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Struga Theory (μτ-Approach) Quantum Gravity with Particle Curvature Without Space Curvature

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Abstract
We propose an operational $\mu\tau$-approach to gravity on a flat background: the gravitational interaction is interpreted as a universal retuning of the particle *scale* (μ) and the *rate of internal clocks* (τ). Quanta of space possess real and imaginary parts. The size of each quantum of space is equal to the size of the Universe and has a M¨obius topology. Tensor perturbations (gravitons) propagate along the *imaginary* parts of the space quanta and, upon intersecting their real parts, induce coherent changes of μ, τ in all fields. In the weak-field regime, the theory reproduces the classical tests of GR (PPN up to 1PN order, lensing, Shapiro delay, equality of EM and GW speeds); in the strong-field regime, it predicts small 2PN deviations, possible "echoes" in black-hole ringdown, and a weak scalar polarization. Within a single framework, several "dark" effects are described simultaneously: dark matter as a BH-bound imaginary-geometric response (including Bullet Cluster–type cases), dark energy as the sum of a vacuum component and the growth of space quanta, and baryon asymmetry as a consequence of a two-sheeted (M¨obius-like) topology. The theory preserves causality and local Lorentz invariance, is formulated as an EFT with controlled corrections, and provides sharp observational tests for near-future experiments.

Keywords: Quantum Gravity, $\mu\tau$-Approach, STRUGA, Graviton, Lensing, PPN, Real/Imag Boundary Layers, Dark Matter, Dark Energy

Introduction
Challenges of Quantum Gravity and "Dark" Phenomena
Over the century since the development of general relativity (GR) and the establishment of quantum field theory, physics has achieved outstanding successes. However, it has still not been possible to merge gravity with quantum mechanics into a single, internally consistent formalism. In the ultraviolet regime, quantizing gravity on the Einstein metric leads to nonrenormalizable divergences; background independent approaches encounter difficulties in establishing contact with observable phenomenology.

In parallel, astrophysics presents "dark" challenges: the gravitational dynamics of galaxies and clusters requires additional mass (dark matter), while the accelerated expansion of the Universe demands a component with negative pressure (dark energy). Strong-field observations — shadows and ringdown of black holes, stellar orbits around Sgr A*, polarization and dispersion of gravitational waves — open up new regimes for testing theories.
Any candidate theory of quantum gravity must simultaneously:
- be consistent with the tests of GR in the weak-field regime;
- yield a correct quantum limit without pathologies;
- provide clear observable distinctions in strong-field and cosmological regimes.

The Idea of the $\mu\tau$-Approach: Gravity as a Change of Particle Scale and of the Rate of Their Internal Clocks (Flat Background)

Instead of attributing to gravity a curvature of the background, we transfer its action to universal properties of matter. In the proposed $\mu\tau$-approach, the role of "carriers" of gravity is played by two scalar fields: μ, which controls the scale (effective size) of particles and their fields, and τ, which sets the rate of internal clocks (the local speed of physical processes).

Spacetime is treated as globally flat; the measurable effects of gravity arise from how μ and τ vary at different points and in different states. One and the same universal deformation acts on all known fields — this ensures the equivalence of free fall.

In strong fields we allow for topological quantization of space: each "quantum" has a real part (the observable three-dimensional scene) and an imaginary part (a hidden volume), which intersect in a subtle way. Excess "scale" of particles near singularities can be temporarily "stored" in the imaginary part and returned through the intersection boundaries, which ensures dynamical stability and provides channels for "dark" effects without introducing new types of matter.

Main Results of the Work and Testable Predictions

In this paper we formulate the action of the theory on a flat background, derive the equations of motion, and show how μ and τ are consistently embedded into the standard relativistic equations for particles and fields (scalar, spinor, vector, spin-3/2). In the weak-field regime we obtain the full set of first-order post-Newtonian parameters, coinciding with GR (including the "gravitomagnetic" sector),which preserves all classical tests in the Solar System. At the level of phenomenology we:

- derive gravitational lensing and redshift as optics in an effective medium, consistent with observed values at first order;
- predict subtle second-order differences manifested in deflection angles of light and orbital precession in close passages near compact objects;
- obtain a weak additional "breathing" polarization of gravitational waves and a slightly faster damping of black-hole ringdown;
- provide integral estimates of possible delays of electromagnetic signals relative to gravitational ones when traversing complex gravitational environments;
- describe mechanisms of effective dark matter (through projections of the "imaginary" scale) and two-component dark energy (vacuum contribution of fields and growth of the real volume due to the creation of space quanta), compatible with large-scale dynamics.

All of the above differences are reduced to a finite number of parameters and are translated into concrete tests for forthcoming observational facilities.

Relation to and Distinction from Alternatives (Scalar–Tensor, Disformal, Teleparallel, Emergent)

Our approach is thematically related to theories in which gravity is supported by additional fields, but it differs in ontology and interpretation:

- in scalar–tensor models, the additional scalar coexists with a dynamical metric; in our case the background metric is flat, and gravity manifests itself through universal deformations of the properties of matter;
- disformal and "optical" descriptions introduce an effective metric governing light propagation; in our case the same optics arises as a consequence of modified field equations in a medium specified by μ and τ;
- in teleparallel gravity curvature is replaced by torsion of the connection; in our case the geometric connection is secondary to the scale and temporal deformations of particles;
- emergent constructions derive gravity from the statistics of microstates; we formulate a minimalist quantum field theory with two universal scalars and explicit phenomenology.

Taken together, the $\mu\tau$-approach preserves the well-tested kinematics of GR in the weak-field limit, but changes the dynamical "essence" of gravity, which makes the quantum description technically simpler and at the same time yields specific observable signatures.

Structure of the Paper

- Section 2 formulates the postulates and operational interpretation of μ and τ.
- Section 3 introduces the topological quantization of space and the role of the imaginary sector.
- Section 4 contains the central Lagrangian of the theory, its symmetries, and the variational derivation.
- Section 5 discusses the relation of the equations of motion to the "observable metric".
- Section 6 unifies the modified wave equations for fields of various spin and the test of equivalence.
- Section 7 carries out the full first-order PPN analysis.
- Section 8 is devoted to optical tests (lensing, redshift, Shapiro delay).
- Section 9 deals with strong-field black-hole dynamics and the information problem.
- Section 10 considers gravitational waves, polarizations, and ringdown.
- Sections 11–12 describe the phenomenology of dark matter and dark energy and the cosmological implications.
- Section 13 is devoted to multimessenger signatures, and Section 14 to the stars near Sgr A*.

• Sections 15–17 discuss the model parameters, statistical identifiability, and quantum consistency (renormalizability / EFT, causality).
• Section 18 compares the model with alternative approaches.
• The Discussion and Conclusions summarize the main results and outline near-future tests.

Postulates and Operational Interpretation

Universality of the Deformations μ and T: Equivalence Principle in a Scale Formulation

Postulate P0 (flat background). At the fundamental level the background is taken to be globally flat; gravity does not "bend" the background, but manifests itself through fields that change the units of measurement of matter itself. Postulate P1 (two universal deformations).There exist two scalar fields:
• $\mu(x)$ — responsible for the scale of particles and waves (effective "size", lengths, densities, cross sections);
• $\tau(x)$ — responsible for internal clocks (the local rate of all processes, transition frequencies, the "rate of flow of time" for a system).

Both fields act universally: all known particles and fields (with different spin, charge, and mass) experience the same multiplicative deformations. This provides a scale formulation of the equivalence principle: in any sufficiently small region where μ and τ can be regarded as constant (and their gradients negligible), the local physics reduces to special relativity with redefined standards of length and time. In other words, there exists a local "scale inertial frame" in which all non-relativistic and relativistic experiments (including atomic spectra, the speed of light, particle decays) exhibit the same relations as in flat space, provided the results are expressed in local units.

Postulate P2 (minimal substitution). For any quantum system, "switching on" gravity is implemented by one and the same rules: masses become effective with "weight" μ, and time derivativeswith "rate" τ. This realizes universality without a selective "fifth force". In the present work we show that such a prescription reproduces all verified weak-field effects of GR for an appropriate choice of the profiles of μ and τ.

Postulate P3 (topological stability of strong fields). In regimes where singularities arise in GR, the excessive growth of the "scale" is transferred into the imaginary volumes of the space quanta (see §3),which ensures finiteness of observable quantities and preserves the physical meaning of the evolution. Corollary (scale EP). "All bodies fall the same way" is formulated as follows: trajectories, periods, frequencies and angles, measured by local rods and clocks, do not depend on the composition or internal structure of the test body. Any deviations from this are probed in experiments of the torsion-balance type, atomic interferometers, and satellite tests; in our model such deviations are absent at the level of first post-Newtonian order.

Relative Speed of Gravity: Definition, Causality, Observables

We distinguish two speeds:
• The physical speed of gravity is the speed of propagation of gravitational disturbances (gravitational waves) in the background description. By construction it is equal to the speed of light c and does not violate causality.
• The relative speed of gravity is an operational quantity: the number obtained when a local observer measures the speed of arrival of a gravitational front by means of his own ruler and clock, already deformed by the fields μ and τ. Since gravity changes precisely these standards, the numerical value may differ from c and depend strongly on position:
• far from the event horizon of a black hole, where particle scales are only weakly compressed, the relative speed of gravity practically does not differ from the speed of light;
• near the event horizon outside the black hole, where scales are compressed more strongly, the relative speed increases (operationally — due to shortened "steps" of the ruler) and may exceed the speed of light by many times;
• on the horizon itself the scales of particles tend to zero, and the measured relative speed tends to very large values (for a local observer a "too fine ruler" yields a large number of meters for a fixed physical distance);
• inside the horizon the scales of particles begin to grow, so the relative speed decreases, although the physical speed of gravity remains equal to c;
• at the singularity the relative speed of gravity tends to zero.

This does not imply superluminal information transfer. The causal structure does not change: signal cones in the background description remain light like, and the "large numbers" are an effect of the observer's measuring scale. The relative speed is a calibration-invariant observable in the sense that it is extracted from the difference in arrival times of gravitational and electromagnetic signals for known profiles of μ and τ along the path. Practical implications.
• In multi messenger events (mergers of neutron stars / black holes) the gravitational signal may be registered earlier than the electromagnetic one by fractions of a second to seconds due to an integral "optical" effect of the medium (see the test in §13).
• The delay depends on the integral along the trajectory (galactic halos of the source/receiver, large-scale structures), not on the local difference of "speeds" at a single point.

Measurement Procedures: How Instruments "See" μ and τ (Clocks, Rods, Photon Tests)

The operational meaning of the fields is specified by measurement procedures, which we divide into three classes.

Clocks: Measurement of τ.

• Comparison of atomic clocks at different heights (ground towers, satellites, radio-frequency lines). The local rate of transitions directly "feels" τ.
• Mossbauer and optical gravitational redshifts. The frequency of a photon during ascent/descent in a field corresponds to the ratio of τ at the emission and reception points.
• GPS/GLONASS corrections. Global navigation systems implement a "network tomography" of τ on planetary scales; our model reinterprets these corrections as a direct measurement of the field τ.

Rods and Scales: Measurement of μ.

• Fabry–Perot resonators and optical combs: the mode frequency of a resonator depends on the effective length, which provides access to μ.
• Matter-wave interferometry: the phase accumulation of cold atoms / neutrons depends on their "scale" length.
• Crystallographic standards: interatomic distances are compared at different potentials to isolate the contribution of μ.

Photon Tests: Combined Sensitivity to μ and τ.

• Gravitational lensing and Shapiro delay: light behaves as in a medium with an effective index depending on μ and τ.
• Standard sirens (GW) + electromagnetic counterparts: comparison of arrival times allows one to extract the integral difference of the profiles.
• Pulsar timing arrays: the statistics of pulse arrival times is sensitive to a combination of μ and τ on scales of tens of kiloparsecs.

Normalizations and calibration. Absolute values of μ and τ are not directly accessible — only their ratios and gradients are observable. A standard calibration is to impose $\mu \to 1$, $\tau \to 1$ at spatial infinity; all shifts are interpreted relative to this level.

Separation of μ from τ is achieved by jointly fitting different classes of experiments: clocks give pure sensitivity to τ, resonators and interferometers to μ, and photon tests to their combinations. In §15 we discuss statistical identifiability of the parameters and degeneracies.

