opposite end of the rod at $t = \frac{r_{AB}}{c-v}$. However, in the rod's system, the signal 'moving backwards reaches the opposite end of the rod at $t = \frac{r_{AB}}{c+v}$. Therefore, events that are simultaneous inthe rest system are not simultaneous in the moving system.

In general, in two systems, the violation of the synchronization condition in Eq. (33) implies the violation of the simultaneous occurrence of the same events in both systems.

Conversely, if the clocks can be synchronized in both systems, then the simultaneous occurrence of the same events in both systems is enabled.

### Simultaneity at Planets

Each planet $P_j$ is a celestial body with self-gravity. As a consequence, for each planet, the Planetary Centered Inertial (PCI) system is an ACS, see section (4).

Consequently, two clocks A and B at the end of a rod, which is at rest at the PCI and at a constant potential $\Phi_1$, *can be synchronized*, see section (9). That is, a light signal that runs from A to B needs the same time interval as a light signal that runs from $B$ to $A$, and this holds permanently. This relation, *can be synchronized*, is transitive for all clocks C that are at rest at the rod. Therefore, all clocks $C$ that are at rest at the rod can be synchronized with each other.

As the rod can be extended to all places of a planet's PCI with a common potential $\Phi_1$, the following holds: All clocks $C_1$, that are at rest at a planet's PCI at a common potential $\Phi_1$, can be synchronized with each other.

Two clocks $C_1$ and $C_2$, that are at rest at a planet's PCI at respective potentials $\Phi_1$ and $\Phi_2$, can be synchronized with each other as follows: These clocks have the following fractional time differences, relative to a point with zero potential:

$$\delta t_{1,grav,frac} = \frac{\Phi_1}{c^2};\ \delta t_{2,grav,frac} = \frac{\Phi_2}{c^2};\ \delta t_{2,grav,frac} - \delta t_{1,grav,frac} = \frac{\Delta\Phi}{c^2}\ \text{ and }\ \Delta\Phi = \Phi_2 - \Phi_1. \tag{36}$$

The rate of the clock $C_2$ is decreased by the fractional time difference $\delta t_{2,grav,frac} - \delta t_{1,grav,frac}$. As a consequence, the clocks $C_1$ and $C_2$ are at the same rate, and they are synchronized. In general, all clocks $C_1$ and $C_2$, that are at rest at a planet's PCI at respective potentials $\Phi_1$ and $\Phi_2$, can be synchronized in the same manner.

More generally, two clocks $C_1$ and $C_2$, that are at different planets $P_1$ and $P_2$, and that are at rest at their planet's PCI at respective potentials $\Phi_1$ and $\Phi_2$, can be synchronized in the same manner.

Therefore, all clocks $C_1$ and $C_2$, that are at planets $P_1$ and $P_2$, and that are at rest at their planet's PCI at respective potentials $\Phi_1$ and $\Phi_2$, can be synchronized in the same manner.

Consequently, all clocks at all planets of the solar system, that are at rest at their planet's PCI, can be synchronized with each other.

More generally, this result can be generalized to all celestial bodies $CB_j$, with a mass $M_j$, that are stabilized by self-gravity: All clocks at all such $CB_j$, that are at rest at their respective $M_j$ Centered Inertial (MCI) system, can be synchronized with each other.

In each set of clocks, that can be synchronized, the concept of simultaneity can be applied.

### Simultaneity Can be Provided to All Points in the Universe

For each point $P_j$ in the universe, the following holds: If the rod in Fig. (5) is at rest in the ACS of $P_j$, then the derivation in section (10) can be applied to this rod.

As a consequence, the result of section (10) can be transferred: All clocks at all such points $P_j$, that are at rest at their respective $P_j$ Centered ACS, can be synchronized with each other.

In each set of clocks, that can be synchronized, the concept of simultaneity can be applied. Therefore, for all points $P_j$ in the universe, the concept of simultaneity can be applied to all clocks, that are at rest at the local ACS of the respective point $P_j$.

### Simultaneity Can be Provided to All Clocks in the Universe

For each point $P_j$ and $P_k$ in the universe, the following holds: There is a local $ACS_j$ of $P_j$, and there is a local $ACS_k$ of $P_k$. A clock $C_j$ can be placed at rest at $ACS_j$, and a clock $C_k$ can be placed at rest at $ACS_k$, so that $C_j$ and $C_k$ can be synchronized.