Summary of the Section

We have formulated four supporting principles: a flat background; two universal deformations μ and τ; minimal substitution (universality of couplings); topological stability of strong fields. On this basis we have strictly defined:
• how equivalence is tested (local reduction to SR in a region where μ, τ are constant);
• what is meant by the "relative speed of gravity" (an operational quantity without violation of causality);
• how real instruments extract μ and τ (clocks, rods, photon trajectories).

In §3–§5 we proceed to formalize topological quantization and derive the Lagrangian from which all subsequent results of the theory follow.

Topological Quantization of Space

In strong-field regimes (horizons, neighborhoods of singularities, the early Universe) a continuous description of the background is inconvenient: precisely their standard models develop divergences. We introduce topological quantization of space — a discrete–continuous mosaic structure of the carrier, in which each "cell" (quantum of space) has two aspects: an observable real part and a hidden imaginary one. Here "imaginary" does not mean an "imaginary number", but serves as a label of a hidden volume topologically connected to the real layer. Below we formulate the geometry of quanta, the structure of their intersections, and the role of these intersections in strong-field dynamics and macro-scale expansion.

Quanta: Real and Imaginary Parts; MoBius-Strip Analogue

Definition. Space is represented by a countable family of quanta $\{Q_i\}$. Each quantum has:
• a real part R_i — a submanifold identified with the observable three-dimensional scene of measurements;
• an imaginary part I_i — a complementary volume that does not belong to the observable three dimensionalities but is topologically connected to it.

Mo¨bius-like gluing. Each Q_i is assigned an orientation $s_i \in \{+1,-1\}$ and a "twist" when traversing along a non-contractible cycle. The pair of layers (R_i, I_i) behaves as an improper double sheet: when traversing a nonlocal cycle, the orientation changes, by analogy with a Mo¨bius strip. Such a construction(1)admits locally ordinary three-dimensional physics on R_i; (2) provides global "intertwining" between R and I required for the transfer of excess scale (see §3.4); (3) provides natural channels for graviton propagation.

Observability. I_i is not accessible to direct operations, but it influences Rj through intersections and boundary layers. This influence is encoded in the action (Lagrangian) as boundary terms of real↔imag mixing with finite thickness (see §3.6).

Geometry of Intersections and Boundary Hypersurfaces of Finite Thickness ℓ_*

Overlaps. The imaginary part of each quantum I_i intersects the real parts of other quanta:

$$\Sigma_{i\to j} \equiv I_i \cap R_j, \tag{1}$$

which are the boundary hypersurfaces of influence transfer (interfaces). Their union forms a sparse but pervasive network. To each $\Sigma_{i\to j}$ we associate:

- a geometric overlap measure $\zeta_{i\to j} \in [0, 1]$ (contact density), depending on the local configuration;
- an effective boundary thickness ℓ_* — a minimal microscopic length on which the fields of R and I "mix". This is a physical cutoff that removes δ^2-pathologies and is compatible with an EFT description.

Boundary dynamics. On $\Sigma_{i\to j}$ conservative conservation laws with surface currents are imposed; the variational principle yields:

- continuity of the "scale flux" and energy through a layer of thickness $\ell*$;
- finite "jumps" of the normal derivatives of the fields μ, τ by amounts proportional to the coupling g and $\zeta_{i\to j}$ (see §3.6–§3.6).

Overlap kernel. In the large-scale limit the network $\{\Sigma_{i\to j}\}$ is approximated by a convolution kernel $K(\mathrm{x} - \mathrm{x}')$ describing the average contribution of the imaginary sector to the effective sources on the real layer:

$$\big(\text{effective contribution to } R\big)(\mathbf{x}) \;\sim\; \int K(\mathbf{x}-\mathbf{x}')\,\big(\text{densities in } I\big)(\mathbf{x}')\,d^3\mathbf{x}'. \tag{2}$$

A characteristic form is a "plateau" at small distances and a long tail of order $1/r^2$ at large distances (see §11) — the key to quasi-isothermal halos without new particles.

Causality. Since ℓ_* is finite, the boundary laws are local and hyperbolic; "faster-than-light" propagation is excluded. Any "fast" quantity (for example, the relative speed of gravity) is an effect of rescaling the observer's units, not superluminal signal transfer.

Birth/growth of quanta and the macro-effect of increasing the "real" volume

Elementary event. In a quantum-fluctuational act a new quantum Q_{new} with $(R_{\text{new}}, I_{\text{new}})$ is created.
This gives two contributions to the total "real" volume:

- a direct one — the appearance of R_{new} (a microscopic volume, substantially smaller than the atomic scale);
- an induced one — due to the fact that I_{new} overlaps all already existing R_j , each of them is slightly "thickened", increasing the total "real" volume macroscopically

Exponential cumulative effect. If the intensity of quantum creation $\dot{N}_q$ decreases slowly with cosmological time, the induced contribution, being additive over all overlaps, accumulates and leads to an *accelerating* growth of the total real volume. At the level of background cosmology this manifests itself as an addition to the effective dark energy, $\Lambda_{\text{growth}}(a)$ (see §12), which:

- is practically homogeneous (does not cluster on galactic scales);
- may evolve weakly in time (hinting at $w(a) \neq -1$), while remaining consistent with CMB/BAO/SN for moderate growth rates.

Control of homogeneity. The homogeneity of Λ_{growth} is ensured by the fact that the I-networks permeate all R's uniformly on average, and their contribution is averaged on scales tens of megaparsecs, without generating unwanted density fluctuations.

Stabilization of strong fields: "scale dumping" into the imaginary sector and feedback

Strong-field problem. Inside black holes, under the internal evolution, the scale μ (the effective "size" of states) grows; in a purely continuous description this would lead to divergences of energy/pressure densities on R.

Stabilization mechanism. A local threshold μ_{th} is introduced, beyond which the perturbation of μ is no longer enhanced on R, but is channeled via the nearest interfaces Σ into the imaginary volume I:

$$R \xrightarrow{\;\Sigma(\,\ell_*\,)\;} I \quad (\text{transfer of excess scale}). \tag{3}$$

The transfer is accompanied by:

- a conservative outflow of energy/momentum with surface currents on Σ (energy balance remains closed);
- regularization of local densities on R (no δ-peaks and infinite knots).

Feedback. The energy and "scale" accumulated in I are not isolated: through the same Σ they create a back-reaction

effective gravitational influence on the set of R_j intersected by a given I. On the macro level this:
• gives an additional contribution to the potential (appearing as "dark mass"), controlled by the kernel $K(r)$;
• remains weak around an isolated BH (local profile $\propto r^{-4}$, rapid convergence of the total mass; see §11.1);
• can be significant when summed over many sources on galactic/cluster scales (halo formation; see §11.2–§11.3).

Physical picture inside a black hole. Inside the horizon, the growth of μ leads to a steady "dumping" onto Σ, so that observable quantities on R remain finite; "information" need not be lost — it is redistributed between R and I and can return via the same interfaces (see §9.4).

Causality and unitarity. The exchange proceeds through layers of finite thickness ℓ_* with local conservation laws; causality is not violated. In the quantum description the boundary terms ensure unitary evolution in the extended state space ($R \oplus I$).

Two-layer Mobius topology and separation of matter/antimatter. Early baryon asymmetry

In the topological quantization of §3, the real part of a quantum of space has a two-layer (Mobius-like) structure, which is conveniently described as two layers A and B glued along boundary regions Σ. The locally observable Universe is a superposition of contributions from both layers, but material excitations are tied to one of them: conventionally, particles to A, antiparticles to B. Such a geometry:
• admits local processes of creation/annihilation when the corresponding excitations from A and B meet;
• admits rare topologically induced "transfers" between the layers via Σ (in regimes of high densities and temperatures), parameterized by an effective rate Γ_Σ;
• in configurations with global Z_2 monodromy (Mobius-like identification along a non-contractible cycle), a single traversal along this cycle carries an excitation from layer A to layer B (a second traversal returns $B \to A$). In an orientable topology, changing the layer requires crossing a boundary Σ and is described by "transfers" with an effective rate Γ_Σ;
• naturally generates a global baryon asymmetry if, in the early Universe, an imbalance of occupancy of the layers arose and was subsequently maintained by cosmological expansion and the finite permeability of Σ.

Thus, the observed "dominance of matter" can be interpreted as a local limit of a two-layer topology, in which the "anti-layer" has been diluted and/or dynamically displaced.

Comment on parameters and links to phenomenology

• ℓ_* is the microscopic cutoff of the theory; it determines the UV boundary of the EFT and the magnitude of effective boundary couplings (see §4, §16).
• $\zeta_{i\to j}$ and the average kernel $K(r)$ are geometric parameters of the intersection network, to be extracted from macrodata (galactic rotation curves, weak lensing).
• the threshold $\mu_{\rm th}$ and the shape of nonlinearities in the potential $V(\mu, \tau)$ set the stabilization regime in strong fields, affecting subtle signatures of ringdown (QNM) and BH shadows (see §10, §9).

Taken together, §3 specifies the micro-geometry of the carrier of the theory, on which the Lagrangian construction (§4), weak-field (PPN) equivalence to GR (§7), and phenomenological consequences for black holes, dark-matter halos, and effective dark energy (§9–§12) are based.

Fields, symmetries and Lagrangian — the core of the formalism

We formalize the $\mu\tau$-approach as a local quantum field theory on a globally flat background with two universal scalar fields — the scale $\mu(x)$ and the internal time $\tau(x)$ — and an auxiliary "shift" $B_i(x)$ ensuring the correct gravito-magnetic limit. All Standard Model (SM) fields receive universal deformations via "minimal substitutions".

Field content: $\mu(x)$, $\tau(x)$, $Bi(x)$, Standard Model fields

• $\mu(x)$ — a dimensionless scalar controlling the scale of particles/waves (effective lengths, masses, cross sections).
• $\tau(x)$ — a dimensionless scalar setting the rate of internal clocks (the local speed of physical processes).
• $B_i(x)$ — a vector under $SO(3)$ (a shift in the (3+1)-decomposition), required for the correct g_{0i} in the weak field (frame dragging).
• $\Phi_{\rm SM}$ — the standard SM fields (fermions, gauge fields, Higgs), on which μ, τ act universally.

For convenient expansions we introduce fluctuations

$$\mu(x) = 1 + \frac{\phi_R(x)}{\Lambda_\mu}, \qquad \tau(x) = 1 + \frac{\sigma(x)}{\Lambda_\tau}, \tag{4}$$

where φ_R, σ are canonical scalars (mass dimension 1), and $\Lambda_{\mu,\tau}$ are the characteristic scales.

Symmetries: local Lorentz invariance, CPT, SM gauge symmetries

We work in tangent frames with standard local Lorentz invariance: in regions where $\partial\mu, \partial\tau$ are small, the dynamics reduces to special relativity with redefined local standards; μ, τ transform trivially. CPT and the SM gauge symmetries $SU(3) \times SU(2) \times U(1)$ are preserved; the "minimal substitutions" are compatible with covariant derivatives. Global translations and rotations of the background are symmetries of the action; they are broken only by specific configurations of $\mu(x)$, $\tau(x)$.

Dimensions and natural scales $\Lambda_\mu, \Lambda_\tau, \ell_*$

We work in units $\hbar = c = 1$.

$$[\phi_R] = [\sigma] = 1, \quad [\Lambda_\mu] = [\Lambda_\tau] = 1, \quad [\mu] = [\tau] = 0, \quad [B_i] = 0. \tag{5}$$

ℓ_* is the microscopic cutoff (the minimal thickness of the real↔imag boundary layers, see §3), which sets the EFT UV scale: $\Lambda_{UV} \sim 1/\ell_*$. With these dimensions the basic interactions are marginal (operator dimension ≤ 4), which ensures the consistency of the EFT.

Constructing the Lagrangian

The full action is

$$S = \int d^4x \, \big(\mathcal{L}_{\mu\tau} + \mathcal{L}_{\rm SM}(\Phi_{\rm SM} \,|\, \mu, \tau, B) + \mathcal{L}_B + \mathcal{L}_{\rm bdry}\big). \tag{6}$$

(a) Kinetics of μ, τ and potential $V(\mu, \tau)$ (vacuum energy).

$$\mathcal{L}_{\mu\tau} = \frac{1}{2}\,\partial_\alpha\phi_R\,\partial^\alpha\phi_R + \frac{1}{2}\,\partial_\alpha\sigma\,\partial^\alpha\sigma - V(\mu,\tau), \tag{7}$$

$$V(\mu,\tau) = \frac{m_\mu^2}{2}\,\phi_R^2 + \frac{m_\tau^2}{2}\,\sigma^2 + \frac{\lambda_\mu}{4}\,\phi_R^4 + \frac{\lambda_\tau}{4}\,\sigma^4 + \frac{\lambda_\times}{2}\,\phi_R^2\sigma^2 + V_0, \tag{8}$$

where V_0 is the vacuum energy (a contribution to the effective $\Lambda_{\rm grav}$ in cosmology). The potential is bounded from below; a small mixing $\lambda\times$ is allowed.

(b) "Minimal substitutions" in the SM: $m \to m\mu$, $\partial_t \to \tau\,(\partial_t + B_i\partial_i)$. For all SM fields:

- Mass/threshold rescaling: $m \mapsto\to m\mu(x)$ (for fermions $m_f\bar\psi\psi$, for vectors $m_V^2 A_\mu A^\mu/2$, for the Higgs — the mass parameter and the VEV).
- Time flow in the kinetic terms:

$$\partial_t \;\longrightarrow\; \tau(x)\,\big(\partial_t + B_i(x)\partial_i\big), \tag{9}$$

while spatial contractions are with δ_{ij} (or with $\mu^2\delta_{ij}$ in the "optical" representation). For gauge fields $\partial_\mu \to D_\mu$; the substitution concerns D_t.

Examples.

$$\mathcal{L}_X = \frac{1}{2}\Big[\tau^2\big(\dot X + B_i\partial_i X\big)^2 - (\nabla X)^2\Big] - \frac{1}{2}\,m_X^2\,\mu^2 X^2, \tag{10}$$

$$\mathcal{L}_\psi = \bar\psi\Big[i\gamma^0\,\tau(\partial_t + B_i\partial_i) + i\gamma^i\partial_i - m_\psi\,\mu\Big]\psi, \tag{11}$$

while for electrodynamics the temporal components are modified (see §8: the "index of refraction" for light).

These rules ensure the universality of deformations and preserve the gauge symmetries.

(c) $\mathcal{L}_B$ (gravito-magnetic shift, PPN limit).

$$\mathcal{L}_B = \frac{M_B^2}{2}\Big[(\partial_i B_j - \partial_j B_i)^2 - (\partial_i B_i)^2\Big] - B_i J^i, \tag{12}$$

where J^i is the effective matter momentum current (in the weak field $J^i \simeq \kappa\,T^{0i}$ with κ determined from PPN matching), and M_B sets the normalization. In the gauge $\partial_i B_i = 0$ the variation gives

$$\nabla^2 B_i = -\kappa\, T^{0i}, \tag{13}$$

which reproduces g_{0i} and frame dragging at first PN order (see §7).

(d) Boundary term real↔imag (thick boundary, dimensionless effective coupling). On each interface Σ between the real and imaginary parts of a space quantum (see §3) we introduce a local contribution

$$\mathcal{L}_{\rm bdry} = \int_\Sigma d^3\xi\; h\; \tilde g(\mu,\tau)\; \mathcal{O}_R\big(\Phi_{\rm SM},\phi_R,\sigma\big)\; \mathcal{O}_I\big[\text{imaginary modes}\big], \tag{14}$$

with smoothing over the thickness ℓ_*. The effective coupling $\tilde g \sim g/\ell_*$ is dimensionless, ensuring the absence of UV pathologies and EFT consistency. This term implements the "dumping of excess scale" and the back-reaction from the imaginary sector (see §3.4).

Energy–momentum tensor, Noether currents and conservation laws

In the bulk the Noether energy–momentum tensor

$$T^{\mu\nu} = \sum_\Phi \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi)}\,\partial^\nu\Phi - \eta^{\mu\nu}\mathcal{L}, \qquad \Phi\in\{\phi_R,\sigma,B_i,\Phi_{\rm SM}\}, \tag{15}$$

obeys $\partial_\mu T^{\mu\nu} = 0$. On the interfaces Σ surface currents arising from $\mathcal{L}_{\rm bdry}$ appear, and the conservation law takes the form

$$\partial_\mu T^{\mu\nu} + \delta_\Sigma\, j^\nu_{\rm surf} = 0, \tag{16}$$

which ensures global conservation under the exchange between the real and imaginary sectors. The SM gauge currents are conserved (their covariant divergences vanish); CPT is preserved due to locality and real coefficients.

Signs of kinetic terms, absence of ghosts; hyperbolicity (Cauchy problem)

The kinetic terms $(\partial\varphi_R)^2$, $(\partial\sigma)^2$ have the correct signs; $M^2_B > 0$. The potential V is bounded from below — there are no ghosts or gradient instabilities in the (φ_R, σ, B_i) subspace.

The principal parts of the equations of motion are second order in time; for matter the operator has the form

$$\tau^2\partial_t^2 - \Delta \quad (\text{in the presence of } B_i \text{ — with a convective correction } \propto B_i\partial_i),$$

which defines a well-posed Cauchy problem. The requirement $\tau(x) > 0$ fixes the orientation of "time" and excludes sign flips of the temporal norm.

The physical characteristic speeds do not exceed the speed of light; "apparently superluminal" quantities are artifacts of local rescaling of standards (see §2.2), and causality is not violated.

The boundary thickness ℓ_* provides a natural UV cutoff $\Lambda UV \sim 1/\ell_*$: boundary interactions do not generate irremovable divergences; in the bulk all vertex coefficients are dimensionless (or of positive dimension), yielding standard EFT control (see §16).

Quantization of the $\mu\tau$ fields and the quasi-particle spectrum (graviton)

Linearizing the scale fields

$$\mu = 1+\delta\mu, \qquad \tau = 1+\delta\tau, \qquad B_i = \delta B_i \tag{17}$$

on the background of the observable metric $ds^2_{\rm obs}$ (see §5), we obtain a set of wave-like perturbations. Physically, the *graviton* corresponds to a massless tensor mode with spins ±2, which is conveniently described in tangent frames through the transverse traceless combinations $\{h_+, h_\times\}$, expressed in terms of $\delta\mu$, $\delta\tau$, δBi and preserving gauge invariance.

At this level:

• the tensor polarizations h_+, $h_\times$ propagate with speed c (like light) and couple to the universal source $T_{\mu\nu}$ of matter (at quadrupole order);
• a weak scalar admixture is possible — a "breathing" mode

$$s \propto \delta(\mu/\tau), \tag{18}$$

with a dimensionless amplitude $\kappa_b \ll 1$ that does not modify 1PN predictions;

• vector combinations are suppressed by gauge conditions and do not form new long-range polarizations.

Canonical normalization of the Lagrangian for the tensor modes leads to the standard energy flux of gravitational waves:

$$\langle F\rangle = \frac{c^3}{32\pi G}\,\langle \dot h_+^2 + \dot h_\times^2 + \kappa_b\,\dot s^{\,2}\rangle, \tag{19}$$

and the universality of the coupling fixes the `scale equivalence principle` and ensures that the $\{h_+, h_\times\}$ sector coincides with the GR predictions at the quadrupole level (see §10).

Summary of the section. We have introduced a minimalist yet complete Lagrangian formalism for the $\mu\tau$-approach: two universal scalars (scale and internal time), a shift B_i for the gravito-magnetic sector, "minimal substitutions" in the SM, and a regularized boundary exchange with the imaginary sector. The construction (i) preserves gauge symmetries and CPT, (ii) defines a well-posed hyperbolic dynamics without ghosts, (iii) yields the correct weak-field (PPN) limit, and (iv) provides a meaningful UV cutoff via ℓ_*. In the following sections we derive the equations of motion and carry out PPN matching with GR.

Equations of motion and the "observable metric"

In this section, starting from the action of §4, we derive the Euler–Lagrange equations for the fields μ, τ, B_i and matter, formulate the operational observation interval (the effective "measurement metric"), and show that in the weak-field regime it reduces to the standard post-Newtonian form, coinciding with GR at the PPN level.

Euler–Lagrange equations for μ, τ, B_i and SM fields

Recall the structure of the action

$$S = \int d^4x\,\Big(\mathcal{L}_{\mu\tau} + \mathcal{L}_{\rm SM}(\Phi_{\rm SM}\,|\,\mu,\tau,B) + \mathcal{L}_B + \mathcal{L}_{\rm bdry}\Big), \tag{20}$$

where $\mathcal{L}_{\mu\tau}$ contains canonical kinetic terms for the fluctuations φ_R, σ (so that $\mu = 1+\varphi_R/\Lambda_\mu$, $\tau = 1+\sigma/\Lambda_\tau$) and the potential $V(\mu,\tau)$, $\mathcal{L}_B$ describes the "gravito-magnetic" sector, and $\mathcal{L}_{\rm bdry}$ encodes boundary contributions at the $R \leftrightarrow I$ interfaces of finite thickness ℓ^* (see §3).

(i) Scale field μ. Varying with respect to Φ_R we obtain

$$\phi_R + \frac{\partial V}{\partial \phi_R} = J_\mu + J_\mu^{(\rm bdry)}, \qquad J_\mu \equiv \frac{\partial \mathcal{L}_{\rm SM}}{\partial \mu}\,\frac{\partial \mu}{\partial \phi_R} = \frac{1}{\Lambda_\mu}\,S_\mu. \tag{21}$$

Here S_μ is the universal "mass" source (a sum over SM fields). For simple fields:

$$S_\mu \supset \begin{cases} -\,m_X^2\,X^2, & \text{scalar } X,\\ -\,m_\psi\,\bar\psi\psi, & \text{fermion } \psi,\\ -\,m_A^2\,A_\alpha A^\alpha, & \text{massive vector } A_\alpha. \end{cases} \tag{22}$$

The boundary contribution $J_\mu^{(\rm bdry)}$ is determined by the variation of $\mathcal{L}_{\rm bdry}$ (see §4.4).

(ii) Internal-time field τ . Varying with respect to σ we obtain

$$\sigma + \frac{\partial V}{\partial \sigma} = J_\tau + J_\tau^{(\rm bdry)}, \qquad J_\tau \equiv \frac{\partial \mathcal{L}_{\rm SM}}{\partial \tau}\,\frac{\partial \tau}{\partial \sigma} = \frac{1}{\Lambda_\tau}\,S_\tau. \tag{23}$$

The source S_τ is the "temporal" density of kinetic energies, i.e. that part of the Lagrangian which is multiplied by □ in the replacement $\partial_t \to \tau\,(\partial_t + B_i\partial_i)$. For example:

$$S_\tau \supset \begin{cases} +\,\tau\,(\dot X + B_i\partial_i X)^2, & \text{scalar } X,\\ +\,\bar\psi\,\gamma^0 i(\partial_t + B_i\partial_i)\psi, & \text{fermion } \psi,\\ +\,\mathbf{E}\cdot\mathbf{E}, & \text{electromagnetic field (in a convenient gauge)}. \end{cases} \tag{24}$$

(iii) Shift B_i (gravito-magnetic sector). Varying with respect to B_i, we obtain an equation of Proca type (with zero mass) sourced by the matter momentum flux:

$$M_B^2\Big(\nabla^2 B_i - \partial_i\partial_j B_j\Big) \;=\; J_i^{(B)} + J_i^{(\mathrm{bdry})}, \qquad J_i^{(B)} \equiv \frac{\partial \mathcal{L}_{\mathrm{SM}}}{\partial B_i}. \tag{25}$$

In the gauge $\partial_i B_i = 0$ and in the weak field $J_i^{(B)} \simeq \kappa\, T^{0i}$; a suitable choice of κ/M_B^2 reproduces the standard g_{0i} and frame-dragging effects (see §5.3).

(iv) Standard Model fields. The "minimal substitution" rules of §4.4 lead to

$$D_t \;\equiv\; \tau\big(\partial_t + B_i\partial_i\big), \qquad m \;\mapsto\; m\,\mu, \tag{26}$$

and to the standard equations of motion with these modifications. For a scalar X:

$$D_t^2 X - \Delta X + m_X^2\,\mu^2 X \;=\; (\text{interactions}), \tag{27}$$

while preserving gauge currents and CPT. Boundary conditions on Σ provide the balance of fluxes between the real and imaginary sectors (see §3.2 and §4.5).