A clock $\tilde{C}_j$ at $P_j$ , that has a velocity $\vec{v}_{\tilde{C}_j,\underline{ACS_j}}$ relative to the $ACS_j$, has a kinematic fractional time difference $\delta t_{\tilde{C}_j,kin,frac}$ , that is determined by the ETD. That time difference $\delta t_{\tilde{C}_j,kin,frac}$ can be compensated by changing the rate of the clock $\tilde{C}_j$, so that $\tilde{C}_j$ becomes synchronized with $C_j$.

More generally, at all points $P_j$, all clocks have such a velocity $\vec{v}_{\tilde{C}_j,\underline{ACS_j}}$ (nonzero or zero), and this velocity can be compensated so that $\tilde{C}_j$ becomes synchronized with $C_j$. Consequently, in this manner, all clocks in the universe can be synchronized with each other, with help of the ACS. Therefore, the concept of simultaneity can be applied to all clocks in the universe.

### Earth Moon System and the Artemis Mission

In this section, as an example, a measurable difference is predicted between the fractional time differences $\delta t_{\oplus,frac}$ of a clock at Earth's ground and the fractional time differences $\delta t_{☾,frac}$ at Moon's ground:

#### Fractional Time Difference at Moon's Ground

Moon's potential in Earth's field is as follows

$$\Phi_{☾,\underline{\oplus}} = \frac{GM_\oplus}{a_{☾,\underline{\oplus}}} = -1.036\,9418 \cdot 10^6 \, \frac{\mathrm{J}}{\mathrm{kg}}. \tag{37}$$

Hereby, $a_{☾,\underline{\oplus}}$ = 384 400 km is the semimajor axis of Moon's orbit around Earth. The corresponding fractional time difference is

$$\delta t_{☾,\underline{\oplus},grav,frac} = \frac{\Phi_{☾,\underline{\oplus}}}{c^2} = -1.153\,7534 \cdot 10^{-11}. \tag{38}$$

The clock's potential in Moon's field at Moon's ground is as follows

$$\Phi_{☾} = \frac{GM_{☾}}{R_{☾}} = -2.820\,944 \cdot 10^6 \, \frac{\mathrm{J}}{\mathrm{kg}}. \tag{39}$$

Hereby, $M_{☾}$ = 7.348 $10^{22}$ kg is Moon's mass and $R_{☾}$ = 1.738 $10^6$ km is Moon's radius. The corresponding fractional time difference is

$$\delta t_{☾,grav,frac} = \frac{\Phi_{☾}}{c^2} = -3.138\,72 \cdot 10^{-11}. \tag{40}$$

Moon permanently shows the same side in Fig. (3) to Earth, so that Moon's rotation can be approximately omitted. The sum of Moon's gravitational fractional time differences is as follows:

$$\delta t_{☾,frac} = \delta t_{☾,grav,frac} + \delta t_{☾,\underline{\oplus},grav,frac} = -4.292\,477 \cdot 10^{-11}. \tag{41}$$

According to the postulate of covariance, Moon's speed $v_{☾}$ = 1023.144603 $\frac{\mathrm{m}}{\mathrm{s}}$ at its orbit around Earth could cause the following hypothetic kinematic time difference

$$\delta t_{☾,kin,frac} = \frac{-v_{☾}^2}{2 \cdot c^2} = -5.823\,75 \cdot 10^{-12} = -0.503\,172 \, \frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{42}$$

### Fractional Time Difference At Earth's Ground

The clock's potential in Earth's field at Earth's ground is as follows

$$\Phi_{\oplus} = \frac{GM_{\oplus}}{r_e} = -6.249\,4807 \cdot 10^{7}\,\frac{\mathrm{J}}{\mathrm{kg}}. \tag{43}$$

The corresponding fractional time difference is

$$\delta t_{\oplus,grav,frac} = \frac{\Phi_{\oplus}}{c^2} = -6.953\,4851 \cdot 10^{-10}. \tag{44}$$

Earth's quadrupolar moment causes the following fractional time difference, see [10,29].

$$\delta t_{\oplus,quad,frac} = -3.764 \cdot 10^{-13}. \tag{45}$$

A clock at Earth's equator has the following kinematic fractional time difference

$$\delta t_{\oplus,kin,frac} = \frac{-v_e^2}{2 \cdot c^2} = -1.203\,4371 \cdot 10^{-12}. \tag{46}$$

Hereby, speed $v_e$ = 465.10114 $\frac{\mathrm{m}}{\mathrm{s}}$ is the clock's speed at the equator relative to the Earth Centered Inertial (ECI) system.

The sum of Earth's fractional time differences is as follows:

$$\delta t_{\oplus,frac} = \delta t_{\oplus,kin,frac} + \delta t_{\oplus,grav,frac} + \delta t_{\oplus,quad,frac} = -6.969\,2834 \cdot 10^{-10}. \tag{47}$$

### Time Difference of Clocks At Moon's and At Earth's Ground

The difference of the fractional time differences of Moon (Eq. 41) and Earth (Eq.47) is as follows:

$$\delta t_{\leftmoon,\oplus,frac} = \delta t_{\leftmoon,frac} - \delta t_{\oplus,frac} = 6.540\,04 \cdot 10^{-10} = 56.5059\,\frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{48}$$

The time difference of the two clocks is determined more precisely by using the minimal distance $r_{min}$ = 356 400 km and the maximal distance $r_{max}$ = 406 700 km between Earth and Moon:
At the minimal distance of Moon and Earth, the time difference of the two clocks is

$$\delta t_{\leftmoon,\oplus,min} = 56.4276\,\frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{49}$$

At the maximal distance of Moon and Earth, the time difference of the two clocks is

$$\delta t_{\leftmoon,\oplus,max} = 56.606\,\frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{50}$$

At its elliptical orbit, the Moon's distance to Earth varies between the minimal and maximal value. Therefore, the time difference between the two clocks at Earth's and Moon's ground is in the following Interval

$$\delta t_{\leftmoon,\oplus} \in [56.4276, 56.606]\,\frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{51}$$

### Proposed Experiment With Predicted Result

In order to test the hypothesis that the motion of the Moon relative to Earth would cause an additional time difference $\delta t_{\leftmoon,kin,frac}$ (see Eq. 42), that value is added to the value in Eq. (41). As a result, the time difference would be as follows:

$$\delta t_{\leftmoon,\oplus,hypo} = 56.0012\,\frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{52}$$

As derived above, as a result of Moon's elliptic orbit, that hypothetic time difference is in the following Interval

$$\delta t_{\leftmoon,\oplus,hypo} \in [55.9244, 56.0574]\,\frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{53}$$

Accordingly, the following experiment is proposed: Two clocks should be implemented, on at Earth's ground at the equator and one at Moon's ground. The predictions are as follows:

- The time difference per day in Eq. (51) is predicted.
- If the hypothetic postulate of covariance would hold, then the time difference per day in Eq. (53) would be predicted.
- In fact, the calculation for the Lunar surface, table 3, proposes the following result [30].

$$\delta t_{☾,\underline{\oplus},Ashby\ and\ Patla} = 56.0199\ \frac{\mu\mathrm{s}}{\mathrm{day}} - 0.10843417 \cdot \cos(f)\ \frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{54}$$

Hereby, the subtrahend describes the effect of the ellipticity of Moon's orbit around Earth. Thereby, $f$ is Moon's true anomaly, see [25]. As the subtrahend is periodic, so it vanishes in a long time average. Such an averaged value, calculated by [30] is

$$\delta t_{☾,\underline{\oplus},Ashby\ and\ Patla,average} = 56.0199\ \frac{\mu\mathrm{s}}{\mathrm{day}}. \tag{55}$$

It is in accordance with the postulated hypothetic covariance, see Eqs. (52 and 53). However, this has been falsified in section (3).

Moreover, the hypothetic result calculated by (Eqs. 54 and 55), falsified in section (3), can be excluded by the realistic value in Eqs. (50 and 51) [30].

Therefore, it will be very interesting to obtain the precise measurement of Lunar time, relative to Earth's time. In fact, orbital velocities have been used in [30] in order to describe clocks at Moon's ground and on Earth's ground [30]. This shows again that the ACS can and should be tested by comparing times, that are shown by clocks on Moon's ground and on Earth's ground.

## Planets

Time at the planets is important, see [4]. Therefore, for each planet, and based on the founded ACS, the time of each planet is derived next.

Each planet in the solar system is stabilized by self-gravity. Therefore, at its pole (with zero rotation), each planet has the kinematic time difference zero. As a consequence, clocks at the ground of these planets can be synchronized so that the same times are shown by the clocks simultaneously.