Effective observation interval and the time-flow operator

Since μ and τ retune the standards (rods and clocks), it is natural to introduce an `operational` interval that is directly "read out" by instruments (atomic clocks, interferometers, photon tracers):

$$\boxed{ds^2_{\mathrm{obs}} = -\,\tau^2 c^2\, dt^2 + 2\,\tau^2 B_i\, dx^i dt + \mu^2\,\delta_{ij}\, dx^i dx^j} \tag{28}$$

This interval is not postulated as a fundamental dynamical metric, but is reconstructed from the equations of motion for matter: phase velocities, fronts and invariants behave as if they were propagating in ds^2_{obs}. The corresponding universal "time-flow" operator is

$$\boxed{D_t \equiv \tau(\partial_t + B_i\partial_i)} \tag{29}$$

and is the same for all fields, realizing the scale formulation of the equivalence principle: locally, for nearly constant μ, τ,B, all microphysics coincides with SR in local units.

Weak field and matching to PPN

For a static weak source with Newtonian potential U we expand

$$\mu = 1 + \frac{U}{c^2} + \mathcal{O}(c^{-4}), \qquad \tau = 1 - \frac{U}{c^2} + \frac{1}{2}\frac{U^2}{c^4} + \mathcal{O}(c^{-6}), \qquad B_i = -\frac{4}{c^3}\,V_i + \mathcal{O}(c^{-5}), \tag{30}$$

where V_i is the standard post-Newtonian vector potential (a convolution of ρv_i with the Newtonian kernel). Substituting into (28), we obtain the components of the "observable metric":

$$g_{00}^{(\mathrm{obs})} = -\tau^2 = -\Big(1 - \frac{2U}{c^2} + \frac{2U^2}{c^4}\Big) + \mathcal{O}(c^{-6}), \tag{31}$$

$$g_{ij}^{(\mathrm{obs})} = \mu^2\,\delta_{ij} = \Big(1 + \frac{2U}{c^2}\Big)\delta_{ij} + \mathcal{O}(c^{-4}), \tag{32}$$

$$g_{0i}^{(\mathrm{obs})} = \tau^2 B_i = -\,\frac{4}{c^3}\,V_i + \mathcal{O}(c^{-5}). \tag{33}$$

Thus, in the 1PN approximation (including the U^2/c^4 term in g_{00}) we obtain the same observable components as in GR in the standard PPN gauge:

$$\gamma = 1, \qquad \beta = 1, \qquad \alpha_{1,2,3} = \xi = \zeta_i = 0,$$

and gravito-magnetism (dragging of inertial frames) is given by $g_{0i} = -4V_i/c^3$. The classical tests — light deflection by the Sun, Shapiro delay, perihelion precession, and Lense–Thirring effect — therefore show no deviations from GR at the level of current accuracy.

Summary. The operational metric $ds^2_{\rm obs}$, derived from the universal deformations μ, τ, B_i in the equations of motion of matter, automatically reproduces weak-field effects and provides a correct starting point for the full PPN analysis (§7). We further use it to derive the optics of gravity (lensing, redshift) and to formulate strong-field predictions (black holes, gravitational waves).

Linearized theory: SCRE ($\mu\tau$ versions of wave equations)

We linearize the theory around a homogeneous background

$$\mu = 1 + \frac{\phi_R}{\Lambda_\mu}, \qquad \tau = 1 + \frac{\sigma}{\Lambda_\tau}, \qquad B_i = \mathcal{O}(v/c),$$

assuming $|\varphi_R|/\Lambda_\mu, |\sigma|/\Lambda_\tau, |B_i| \ll 1$ and keeping only first order in the gradients $\partial\mu, \partial\tau, \partial B$. The `resulting Scale-Clock Relativistic Equations` (SCRE) are standard relativistic wave equations with the substitutions

$$m \;\to\; m\,\mu(x), \qquad \partial_t \;\to\; D_t \equiv \tau(x)\big(\partial_t + B_i(x)\partial_i\big),$$

while spatial derivatives ∂_i remain unchanged (see §4.4).

Scalar (Klein–Gordon): mass ↔ scale, time ↔ rate

For a complex scalar X with mass m_X the Lagrangian of §4.4 gives

$$D_t^2 X - \nabla^2 X + m_X^2\,\mu^2\,X = 0, \qquad D_t = \tau(\partial_t + B_i\partial_i). \tag{34}$$

For a plane wave $\sim e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$ we obtain

$$\big[\tau(\omega - \mathbf{B}\cdot\mathbf{k})\big]^2 = k^2 + m_X^2\mu^2. \tag{35}$$

In the laboratory frame (B = 0):

$$\omega^2 = \frac{k^2 + m_X^2\mu^2}{\tau^2}, \qquad v_{\rm ph} = \frac{\omega}{k} = \frac{1}{\tau}\sqrt{1 + \frac{m_X^2\mu^2}{k^2}}, \qquad v_{\rm gr} = \frac{\partial\omega}{\partial k} = \frac{1}{\tau}\frac{k}{\sqrt{k^2 + m_X^2\mu^2}}. \tag{36}$$

Thus, μ retunes the effective mass, while τ retunes the time scale of the dynamics.

Fermions (Dirac/Weyl/Majorana): spinor consistency, CPT

For a Dirac fermion ψ:

$$\big[i\gamma^0 D_t + i\gamma^i\partial_i - m_f\,\mu\big]\psi = 0, \qquad D_t = \tau(\partial_t + B_i\partial_i). \tag{37}$$

Squaring the operator (or working with a plane wave) yields the same dispersion relation:

$$\big[\tau(\omega - \mathbf{B}\cdot\mathbf{k})\big]^2 = k^2 + m_f^2\mu^2, \tag{38}$$

which excludes spin-dependent "fifth forces" and preserves equivalence at the 1PN level.

Chiral fields. For massless spinors the replacement $\partial_t \to D_t$ gives $\tau^2(\omega - \mathrm{B}\cdot\mathrm{k})^2 = k^2$, i.e. lightlike characteristics with respect to $ds^2_{\rm obs}$ (§5.2).

Majorana masses. Terms $mM\mu\,\psi^T C\psi + \mathrm{h.c.}$ are compatible with CPT and SM gauge symmetries; μ is a common scale "handle" for mass parameters.

Vectors (Maxwell/Proca): "optics of gravity", index of refraction

Maxwell. On a homogeneous background ($\nabla\mu = \nabla\tau = 0$, B_i = const) the Maxwell equations lead to the dispersion relation

$$\omega \simeq \frac{\tau}{\mu}\,|\mathbf{k}| \;+\; \tau\,\mathbf{B}\!\cdot\!\hat{\mathbf{k}} \quad\Rightarrow\quad v_{\rm ph} = \frac{\omega}{|\mathbf{k}|} \simeq \frac{\tau}{\mu}, \qquad n \equiv \frac{c}{v_{\rm ph}} = \frac{\mu}{\tau}, \tag{39}$$

and the term $\propto \mathbf{B}\!\cdot\!\hat{\mathbf{k}}$ is the "drift" of the medium (gravito-magnetism). Hence, the optics of gravity in our approach is optics in a weakly inhomogeneous medium with

$$n(\mathbf{x}) = \frac{\mu(\mathbf{x})}{\tau(\mathbf{x})}, \tag{40}$$

from which lensing and Shapiro delay are derived (see §8).

Proca. For a massive vector A_α with mass m_V :

$$D_t^2 A_i - \nabla^2 A_i + m_V^2 \mu^2 A_i - \partial_i\big(D_t A_0 + \partial_j A_j\big) = 0, \tag{41}$$

and in the gauge $\partial_i A_i + D_t A_0 = 0$ the dispersion relation

$$\big[\tau(\omega - \mathbf{B}\!\cdot\!\mathbf{k})\big]^2 = k^2 + m_V^2 \mu^2 \tag{42}$$

coincides with the general form; the longitudinal degree of freedom is well-behaved for the kinetic signs chosen in §4.6.

Spin 3/2 (Rarita–Schwinger): consistency and limitations

The Rarita–Schwinger Lagrangian with the "minimal substitutions"

$$\mathcal{L}_{3/2} = \bar{\psi}_\mu\,\gamma^{\mu\nu\rho} D_\nu \psi_\rho - m_{3/2}\mu\,\bar{\psi}_\mu\,\gamma^{\mu\nu}\psi_\nu, \qquad D_\nu = (D_t, \partial_i), \tag{43}$$

for weak gradients $\partial\mu, \partial\tau$ satisfies the standard conditions $\gamma_\mu \psi^\mu = 0$, $\partial_\mu \psi^\mu \approx 0$. Since the background is specified by the scalars μ, τ (and a weak drift B_i), there are no EM tensor couplings that would lead to Velo Zwanziger pathologies; the principal part of the operator is hyperbolic, and the characteristic cones coincide with the null cones of $ds^2_{\rm obs}$ (§5.2). For the massive case there remain $2s+1$ = 4 physical polarizations; the rest are removed by subsidiary constraints.

Domain of applicability. Strong inhomogeneities $\partial\mu, \partial\tau$ can spoil exact subsidiary conditions beyond linear order — this is the natural EFT limit; the conditions of §4.6 are necessary to avoid ghosts/gradient instabilities.

Equivalence: universality of deformations, absence of a selective "fifth force"

All spins (0, 1/2, 1, 3/2) obey the same deformation structure:

$$m \to m\,\mu(x), \qquad \partial_t \to D_t = \tau(\partial_t + B_i \partial_i),$$

which leads to a single dispersion relation

$$\boxed{\tau^2\big(\omega - \mathbf{B}\!\cdot\!\mathbf{k}\big)^2 = k^2 + m_{\rm eff}^2, \qquad m_{\rm eff} = m\,\mu} \tag{44}$$

(for massless fields m = 0). Hence:

- **No selective "fifth force"**. The couplings are composition-independent and universal ⇒ a vanishing Eotvos parameter at 1PN order and agreement with GR in standard tests (§7–§8).
- **Unified optics**. For light $n = \mu/\tau$, and for matter the geodesics coincide with the trajectories of the "observable metric" (§5.2); refraction, Shapiro delay and precessions are reproduced at 1PN order.
- Causality and unitarity. The characteristics of SCRE coincide with the null cones of $ds^2_{\rm obs}$; $\tau > 0$ fixes the "arrow of time", and positive kinetic terms (§4.6) exclude ghosts.

Thus, the linearized $\mu\tau$ versions of the wave equations provide a universal, gauge-compatible and causal weak-field limit, coinciding with GR in tested domains and forming the basis for strong-field phenomenology (lensing, GW polarizations, QNM) in the subsequent sections.

Quantization of waves: operators and the graviton propagator

In the linear theory the tensor perturbation is represented by an expansion in plane waves:

$$h_{ab}(x) = \sum_{\lambda=+,\times} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[a_k(\lambda)\, \epsilon_{ab}(\lambda)\, e^{-ik\cdot x} + \text{h.c.} \right], \tag{45}$$

where $\epsilon_{ab}(\lambda)$ are transverse traceless polarization tensors, and $\omega_k = c\,|k|$. The canonical commutators

$$\left[a_k(\lambda),\, a^\dagger_{k'}(\lambda') \right] = (2\pi)^3 \delta^{(3)}(k-k')\, \delta_{\lambda\lambda'} \tag{46}$$

ensure microcausality with respect to the "observable" light cone (§17).

The propagator of the massless tensor mode in Fourier space is taken in the transverse–traceless gauge and has the standard projector onto the spin-2 subspace, so that the Feynman rules for graviton exchange with sources $T_{\mu\nu}$ coincide with the familiar ones of linearized GR. The scalar admixture s is quantized as a free scalar field with a suppressed coupling $\propto \kappa_b$.

Weak-field limit: full PPN analysis from the Lagrangian

We now connect the formalism of §4–§5 with the observable metric coefficients in the post-Newtonian (PPN) expansion and show that, at first PN order, the theory coincides with GR:

$$\gamma = 1, \qquad \beta = 1, \qquad \alpha_i = \xi = \zeta_i = 0.$$

We then extract constraints on possible deviations of the profiles μ, τ, B_i from the classical tests (Cassini, LLR, VLBI, GP B/LAGEOS).

Expansions of μ, τ, B_i in the Newtonian potential and velocities

Let $U(x,t)$ be the Newtonian potential of the source, and v the characteristic velocity of matter. In the weak-field regime, $\epsilon \sim U/c^2 \sim v2/c^2 \ll 1$, we expand

$$\mu(x,t) = 1 + a_1 \frac{U}{c^2} + a_2 \frac{U^2}{c^4} + \mathcal{O}(\epsilon^3), \tag{47}$$

$$\tau(x,t) = 1 + b_1 \frac{U}{c^2} + b_2 \frac{U^2}{c^4} + \mathcal{O}(\epsilon^3), \tag{48}$$

$$B_i(x,t) = \frac{b_V}{c^3} V_i + \frac{b_W}{c^3} W_i + \mathcal{O}(\epsilon^{5/2}), \tag{49}$$

where V_i, W_i are the standard PPN vector potentials. The coefficients $(a_1, a_2, b_1, b_2, b_V, b_W)$ are fixed by varying the action (§4, §5) and matching to the "observable" metric (§5.2):

$$ds^2_{\text{obs}} = -\tau^2 c^2 dt^2 + 2\,\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij}\, dx^i dx^j. \tag{50}$$

The requirement of agreement with GR at 1PN order yields

$$\boxed{a_1 = +1, \quad a_2 = 0; \qquad b_1 = -1, \quad b_2 = \tfrac{1}{2}; \qquad b_V = -4, \quad b_W = -\tfrac{1}{2}}. \tag{51}$$

From this we obtain the components of the "observable" metric:

$$g^{(\text{obs})}_{00} = -\tau^2 = -\left(1 - \frac{2U}{c^2} + \frac{2U^2}{c^4}\right) + \mathcal{O}(c^{-6}), \tag{52}$$

$$g^{(\text{obs})}_{ij} = \mu^2 \delta_{ij} = \left(1 + \frac{2U}{c^2}\right)\delta_{ij} + \mathcal{O}(c^{-4}), \tag{53}$$

$$g^{(\text{obs})}_{0i} = \tau^2 B_i = -\frac{4}{c^3} V_i - \frac{1}{2c^3} W_i + \mathcal{O}(c^{-5}), \tag{54}$$

which coincides with the standard PPN form of GR (Will's gauge) at 1PN order.

Variation with respect to $B_i \Rightarrow$ equation for $g0i$ (frame dragging)

From $\mathcal{L}_B$ (§4.4) and the "minimal substitutions" (§4.4), variation with respect to B_i in the gauge $\partial_i B_i = 0$ gives

$$\nabla^2 B_i = -\,\kappa\, T^{0i} + \mathcal{O}(\epsilon^{5/2}), \qquad T^{0i} \simeq \rho\, v^i. \tag{55}$$

Choosing the normalization κ/M_B^2 such that $b_V = -4$ and $b_W = -\frac{1}{2}$ leads to

$$\boxed{g_{0i}^{(\mathrm{obs})} = \tau^2 B_i = -\frac{4}{c^3} V_i - \frac{1}{2c^3} W_i}, \tag{56}$$

which reproduces the GR frame-dragging (Lense–Thirring effect) at 1PN order.

Mapping to U,Φ_1..4, V_i,W_i; values of γ, β, αi, ξ, ζ_i

Comparing $g_{\mu\nu}^{(\mathrm{obs})}$ with the general PPN form:

$$g_{00} = -1 + \frac{2U}{c^2} - \frac{2\beta U^2}{c^4} + \frac{2}{c^4}\big(\Phi_1 + \Phi_2 + \Phi_3 + 3\Phi_4\big) + \cdots, \tag{57}$$

$$g_{ij} = \Big(1 + \frac{2\gamma U}{c^2}\Big)\delta_{ij} + \cdots, \tag{58}$$

$$g_{0i} = -\frac{1}{c^3}\Big[\tfrac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2)\, V_i + \tfrac{1}{2}(1 + \alpha_2)\, W_i\Big] + \cdots, \tag{59}$$

we obtain the PPN parameters of our theory:

$$\boxed{\gamma = 1, \qquad \beta = 1, \qquad \alpha_{1,2,3} = 0, \qquad \xi = 0, \qquad \zeta_{1,2,3,4} = 0}. \tag{60}$$

Thus: (i) the spatial curvature per unit mass (γ) and the nonlinearity of superposition (β) coincide with GR; (ii) the preferred-frame parameters α_i and the momentum/energy nonconservation parameters ζ_i, ξ vanish, as in metric theories without a preferred frame; (iii) the coefficients in front of $\Phi_{1..4}$, V_i, W_i match those of GR, which is crucial for Shapiro delay, perihelion precession, and frame dragging

Comparison with Cassini, LLR, VLBI, GP B/LAGEOS: numerical bounds and parameter limits

Since $\gamma = \beta = 1$ and $\alpha_i = \xi = \zeta i = 0$, all classical tests at the 1PN level are automatically satisfied. Their accuracy provides bounds on deviations of the calibration coefficients (a_1, a_2, b_1, b_2, b_V , b_W). Introduce

$$\delta a_1 \equiv a_1 - 1, \quad \delta b_1 \equiv b_1 + 1, \quad \delta b_2 \equiv b_2 - \tfrac{1}{2}, \quad \delta b_V \equiv b_V + 4, \quad \delta b_W \equiv b_W + \tfrac{1}{2}.$$

Then:

- **Cassini (Shapiro delay):** $|\gamma - 1| 2 \times 10^{-5} \Rightarrow |\delta a_1 + \delta b_1| 2 \times 10^{-5}$.
- **LLR (Nordtvedt parameter):** $|\beta - 1| 10^{-4} \Rightarrow |\delta b_2| 10^{-4}$ (taking into account quadratic combinations of a_1, b_1).
- VLBI: refines the constraint on γ to $\sim 10^{-4}$ using independent light paths $\Rightarrow$ joint bounds on δa_1, δb_1.
- GP-B, LAGEOS (frame dragging): agreement with GR at the level of several percent $\Rightarrow$ $|\delta b_V|$, $|\delta b_W|$few% (the normalization of the gravito-magnetic sector, κ/M_B^2, is fixed at this level).

In summary,

$$|\delta a_1|, |\delta b_1| 10^{-5} - 10^{-4}, \qquad |\delta b_2| 10^{-4}, \qquad |\delta b_V|, \quad |\delta b_W| \text{a few percent.}$$

These bounds are compatible with natural values of the Lagrangian parameters (§4.3) and confirm that the $\mu\tau$-approach preserves the tested phenomenology of the Solar System. Nonzero deviations are expected only at second PN order and/or in strong-field regimes — the subject of the following sections (optics, GW polarizations, QNMs, S-stars).

Light and time in a gravitational field (observable tests)

In this section we apply the "$\mu\tau$–Maxwell" theory and the "observable metric" (§5) to key effects: lensing, Shapiro delay, gravitational redshift, and perihelion precession. At 1PN order our theory coincides with GR; we then write controlled 2PN corrections in terms of the expansion coefficients of μ, τ,B_i (§7).

Lensing in $\mu\tau$–Maxwell (deflection angle, subtle corrections)

In the $\mu\tau$ formulation light propagates as in a weakly inhomogeneous medium with effective index of refraction

$$n(\mathbf{x}) = \frac{\mu(\mathbf{x})}{\tau(\mathbf{x})}. \tag{61}$$

For a static spherically symmetric source with Newtonian potential $U(r) = GM/r$ we use the expansions of §7.1

$$\mu = 1 + a_1\frac{U}{c^2} + a_2\frac{U^2}{c^4}, \qquad \tau = 1 + b_1\frac{U}{c^2} + b_2\frac{U^2}{c^4}, \tag{62}$$

and obtain (up to $\mathcal{O}(c^{-4})$)

$$n = 1 + \underbrace{(a_1 - b_1)}_{=2}\frac{U}{c^2} + \underbrace{\left[a_2 + (b_1^2 - b_2) - a_1 b_1\right]}_{=3/2}\frac{U^2}{c^4} + \mathcal{O}(c^{-6}), \tag{63}$$

where the numerical values of the coefficients are fixed by the PPN calibration (§7.1): $a_1 = +1, b_1 = -1, a_2 = 0, b_2 = \frac{1}{2}$.

The deflection angle for a light ray with impact parameter b is given by the "optical" formula

$$\alpha = 2\int_{-\infty}^{+\infty} \partial_\perp (n-1)\, dz. \tag{64}$$

Integrating successively the terms $\propto U$ and $\propto U^2$ along the unperturbed straight line (with error $\mathcal{O}(c^{-6})$), we obtain

$$\boxed{\alpha = \frac{4GM}{b\,c^2} + \frac{15\pi}{4}\frac{G^2M^2}{b^2c^4} + \mathcal{O}\left(\frac{G^3M^3}{b^3c^6}\right)} \tag{65}$$

i.e. at 1PN and 2PN orders the coefficients coincide with the Schwarzschild result in isotropic gauge. Any deviations are possible only if a_2, b_2 depart from their calibrated values (§7.4).

Shapiro delay and gravitational redshift (up to second order)

Shapiro delay. The phase velocity is $v_{ph} = c/n$. The additional travel time along a ray trajectory L is

$$\Delta t = \frac{1}{c}\int_L (n-1)\, dl. \tag{66}$$

Substituting $n - 1 = \frac{2U}{c^2} + \frac{3}{2}\frac{U^2}{c^4}$ and integrating along the unperturbed path with impact parameter b, we obtain

$$\boxed{\Delta t = \frac{2GM}{c^3}\ln\left(\frac{4\,r_E r_R}{b^2}\right) + \frac{3\pi}{2}\frac{G^2M^2}{b\,c^5} + \mathcal{O}\left(\frac{G^3M^3}{b^2c^7}\right)} \tag{67}$$

where r_E, r_R are the distances from the source (of mass M) to the receiver and emitter. The first, logarithmic term is the classical Shapiro formula for $\gamma = 1$; the second is the 2PN correction, which coincides with GR for our choices $a_2 = 0, b_2 = \frac{1}{2}$.

Gravitational redshift. In the $\mu\tau$-approach the photon frequency scales as $\nu \propto \tau$. For emission at point A and reception at B:

$$\frac{\nu_B}{\nu_A} = \frac{\tau_A}{\tau_B}. \tag{68}$$

With the expansion $\tau = 1 - \frac{U}{c^2} + \frac{1}{2}\frac{U^2}{c^4}$ up to second order we obtain

$$\boxed{z \equiv \frac{\nu_A - \nu_B}{\nu_B} = \frac{U_A - U_B}{c^2} + \frac{U_A^2 - U_B^2}{2\,c^4} + \mathcal{O}(c^{-6})} \tag{69}$$

i.e. the 1PN and 2PN terms coincide with the standard GR prediction for our calibrated b_2.

Pericenter precession (Mercury, S-stars); comparison with GR

Using the observable metric (§5.2) with the coefficients from §7.1, the equations of motion for a test particle on a quasi-Keplerian orbit yield the standard 1PN shift of pericenter per orbit:

$$\boxed{\Delta\varpi_{1\mathrm{PN}} = \frac{6\pi GM}{a(1-e^2)c^2}} \tag{70}$$

where a is the semi-major axis and e the eccentricity. This coincides with GR and describes both Mercury's precession and the average pericenter shift of stars S2/S62/S4714/S4716 for sufficiently large pericenters.

At 2PN order the shift can be parametrized via $\epsilon \equiv GM/[a(1-e^2)c^2]$:

$$\Delta\varpi = \Delta\varpi_{1\mathrm{PN}}\left[1 + \kappa_{2\mathrm{PN}}\,\epsilon + \mathcal{O}(\epsilon^2)\right], \tag{71}$$

where κ_{2PN} depends on the 2PN parts of g_{00}, g_{ij} , i.e. in our notation on (a_2, b_2). For the calibrated values, κ_{2PN} coincides with the isotropic GR result; possible deviations appear only when $|\delta b_2|, |\delta a_2| \neq 0$ and, for Mercury, lie below current sensitivity (10−3 of the 1PN term). For S-stars near Sgr A* the 2PN correction scales as $\sim \epsilon$ with $\epsilon \sim 10^{-3} - 10^{-2}$ at pericenters $100Rs$; with current astrometry this is at the edge of detectability, making S-orbits a promising test of δa_2, δb_2 (§14).

Summary of the section.

- **Lensing:** $\alpha = \dfrac{4GM}{bc^2} + \dfrac{15\pi}{4}\dfrac{G^2M^2}{b^2c^4} + \ldots$ — agreement with GR up to 2PN.
- **Shapiro delay:** the standard logarithmic term (Cassini) and a 2PN term $\propto (GM)^2/(bc^5)$ withthe GR coefficient for $a_2 = 0, b_2 = \frac{1}{2}$.
- **Redshift:** 1PN and 2PN orders coincide with GR for our coefficients.
- **Pericenter precession:** 1PN as in GR; 2PN deviations depend on (a_2, b_2) and are testable with S-stars.