For it, for each planet $j$, the clock's gravitational time difference $\delta t_j$ per day is determined, relative to a point with zero gravitational potential caused by Sun. With it, at each planet $j$, the clock at the planet's ground must be corrected by that time difference $\delta t_j$. As a consequence, the clocks at the planet's ground are synchronized and show the same time simultaneously.

Each planet has an elliptic orbit around Sun. Accordingly, for each planet, a minimal and maximal value of the gravitational time difference is determined, see table (1). These values are determined as follows:

### For Each Planet J, the Potential Caused by Sun is Determined

$$\Phi_{j,\underline{\odot}} = \frac{-GM_{\odot}}{r_{j,\underline{\odot}}}. \tag{56}$$

Hereby, $r_{j,\odot}$ is the planet's distance to Sun. The maximal value of the distance is

$$r_{j,\underline{\odot},max} = a_j \cdot (1 + \varepsilon_j), \tag{57}$$

whereby $a_j$ is the planet's semimajor axis, and $\varepsilon_j$ is the planet's eccentricity. The minimal value of the distance is

$$r_{j,\underline{\odot},min} = a_j \cdot (1 - \varepsilon_j). \tag{58}$$

### For Each Planet $J$, the Potential Caused By That Planet At the Planet's Ground is Determined:

$$\Phi_{j,\underline{j}} = \frac{-GM_j}{R_j}. \tag{59}$$

Hereby, $R_j$ is the planet's radius, and $M_j$ is the planet's mass. For each planet, the values of $a_j$, $\varepsilon_j$, $R_j$ and $M_j$ are taken from [25].

**For Each Planet J, the Sum of The Above Potentials is Formed:**

$$\Phi_j = \Phi_{j,\underline{\odot}} + \Phi_{j,\underline{j}}. \tag{60}$$

For Each Planet **J**, and For the Maximal And Minimal Values of Its Distance To Sun, the Gravitational Fractional Time Difference is Calculated

$$\delta t_{j,grav,frac} = \frac{\Phi_j}{c^2}. \tag{61}$$

These gravitational time differences per day are presented in table (1).

| $j$ | planet | $\delta t_{j,min}$ in $\frac{\mu s}{\text{day}}$ | $\delta t_{j,max}$ in $\frac{\mu s}{\text{day}}$ |
|---|---|---|---|
| 1 | Mercury | $-2782.91$ | $-1836.57$ |
| 2 | Venus | $-1239.04$ | $-1223.07$ |
| 3 | Earth | $-927.71$ | $-899.21$ |
| 4 | Mars | $-629.70$ | $-524.18$ |
| 5 | Jupiter | $-1877.48$ | $-1861.57$ |
| 6 | Saturn | $-700.33$ | $-690.62$ |
| 7 | Uranus | $-264.05$ | $-259.84$ |
| 8 | Neptune | $-295.48$ | $-294.996$ |

**Table 1: Planets and Their Minimal and Maximal Time Differences Per Day $\delta t_{j,min}$ and $\delta t_{j,max}$. the Reference Point is At Zero Gravitational Potential Of Sun (i. e. at Large Distance)**

**Predictions**
For each pole at each planet $j$, except Earth, the time difference relative to Earth's pole is predicted as follows:

$$\delta t_{j,\oplus,min} = \delta t_{j,min} - \delta t_{j=3,max}, \text{ and} \tag{62}$$

$$\delta t_{j,\oplus,max} = \delta t_{j,max} - \delta t_{j=3,min}. \tag{63}$$

These predicted time differences per day are presented in table (2). The time differences per day are relatively large and could be tested in the future. For instance, a rover at Mars could have an onboard atomic clock which measures the time difference relative to Earth.

**Application**
As the kinematic time difference is zero at a planet's pole, at the pole, the measured time difference is equal to the gravitational time difference. This provides a valuable and precise measurement of the potential difference to the potential of a reference clock, see section (6).