Thus, the $\mu\tau$-approach preserves all classical optical and orbital tests in the Solar System and predicts measurable (though small) 2PN differences in strong-field regimes, defining an observational test program in §13–§14.

Black holes and strong fields

In the strong-field regime, $\mu(x), \tau\,(x)$ cease to be small perturbations and determine the dynamics of the "observable" causal structure via

$$\boxed{ds^2_{\mathrm{obs}} = -\,\tau^2c^2\,dt^2 + 2\,\tau^2B_i\,dx^i dt + \mu^2\delta_{ij}\,dx^i dx^j} \tag{72}$$

(see also §5.2). Below we describe: (i) radial profiles μ, τ for a static black hole, (ii) the mechanism of strong-field stabilization via a "scale dump" into the imaginary sector, (iii) the fate of inextendible geodesics in the $\mu\tau$ language, and (iv) the resolution of the information paradox.

Exterior region, horizon, interior: evolution of scale and "clocks"

Consider, for definiteness, a stationary spherically symmetric non-rotating black hole (outside 1PN corrections we may set $B_i = 0$). Let r be the radial coordinate of the flat background.

Exterior region ($r \gg r_s$). As in §7.1,

$$\mu(r) \simeq 1 + \frac{GM}{rc^2}, \qquad \tau(r) \simeq 1 - \frac{GM}{rc^2} + \frac{1}{2}\frac{G^2M^2}{r^2c^4}, \tag{73}$$

so that the "optics of gravity" is encoded in

$$n(r) = \frac{\mu}{\tau} \simeq 1 + \frac{2GM}{rc^2} + \frac{3}{2}\frac{G^2M^2}{r^2c^4}. \tag{74}$$

Horizon. The operational horizon is defined by the condition

$$\tau(r_h) = 0. \tag{75}$$

Near r_h the behavior of clocks dominates over the scale: $n(r) \to \infty$, and for radial null curves we have

$$\frac{dr}{dt} = \pm c \frac{\tau}{\mu} \longrightarrow 0, \tag{76}$$

i.e. for an external observer the light ray "freezes" at the horizon, while the infaller's proper time remains finite.

Interior region $(r < r_h)$. $\tau(r)$ becomes positive again, but the orientation of the time-flow operator

$$D_t = \tau \, \partial_t \tag{77}$$

is opposite to the Killing time of the external stationary region: processes inside count phases and energies with respect to the reversed time orientation of the external observer. At the same time $\mu(r)$ grows, enhancing the boundary effects of §9.2.

Physical picture. In the $\mu\tau$ description the "interchange of space and time axes" of the GR interior is realized via the zero level of τ (the horizon) and the change of orientation of D_t while keeping $\tau > 0$, which ensures a well-posed Cauchy problem and hyperbolicity (§4.6).

Scale dump" into the imaginary part: finiteness and stability

As $r \to 0$, the growth of $\mu(r)$ increases the "effective sizes/masses" of states. If all this growth remained on the real layer R, the energy densities and invariants from $\mathcal{L}_{\mu\tau}$ would diverge. This does not happen due to boundary exchange with the imaginary sector I:

- On the interfaces Σ (of thickness ℓ_*) there is a boundary term $\mathcal{L}_{\text{bdry}}$ (§4.4), which induces surface currents of scale and energy j^{ν}_{surf} .
- The conservation law takes the form

$$\partial_\mu T^{\mu\nu}_{(R)} + \delta_\Sigma \, j^{\nu}_{\text{surf}} = 0, \tag{78}$$

so that the excess contribution $\propto \mu^2$ is channeled into I once the threshold $\mu \geq \mu_{\text{th}}$ is reached.

- Constructively,

$$j^{\nu}_{\text{surf}} \;\sim\; \tilde{g}\, \Theta(\mu - \mu_{\text{th}}) \, \frac{\mu - \mu_{\text{th}}}{\ell_*} \, n^{\nu}_{\Sigma}, \tag{79}$$

where $\tilde{g} \sim g/\ell_*$ is a dimensionless effective coupling and n^{ν}_{Σ} is the normal to Σ.

Consequences. *Finiteness*: the densities $T^{(R)}_{00}$, invariants from $V(\mu, \tau)$, and $(\partial\mu)^2$ are bounded by a value of order $\sim \mu^2_{\text{th}} \Lambda^2_{\mu}$ (with exact coefficients determined by the form of V). *Stability*: switching on an outflow whenever $\mu \geq \mu_{\text{th}}$, linear in $(\mu - \mu_{\text{th}})$ on the scale ℓ_*, drives the system toward a stationary profile without δ-like spikes. *Feedback*: energy/scale accumulated in I generates, via Σ, an effective potential on the set of real patches R_j , which at large r manifests itself as "dark mass" with kernel $K(r)$ (see §11).

Thus, the singularity is replaced by a regular region with active boundary exchange, and the observable dynamics remains finite and hyperbolic.

Clusters. Since the "scale dump" into the imaginary sector is enhanced in the vicinity of black holes, the integral dark-matter-like subsystem in clusters is traced by the population of black holes (including IMBH and stellar-mass BHs in galaxies). In dynamical events (such as the Bullet Cluster) this produces a ballistic behavior of the dark component, whereas the hot gas is decelerated by hydrodynamical interactions — the source of the observed mass offsets.

Inextendible geodesics: $\mu\tau$ interpretation and physical meaning

In GR, geodesics become inextendible upon reaching the singularity. In the $\mu\tau$ approach:

1. Trajectories on R are determined by varying the action with the metric ds^2_{obs}; when $\mu \to \mu_{\text{th}}$, the boundary exchange switches on and the worldline hits Σ in finite proper time.
2. Beyond that point, two scenarios are possible:

- *Continuation in the enlarged space* $R \oplus I$: the worldline continuously enters I (with energy– momentum and the normal preserved), and the evolution remains unitary.
- *Effective reflected/scattered dynamics*: a part of the amplitude returns to R (possible "echoes"), and a part escapes into I.

We define *$\mu\tau$-completeness* as follows: a solution is complete if the affine parameter along a worldline is unbounded from above when the evolution is considered in $R \oplus I$ with the boundary law on Σ. Under the conditions of §4.6 ($\tau >$ 0, hyperbolicity, finite currents on ℓ*) the solutions are $\mu\tau$-complete, although the projection onto the real layer R alone may terminate at Σ. Partial reflection at Σ produces weak GW "echoes" in the ringdown (see §10), controlled by the parameters $\tilde{g}, \ell_*$.

Information paradox: channels through the imaginary sector and recoverability

The classical paradox (thermal Hawking radiation with an apparent "disappearance" of the carrier) is resolved by the fact that:

- The full dynamics is described by an action on $R \oplus I$ with a boundary term $\mathcal{L}_{\text{bdry}}$.
- Unitarity is realized in the enlarged state space: the evolution operator $U(t)$ is unitary when channels $R \leftrightarrow I$ are included.
- The "thermality" of the early radiation is a result of tracing over I (and over hidden degrees of freedom on Σ); as evolution proceeds, feedback channels return information to R (subtle correlations and spectral deviations suppressed as $\propto \tilde{g}^{\,2}$).

Qualitatively, the Page curve emerges as follows: at early times the entropy of the radiation increases (due to tracing over I); at a time $t_{\text{Page}} \sim t_{\text{Page}}(g, \tilde{\ell}_*, V)$ it reaches a maximum and then decreases as correlations leak back into R. A small but finite permeability of Σ leads to small spectral deviations from an ideal black-body law and to delayed GW "echoes" in the ringdown — both signals are quantitatively predictable for given $\tilde{g}$, ℓ_* (see §10).

Summary. In the $\mu\tau$ approach singularities are replaced by regular boundary dynamics at the interfaces $R \leftrightarrow I$; geodesic incompleteness disappears in the enlarged state space, and the information flow is closed by unitary boundary exchange. Causality is preserved, and the construction is compatible with weak-field phenomenology, while predicting subtle but testable strong-field effects (GW echoes, small spectral deviations, correlations in late-time radiation).

Gravitational waves

Gravitational perturbations are described as small oscillations of the fields

$$\mu = 1 + \frac{\phi_R}{\Lambda_\mu}, \qquad \tau = 1 + \frac{\sigma}{\Lambda_\tau}, \qquad B_i = b_i, \tag{80}$$

on the background $(\mu, \tau, B_i) = (1, 1, 0)$. The relation to measurable quantities is encoded in the "observable" line element (§5):

$$ds^2_{\text{obs}} = -\,\tau^2 c^2 dt^2 + 2\,\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij}\, dx^i dx^j, \tag{81}$$

so that linear metric perturbations accessible to detectors are

$$\delta g_{00} = -\frac{2\sigma}{\Lambda_\tau}, \qquad \delta g_{0i} = b_i, \qquad \delta g_{ij} = \frac{2\phi_R}{\Lambda_\mu}\,\delta_{ij}\ (+\text{ gauge terms}). \tag{82}$$

Below we write down the modes, polarizations, and the radiation law.

Scalar graviton mode (imaginary sector): excitation and energy flux

The kinetic term $\mathcal{L}_{\mu\tau}$ (§4.4) gives, in the wave zone (vacuum, far from sources),

$$\phi_R = 0, \qquad \sigma = 0, \qquad \partial_i b_i = 0, \qquad b_i = 0, \quad \text{with } \equiv \frac{\partial_t^2}{c^2} - \nabla^2. \tag{83}$$

The linear combination

$$\psi_b \;\equiv\; \alpha_\mu \frac{\phi_R}{\Lambda_\mu} + \alpha_\tau \frac{\sigma}{\Lambda_\tau} \tag{84}$$

describes the scalar ("breathing") mode: it produces an isotropic stretching $\propto \delta_{ij}$ in the plane perpendicular to the propagation direction. This mode couples to the imaginary sector via the boundary term $\mathcal{L}_{\text{bdry}}$ (§4.4): it allows a weak "feeding" of the scalar wave as it crosses the interfaces $R \leftrightarrow I$, without violating causality (finite thickness ℓ_*).

The energy flux density of waves from the Noether T_{00} (period-averaged, $\langle\cdot\rangle$) has the form

$$F_{\text{GW}} = \frac{c^3}{32\pi G_N}\,\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle + \frac{c^3}{8\pi G_N}\,\kappa_b\,\langle \dot{\psi}_b^2 \rangle, \tag{85}$$

where $h+,\times$ is the tensor part of δg^{ij} in the TT representation, "dot" denotes ∂_t, G_N is Newton's constant (fixed by weak-field tests, §7), and $\kappa_b \ll 1$ is the effective (dimensionless) strength of the scalar channel, depending on $\Lambda_{\mu,\tau}$ and the boundary coupling $\tilde{g}$. In the limit $\kappa_b \to 0$ the flux coincides with that of GR.

Polarizations (+,× and a weak "breathing" mode); detector response

The combinations of b_i and derivatives of φ_R, σ form a gauge-invariant tensor variable $h^{TT}{}_{ij}$, satisfying $h^{TT}{}_{ij} = 0$ in the wave zone and decomposing into two tensor polarizations (+,×) with the same projection tensors as in GR. Simultaneously, the isotropic component $\propto \delta_{ij}$ is controlled by the scalar ψ_b and yields a weak breathing polarization. The full set is (+,×, b), where the b-mode is suppressed by κ_b.

An interferometer with orthogonal arms along unit vectors e_1, e_2 responds as

$$h(t) = \frac{1}{2}\left(\mathbf{e}_1 \otimes \mathbf{e}_1 - \mathbf{e}_2 \otimes \mathbf{e}_2\right) : h_{ij}^{\mathrm{TT}} + \frac{1}{2} F_b(\hat{\mathbf{n}})\, \psi_b, \tag{86}$$

where $\hat{\mathbf{n}}$ is the propagation direction and F_b is the azimuthally symmetric beam pattern for the breathing mode. For the terrestrial network: (i) the antenna patterns for +,× are identical to GR; (ii) the resolvability of the scalar admixture is determined by κ_b and the geometry of the network. Pulsar-timing arrays and multi-messenger events provide additional constraints on κ_b.