More generally, a clock at rest at the ACS can measure the potential difference to the potential of a reference clock, at each point $P$ and instant of time $t_I$ in the universe.

| $j$ | planet | $\delta t_{j,\underline{\oplus},min}$ in $\frac{\mu s}{\text{day}}$ | $\delta t_{j,\underline{\oplus},max}$ in $\frac{\mu s}{\text{day}}$ |
|---|---|---|---|
| 1 | Mercury | $-1883.70$ | $-908.86$ |
| 2 | Venus | $-339.83$ | $-295.356$ |
| 4 | Mars | 269.509 | 403.53 |
| 5 | Jupiter | $-978.27$ | $-933.87$ |
| 6 | Saturn | 198.875 | 237.09 |
| 7 | Uranus | 635.16 | 667.86 |
| 8 | Neptune | 603.725 | 632.71 |

**Table 2: Planets and Their Minimal and Maximal Time Differences Per Day $\delta t_{j,\oplus,min}$ and $\delta t_{j,\oplus,max}$, Relative to Earth**

## Pulsar Time

Beyond the planetary system, pulsars provide very useful time signals, see [31].

As each pulsar is stabilized by self-gravity, it provides its own ACS. Consequently, in principle, at each pole of each pulsar, a clock can be synchronized with a clock on Earth. Therefore, such clocks show the same time simultaneously, such clocks exhibit zero kinematic time difference, and simultaneity is provided between Earth and each pulsar, in principle. As a consequence, time signals that come from a pulsar and that are received at Earth exhibit no errors related to simultaneity.

Furthermore, time signals that come from pulsars exhibit stability and precision for long time intervals, such as three years, see [31]. Accordingly, atomic clocks can be steered with time signals from pulsars, so that the fractional daily drift is reduced to 4.74 ns, i. e. $\delta t_{pulsar\ steered\ atomic\ clock} = \frac{4.74\ \mathrm{ns}}{\mathrm{day}}$.

Moreover, time signals that come from a pulsar have an extraterrestrial origin. Therefore, they can be used as a reference for the investigation of terrestrial processes. For instance, the hypothesis of covariance can be tested again as follows:

Earth's velocity at its orbit around Sun is

$$v_{\oplus,\underline{\odot}} = \sqrt{G \cdot M_{\odot} \cdot \left(\frac{2}{r} - \frac{1}{a}\right)}. \tag{64}$$

Hereby, $r$ is Earth's distance to Sun, and a is the semimajor axis of the orbit.

$$v_{\oplus,\underline{\odot},max} = 30\ 290\ \frac{\mathrm{m}}{\mathrm{s}}. \tag{65}$$

At $r = a(1 - \varepsilon)$, the maximal speed occurs as follows:

$$v_{\oplus,\underline{\odot},max} = 29\ 290\ \frac{\mathrm{m}}{\mathrm{s}}. \tag{66}$$

According to the postulate of covariance, these speeds could cause the following hypothetic kinematic time differences

$$\delta t_{\oplus,kin,frac,hypo,max} = \frac{-v^2_{\oplus,\underline{\odot},max}}{2c^2} = -441.002\ \frac{\mu\mathrm{s}}{\mathrm{day}}, \tag{67}$$

$$\delta t_{\oplus,kin,frac,hypo,min} = \frac{-v^2_{\oplus,\underline{\odot},min}}{2c^2} = -412.364\ \frac{\mu\mathrm{s}}{\mathrm{day}}, \tag{68}$$

The difference of these hypothetic time differences is

$$\delta t_{hypo} = \delta t_{\oplus,kin,frac,hypo,max} - \delta t_{\oplus,kin,frac,hypo,min} = 28.638\ \frac{\mu\mathrm{s}}{\mathrm{day}}, \tag{69}$$

This difference occurs during six months or 183 days.

During 183 days, the drift of an atomic clock steered by pulsars is as follows:

$$\delta t_{pulsar\ steered\ atomic\ clock} \cdot 183\ \mathrm{days} = \frac{4.74\ \mathrm{ns}}{\mathrm{day}} \cdot 183\ \mathrm{days} = \frac{0.867\ \mu\mathrm{s}}{1} \tag{70}$$

Consequently, with an atomic clock steered by pulsars, a precise time signal with extraterrestrial origin would be available, that could measure the hypothetic terrestrial time difference in Eq. (69). Therefore, the hypothetic postulated covariance could be tested with the experiment proposed here

Additionally, it is possible that the necessary measurements have already been performed, whereby measured times of Earth based clocks and of pulsars should have been documented simultaneously.

## Discussion

The International Astronomical Union (IAU) realized, that General Relativity Theory (GRT) combined with its postulated and hypothetical coordinate systems, see [1], is insufficient for space navigation. Accordingly, the IAU proclaimed the ACS problem, to find and Adequate Coordinate System (ACS), see [2].