Binary emission: energy balance; agreement with GR at the quadrupole level

From $\mathcal{L}_B$ and the "minimal substitutions" (§4.4) it follows that the source of tensor waves is the quadrupole combination of momentum flux and energy density, as in GR. In the wave zone,

$$h_{ij}^{\mathrm{TT}}(t, \mathbf{x}) = \frac{2G_N}{c^4 R}\, \ddot{Q}_{ij}^{\mathrm{TT}}(t - R/c) + \ldots, \tag{87}$$

and the quadrupole energy-loss law is

$$\left.\frac{dE}{dt}\right|_{\mathrm{tens}} = -\frac{G_N}{5c^5}\left\langle \dddot{Q}_{ij}\, \dddot{Q}_{ij} \right\rangle, \tag{88}$$

which exactly coincides with GR. Hence the same leading law for orbital evolution and the familiar −5/3 chirp exponent.

Because of the universality of couplings (no composition-dependent "charge" of sources), the scalar channel does not produce dipole radiation for binary systems; the leading contribution appears no lower than quadrupole order:

$$\left.\frac{dE}{dt}\right|_{\mathrm{scal}} = -\,\kappa_b\, \frac{G_N}{5c^5}\left\langle \dddot{\mathcal{Q}}^{\,2} \right\rangle + \ldots, \tag{89}$$

where $\mathcal{Q}$ is a scalar quadrupole (with weights derived from μ, τ). Since $\kappa_b \ll 1$, this contribution is suppressed and consistent with observations of tight binaries (from pulsar timing to LIGO/Virgo/KAGRA); possible differences appear only as small PN corrections to the phase.

Black-hole ringdown (QNM): predicted shifts of frequencies and damping

After the merger of compact objects, quasinormal modes (QNMs) are excited. In the $\mu\tau$ approach their spectrum is sensitive to two new ingredients:

1. Boundary dynamics on the interface Σ (thickness ℓ_*, coupling $\tilde{g}$) near the operational horizon $\tau(r_h) = 0$ (§9): effectively, a small reflectivity $\mathcal{R} \sim \tilde{g}\, 2$ at a distance $\sim \ell_*$ from the horizon.
2. A weak scalar mode ψ_b excited together with the tensorial one.

For the dominant tensor mode (ℓ, m, n) = (2, 2, 0) the frequency and damping time receive corrections

$$\frac{\delta f_{220}}{f_{220}^{\mathrm{GR}}} \sim \mathcal{O}\!\left(\frac{\ell_*}{r_s}\right) + \mathcal{O}(\tilde{g}^{\,2}), \qquad \frac{\delta \tau_{220}}{\tau_{220}^{\mathrm{GR}}} \sim \mathcal{O}\!\left(\frac{\ell_*}{r_s}\right) + \mathcal{O}(\tilde{g}^{\,2}), \tag{90}$$

where $r_s = 2GM/c^2$. For $\ell_*/r_s \ll 1$ and $\tilde{g} \ll 1$ the shifts are small, but in principle observable with high-precision ringdown measurements (LIGO A+/ET/LISA). In addition, a train of "echoes" with time separation

$$\Delta t_{\text{echo}} \;\approx\; 2\left|r_*(r_h+\varepsilon)\right| \;\sim\; 2\,r_s\,\ln\frac{r_s}{\ell_*}, \tag{91}$$

may appear if the effective reflectivity $\mathcal{R} \neq 0$; the echo amplitude is $\propto \mathcal{R}$ and decays rapidly.

The scalar mode ψ_b excites a weak breathing ringdown with frequencies close to those of spherically symmetric (s-mode) perturbations. Its contribution is small ($\propto \kappa_b$), but provides correlated fine structure in the late-time ringdown and a potential target for polarization tests.

Graviton picture of emission and ringdown

Quadrupole emission from a compact binary system can be interpreted as coherent emission of gravitons with + and × polarizations; the leading energy-loss law matches the GR prediction.

Near the horizon, the real↔imag boundary layer with finite thickness ℓ_* and "permeability" $\tilde{g}$ modifies the effective inner boundary condition of the scattering problem: this manifests itself as small shifts of the QNM resonant frequencies and damping times and, for $|\tilde{g}| > 0$, as partially reflected echoes.

In the quasiparticle language this corresponds to weak mixing of tensor states with surface modes of the interface; the magnitude of the effects is proportional to ℓ_*/r_s and $\tilde{g}^2$ (see §10, §16).

Summary. In the wave zone, the tensor modes (+,×) obey the same wave dynamics and quadrupole radiation law as in GR; the propagation speed is c. A weak scalar breathing polarization appears, suppressed by $\kappa_b \ll 1$, with no dipole radiation thanks to the universality of couplings. Black-hole ringdown is almost GR-like; small shifts $\propto \ell_*/r_s$, $\tilde{g}^2$ and possible echoes provide sharp tests for future detectors. These results are consistent with the PPN agreement (§7) and the strong-field regularization (§9) and translate into concrete observable predictions.

Dark matter

In the $\mu\tau$ approach, "dark" gravity does not arise from new particles, but as an effective contributionfrom the imaginary sector and boundary interfaces Σ (§3, §4.4). On small scales (in the vicinity of black holes, BH) this produces local shrouds with a steeply falling density; on galactic scales it appears as the convolution response of a kernel $K(r)$ to baryonic sources, leading to quasi–isothermal halos and flat rotation curves.

Local shrouds around BH: $\rho(r) \sim r^{-4}$ and saturation of enclosed mass

The "scale-dump" mechanism (§9) induces surface currents on Σ near the operational horizon $\tau\,(r_h) = 0$ and transfers part of the energy/scale into the imaginary part of the quantum. The feedback on the real layer R manifests itself as an additional potential equivalent to a "dark" density $\rho_{\text{DM}}(r)$. For an isolated BH in a stationary regime the solution of the flux-balance equation yields a universal asymptotic profile

$$\rho_{\text{shroud}}(r) = \frac{A}{r^4} \qquad (r \gg r_c), \tag{92}$$

where A is a constant determined by the local permeability of the interfaces $\tilde{g}$, the threshold μ_{th} and the microscopic cutoff ℓ_*; $rc \sim \max(\ell_*, \varepsilon r_s)$ is the radius where the stationary boundary flow switches on.

Mass saturation. The mass of the shroud enclosed within radius R is finite:

$$M_{\text{shroud}}(<R) = \int_{r_c}^{R} 4\pi r'^2\,\frac{A}{r'^4}\,dr' = 4\pi A\left(\frac{1}{r_c} - \frac{1}{R}\right) \xrightarrow{R\to\infty} \frac{4\pi A}{r_c}. \tag{93}$$

Thus the contribution of the local shroud saturates and remains $\ll M_{\text{BH}}$ for reasonable A/r_c. Therefore, for S-stars near Sgr A* and for orbits at several tens–hundreds of r_s the additional mass is negligibly small and does not distort the 1PN phenomenology, preserving agreement with observations.

Galactic halos: convolution kernel $K(r)$, quasi–isothermal profile, flat rotation curves

On larger scales, the ensemble of interfaces {Σ} defines the average convolution response of the imaginary sector to baryonic mass:

$$\Phi_{\text{eff}}(\mathbf{x}) = \Phi_{\text{bar}}(\mathbf{x}) + G\int d^3x'\,K(|\mathbf{x}-\mathbf{x}'|)\,\rho_{\text{bar}}(\mathbf{x}'), \qquad \rho_{\text{DM}}(\mathbf{x}) = \int d^3x'\,K(|\mathbf{x}-\mathbf{x}'|)\,\rho_{\text{bar}}(\mathbf{x}'). \tag{94}$$

The shape of the kernel is determined by the statistics of intersections $I \to R$, their thickness ℓ_* and the threshold μth. A minimal two-parameter approximation that simultaneously yields quasi–isothermal halos and flat rotation curves is a "pseudo–isothermal" kernel

$$K(r) = \frac{\kappa}{4\pi}\,\frac{1}{r^2 + r_c^2}, \tag{95}$$

where r_c is the core scale (related to ℓ_* and the typical thickness of interfaces), and κ is a dimensionless feedback strength (a function of $\tilde{g}, \mu_{\rm th}$). For a point-like baryonic source of mass M this gives

$$\rho_{\rm DM}(r) = \frac{\kappa M}{4\pi}\,\frac{1}{r^2 + r_c^2}, \tag{96}$$

i.e. $\rho_{\rm DM} \propto r^{-2}$ at $r \gg r_c$ (isothermal tail) and a central plateau at rr_c. The enclosed mass

$$M_{\rm DM}(< R) = \kappa M\Big[R - r_c \arctan(R/r_c)\Big] \tag{97}$$

grows $\propto R$ when $R \gg r_c$, so the circular velocity

$$v_{\rm circ}^2(R) = \frac{G\,M_{\rm tot}(< R)}{R} \;\longrightarrow\; G\,\kappa M \quad (R \gg r_c) \tag{98}$$

approaches a constant — i.e. flat rotation curves. For extended disks, convolution with the same $K(r)$ produces the familiar quasi–isothermal halo profile:

$$\rho_{\rm halo}(r) \simeq \frac{\rho_0}{1 + (r/r_c)^2}, \qquad v_{\rm flat}^2 \simeq 4\pi G\,\rho_0\,r_c^2, \tag{99}$$

where ρ_0 and r_c are effective core density and scale, expressible through $\kappa, \ell_*, \tilde{g}$ and the baryon distribution.

Physical meaning of the parameters. κ is the integral "transparency/response" of the network Σ (the fraction of the scale flux that is returned); r_c is the average "thickness/correlation radius" of interfaces in the real layer. Moderate values of these parameters naturally yield quasi–flat rotation curves without introducing new particle species.

Consistency with the Milky Way: local density, weak lensing, satellites
(i) Local density. For the Milky Way with $v_{\rm flat} \approx 230$ km s^{-1} we obtain

$$\rho_0\,r_c^2 = \frac{v_{\rm flat}^2}{4\pi G} \;\Rightarrow\; \rho_0 \simeq 9.8 \times 10^6\,\frac{M_\odot}{\rm kpc^3}\left(\frac{10\ {\rm kpc}}{r_c}\right)^2. \tag{100}$$

For $r_c \sim 10$ kpc this gives $\rho_0 \sim 9.8 \times 10^6\ M_\odot/{\rm kpc}^3 \approx 9.8 \times 10^{-3}\ M_\odot/{\rm pc}^3 \approx 0.4$ GeV cm^{-3}, i.e. the canonical range of the local dark-matter density near the Sun. Thus a choice of (κ, r_c) in a reasonable corridor is consistent with the disk kinematics of the Milky Way and with local constraints.

(ii) Weak lensing. Projection of a quasi–isothermal halo yields a surface density $\Sigma(R)$ with $\Sigma \propto R^{-1}$ at $R \gg r_c$; the mean tangential shear

$$\gamma_t(R) \propto \Sigma(< R) - \Sigma(R) \;\propto\; R^{-1} \tag{101}$$

agrees with the observed slope of weak-lensing profiles for L_* galaxies and for the Milky Way. The normalization of γ_t fixes the combination $\rho_0 r_c$ and is consistent with the same pair of parameters that fit rotation curves — a joint two-parameter fit is possible.

(iii) Satellites and inner slopes. The presence of a core with $r_c \sim$ a few–tens of kpc softens the central slope of the potential, easing the tension between velocity dispersions of dwarf satellites and overly concentrated NFW-like halos (the "too-big-to-fail" problem). In the $\mu\tau$ picture this is directly tied to the finite interface thickness ℓ_* and permeability $\tilde{g}$: both parameters simultaneously control r_c and the normalization κ.

(iv) Case study: Bullet Cluster (1E 0657–56). In the $\mu\tau$ approach the "DM signal" in clusters is the convolution response $K(r)$ tied to black holes. During the collision of subclusters the "imaginarygeometric" mass associated with the BH population decouples from the baryonic gas (which is slowed down by shocks) and follows the trajectories of compact systems, forming a lensing-mass peak displaced from the X-ray emitting gas. Thus the observed lensing–gas offsets are naturally reproduced without introducing a separate field of free cold matter.

Summary. Around an isolated BH, $\mu\tau$ feedback generates a local shroud $\rho \propto r^{-4}$ with a saturating mass — a dynamically safe effect compatible with orbital phenomenology. On galactic scales the convolution response $K(r)$ with two parameters (κ, r_c) yields quasi–isothermal halos and flat rotation curves without new matter. For the Milky Way the same (κ, r_c) simultaneously match the local density, weak lensing and satellite dynamics. Thus, in the $\mu\tau$ approach dark matter is an effective geometric projection of the imaginary sector onto the real layer, naturally reproducing the key observed properties of halos.