Based on a general measurement process, provided a first general solution to the ACS problem: At each point $P$ in the universe, there exists an ACS with a uniquely determined velocity $\vec{v}_{ACS,\underline{UCS}}$, relative to an arbitrary Unspecific Coordinate System (UCS) [3]. Moreover, near a large celestial body generating a field $\vec{G}^*$, the ACS is at rest relative to $\vec{G}^*$. Additionally, this result is in accordance with an observation at Earth's ground.

**In This Paper, the ACS is Derived Analytically for Two Cases**

- For each mass $M_j$ that is stabilized by self-gravity, the ACS is at rest at $M_j$, and the Equation of Time Dilation (ETD) is derived, and the ACS velocity is determined in a unique manner, see section (5). Hereby, the scalar product $ds^2$ in four-dimensional spacetime and the Schwarzschild metric are used in order to derive the ETD and the ACS.
- For each point $P$ in the universe and for each instant of time $t_I$ since the Big Bang, the ETD holds for an ACS. Thereby, the ACS - velocity is equal to each pilot's velocity, at each state, at which that pilot's time $d\tau_p$ is maximal, at fixed ACS time $dt_{ACS}$, see section (8). Hereby, the scalar product $ds^2$ in four-dimensional spacetime has been used to derive the ETD, and the analyzed maximum of the pilot's time has been used in order to derive the ACS.

(1 and 2) In both cases, the ACS moves together (i. e. the relative velocity is zero) with that part of spacetime, that causes the kinematic time difference. Therefore, the ACS is a *Natural Coordinate System of Space and Time* (*NCSST*), see section (7).

**Moreover, in This Paper, the Simultaneity Problem is Solved As Follows**

Derived that problem: In general, two clocks are not synchronized, and the concept of simultaneity cannot be applied to such clocks [8]. Therefore, the simultaneity problem occurs to find systems, in which simultaneity is applicable.

Based on the existence of the ACS and the uniqueness of the ACS - velocity, see sections (5 and 8), it is shown step-by-step that all clocks at the pole of a planet can be synchronized, all clocks at the pole of a celestial body stabilized by self-gravity can be synchronized, all clocks at rest of an ACS can be synchronized, and all clocks can be synchronized, see sections (4, 9, 10, 11 and 12). Therefore, the concept of simultaneity can be applied to all clocks, with help of the ACS and its uniquely determined velocity.

**Furthermore, in This Paper, Tests and Applications Have Been Elaborated**

Lunar time has been derived in two ways, see section (13): It has been derived on the basis of the founded ACS. Additionally, Lunar time has been derived on the basis of the postulated and hypothetic covariance, which has been falsified, see section (3). The two results differ significantly, so Lunar time should be measured, for instance by the Artemis mission, see [32].

Planetary times have been derived with help of the ACS, see section (14). These times should be measured with help of clocks at a planet's surface, for instance by a Mars rover, see [33]. Thereby, potential differences can be measured with help of the ACS and clocks on a planet's pole or on a planet, more generally.

As a result of the ACS, time signals generated by a pulsar can be used in a reliable manner with help of the concept of simultaneity, see sections (12 and 14). Such signals have two significant advantages: They provide a very precise long time stability, and they can be used as a reference with extraterrestrial origin. With these signals, the postulated and hypothetic covariance can be tested again by using clocks on Earth and the ellipticity of Earth's orbit around Sun, see section (14).

The present paper shows that for each point $P$ in the universe and at each time $t_I$, the ACS and its uniquely determined ACS velocity is derived analytically in a very founded manner. Therefore, the ETD holds in its ACS everywhere in the universe. Conversely, in general, as the ACS velocity is uniquely determined, the ETD does not hold in a coordinate system that has a velocity different from the ACS velocity. The corresponding proofs provide a deeper understanding of the role of the scalar product $ds^2$ for the ETD and the ACS, as well as the field's role, $\vec{G}^*$, for the ACS-velocity.

With help of the ACS, all clocks can be synchronized. As a consequence, the concept of simultaneity can be applied to all clocks, in their synchronized state. When the clocks have not been synchronized, in general, the concept of simultaneity cannot be applied to the clocks.

The derived ACS provides calculated Lunar time and times at planets, as well as founded simultaneity for pulsar time signals. These results provide various tests of the postulated and hypothetic covariance. These tests should be performed, and it will be interesting to see the results.

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