Dark energy and cosmology

The key idea of background (FRW-like) cosmology in the $\mu\tau$ approach is that the observer deals with an effective line element

$$ds^2_{\text{obs}} = -\tau^2(a)\, c^2 dt^2 + \mu^2(a)\, a^2(t)\, d\mathbf{x}^2, \tag{102}$$

so that the "observed" scale factor and time are

$$a_{\text{obs}}(t) = \mu(a)\, a(t), \qquad dt_{\text{obs}} = \tau(a)\, dt. \tag{103}$$

Hence the "observed" expansion rate is

$$H_{\text{obs}}(a) \equiv \frac{1}{a_{\text{obs}}} \frac{da_{\text{obs}}}{dt_{\text{obs}}} = \frac{1}{\tau}\Big(H + \frac{\dot{\mu}}{\mu}\Big), \qquad H \equiv \frac{\dot{a}}{a}. \tag{104}$$

The fields $\mu(a)$, $\tau(a)$ obey background Euler–Lagrange equations (§4.4) with sources from mean matter/ radiation densities and slow kinetics, re-anchoring local clocks and rods and generating an effective dark-energy contribution to H_{obs}.

Λ_{eff} as a sum of vacuum and "quantum-growth" parts

The background dark-energy density consists of two components:

$$\rho_\Lambda^{\text{eff}}(a) = \underbrace{V_0}_{\text{vacuum from } L_{\mu\tau}} + \underbrace{\rho_{\text{growth}}(a)}_{\text{growth of real volume, §3}}. \tag{105}$$

Vacuum part V_0. This is the constant contribution of the potential $V(\mu, \tau)$ (§4.4); it plays the role of a "true" cosmological constant $\Lambda_{\text{grav}} = 8\pi G V_0 / c^4$.

Growth of real volume $_{\rho\text{growth}}(a)$. The creation of new quanta and the overlap of their imaginary parts with the real layer (§3) behave like a homogeneous, slowly evolving component with

$$w_{\text{growth}}(a) \equiv \frac{p_{\text{growth}}}{\rho_{\text{growth}}} \simeq -1 + \varepsilon(a), \qquad |\varepsilon(a)| \ll 1, \tag{106}$$

where $\varepsilon(a)$ is controlled by the statistics of interfaces Σ (parameters $r_c, \tilde{g}, \ell_*$). A convenient parametrization is

$$\rho_{\text{growth}}(a) = \rho_{\text{growth},0}\, a^{-3(1+w_0+w_a)} \exp[-3w_a(1-a)]\,, \qquad (w_0 \approx -1,\ |w_a| \ll 1). \tag{107}$$

Then $\Lambda_{\text{eff}} \equiv 8\pi G\ \rho_\Lambda^{\text{eff}}/c^4$ is the sum of a constant and a mildly evolving part.

Background $H(a)$ with $\mu(a), \tau(a)$; slow evolution of $w(a)$

The standard Friedmann equation for the "bare-FRW" expansion $a(t)$ reads

$$H^2(a) = \frac{8\pi G}{3}\left(\rho_m a^{-3} + \rho_r a^{-4} + \rho_\Lambda^{\text{eff}}(a)\right) - \frac{k}{a^2}. \tag{108}$$

The observed expansion rate is

$$H_{\text{obs}}(a) = \frac{H + \dot{\mu}/\mu}{\tau}. \tag{109}$$

For slowly varying backgrounds $\mu = 1+\delta_\mu(a)$, $\tau = 1+\delta_\tau(a)$ with $|\delta_{\mu,\tau}| \ll 1$ and $|d\delta/d\ln a| \ll 1$ we have

$$H_{\rm obs}(a) \simeq H(a)\,[1-\delta_\tau(a)] + \frac{d\ln\mu}{dt}. \tag{110}$$

Equivalently, this can be absorbed into a redefinition of the effective equation of state of dark energy:

$$w_{\rm eff}(a) \equiv -1 + \varepsilon(a) + \frac{2}{3}\frac{d\ln\mu}{d\ln a} - \frac{2}{3}\frac{d\ln\tau}{d\ln a}, \tag{111}$$

where the last two "μ/τ brackets" are purely $\mu\tau$ effects (present even if $\rho_{\rm growth}$ is strictly constant). The internal dynamics is governed by

$$\ddot\phi_R + 3H\dot\phi_R + V'_\mu = S_\mu(a), \qquad \ddot\sigma + 3H\dot\sigma + V'_\tau = S_\tau(a), \tag{112}$$

with sources $S_{\mu,\tau}$ given by averaged kinetic energies of matter/radiation (§4.4). In a quasi-stationary regime, $d\ln\mu/d\ln a$, $d\ln\tau/d\ln a$ and $\varepsilon(a)$ are small, keeping the model close to ΛCDM.

ISW, structure growth, linear perturbations

Late-time ISW. The CMB temperature shift

$$\left.\frac{\Delta T}{T}\right|_{\rm ISW} = 2\int_{\eta_*}^{\eta_0} \Phi'(\eta,\hat n)\,d\eta \tag{113}$$

is sensitive to the evolution of the gravitational potential Φ. In the $\mu\tau$ approach, the lensing "optics" is controlled by

$$\Xi(a) \equiv \frac{d}{d\ln a}\ln\!\Big(\frac{\mu}{\tau}\Big), \tag{114}$$

since the refractive index is $n = \mu/\tau$. For $|\Xi| \ll 1$ the additional contribution to the late-time ISW is of order $\mathcal{O}(\Xi)$.

Structure growth. For sub-horizon modes, with $\delta \equiv \delta\rho_m/\rho_m$:

$$\delta'' + \Big(2 + \frac{H'}{H} + D(a)\Big)\,\delta' - \frac{3}{2}\,\Omega_m(a)\,\frac{G_{\rm eff}(a)}{G}\,\delta = 0, \qquad ({}' \equiv d/d\ln a\,), \tag{115}$$

where

$$D(a) \simeq -\frac{d\ln\tau}{d\ln a}, \qquad \frac{G_{\rm eff}}{G} \simeq 1 + \mathcal{O}\Big(\frac{d\ln\mu}{d\ln a}\Big). \tag{116}$$

Thus the "clock" field τ modifies the effective friction term, while the "length" field μ modifies the source only subdominantly. In the limit $\mu = \tau = 1$ one recovers ΛCDM. Observational control comes from $f\sigma_8(a)$ (RSD) and weak lensing; current data allow $|D|$few × 10^{-2} at $z1$.

Lensing. The combination Φ + Ψ in weak lensing is sensitive to $n = \mu/\tau$; the leading corrections to the cosmic shear–lensing spectrum

$$\Delta C_\ell^\kappa \propto \int dz\,\Xi(a)\,W(z) \tag{117}$$

(with weight W) remain at the percent level or below for $|\Xi|10^{-2}$.

Consistency with CMB/BAO/SN; numerical fitting scheme

We use the standard data set: CMB (TTTEEE + lensing), BAO, SNe Ia, RSD, WL. Two levels of modification suffice.

Background. We replace $H(a) \to H_{\rm obs}(a)$ and parametrize

$$\{\Omega_m, \Omega_b h^2, H_0, n_s, A_s, \tau_{\rm reio}\} \,\cup\, \{\Omega_{\Lambda,0}^{\rm grav}, w_0, w_a\} \,\cup\, \{\varepsilon_\mu \equiv d\ln\mu/d\ln a,\ \varepsilon_\tau \equiv d\ln\tau/d\ln a\}, \tag{118}$$

with priors $|w_0 + 1|, |w_a|, |\varepsilon_{\mu,\tau}| \ll 1$.

Linear perturbations. In CAMB/CLASS it is sufficient to introduce

$$w_{\text{eff}}(a) = -1 + w_0 + w_a(1-a) + \frac{2}{3}(\varepsilon_\mu - \varepsilon_\tau), \tag{119}$$

and to modify the friction term $D(a) \simeq -\varepsilon\tau$ in the growth equation. For lensing/ISW, one includes $\Xi(a) = \varepsilon_\mu - \varepsilon_\tau$ into the projection kernels.

Algorithm. (i) Fit distances $D_A(z)$, $D_L(z)$ from SN/BAO ⇒ posteriors for $(w_0, w_a, \varepsilon_\mu - \varepsilon_\tau)$; (ii) add CMB ($\theta_*$) and CMB lensing ⇒ tight constraints on (H_0, Ω_m) and on the early background (the early fraction of $\rho^{\text{eff}}_\Lambda$ must be a few percent at recombination); (iii) add $f\sigma_8$ and WL ⇒ bounds on ε_τ (via D) and on Ξ.

Expected bounds (conservatively).

$$|w_0 + 1|,\ |w_a| 0.1, \qquad |\varepsilon_\mu|,\ |\varepsilon_\tau| 10^{-2} \quad (z 1), \tag{120}$$

consistent with PPN/Solar System tests (§5.3) and allowing for subtle but observable cosmological deviations (late ISW, shifts in $f\sigma_8$, WL).

Early Universe and baryogenesis from topology

In the hot plasma of the early epochs the connections between layers A and B are enhanced: quasiparticles cross the boundary regions Σ many times, and effective "leakage" of baryon number with an asymmetry $\varepsilon\Sigma$ (a topological analogue of CP violation) arises. A minimal kinetic scheme for the baryon numbers $n^A{}_B$ and $n^B{}_B$ is

$$\dot{n}_B^A + 3Hn_B^A = -\Gamma_\Sigma(n_B^A - n_B^B) + \varepsilon_\Sigma\,\Gamma_\Sigma\,n_*, \tag{121}$$

$$\dot{n}_B^B + 3Hn_B^B = -\Gamma_\Sigma(n_B^B - n_B^A) - \varepsilon_\Sigma\,\Gamma_\Sigma\,n_*, \tag{122}$$

where n_* is the density of relevant carriers. At freeze-out, when $\Gamma_\Sigma \sim H$, the asymmetry

$$\eta_B \equiv \frac{n_B^A - n_B^B}{n_\gamma} \ \sim\ \varepsilon_\Sigma \left(\frac{\Gamma_\Sigma}{H}\right)_{\text{freeze}} \times \mathcal{O}(10^{-2}\text{–}10^{-1}), \tag{123}$$

naturally yields the correct order of magnitude $\eta_B \sim 10^{-10}$ for moderate values of ε_Σ and Γ_Σ/H.

Compatibility with BBN/CMB is ensured by the smallness of the residual contribution of the "antilayer" and by the absence of long-lived anomalous relativistic degrees of freedom. Possible consequences include a weak primordial gravitational-wave background and subtle anisotropies depending on the history of Σ.

Summary. The effective dark energy Λeff has a twofold origin: a vacuum contribution from the Lagrangian and a contribution from the growth of space quanta (of topological origin). The observed expansion is described by $H_{\text{obs}} = (H + \dot{\mu}/\mu)/\tau$; slow variations of μ, τ give $w_{\text{eff}}(a) \approx -1 + \mathcal{O}(10^{-2})$ according to (111). Linear perturbations receive minimal corrections: an additional growth friction through τ and a weak "optical" effect through μ/τ. The standard data stack CMB/BAO/SN/RSD/WL allows a joint fit of $(w_0, w_a, \varepsilon_\mu, \varepsilon_\tau)$ while maintaining stringent local tests.

Multimessenger signatures

We formalize the observable difference in arrival times of electromagnetic (EM) and gravitational-wave (GW) signals during cosmological propagation through gravitational structures. In the $\mu\tau$ approach, light propagates in an effective medium with refractive index $n = \mu/\tau$, whereas GW tensor modes propagate with speed c and experience the same Newtonian potentials via τ and B_i. To accommodate the observed second-scale delays we introduce a parameter for the universality of the GW coupling:

$$n_{\text{gw}} \equiv 1 + \xi_{\text{gw}} \frac{2U}{c^2} + \mathcal{O}(c^{-4}), \tag{124}$$

where U is the gravitational potential along the line of sight (including the halos of the source, the receiver, and the large-scale structure, LSS). In a GR-matching calibration one has $\xi_{\text{gw}} = 1$: EM and GW signals experience the same "gravitational optics", and the purely geometrical delay coincides. Any $(1 - \xi_{\text{gw}}) \neq 0$ produces an integrated difference in arrival times.

Integrated EM–GW delay along cosmological lines of sight

In observational units ($dt_{\text{obs}} = \tau\, dt$) the travel time for an EM photon is

$$t_{\text{EM}} = \frac{1}{c}\int_{\text{LOS}} n_{\text{em}}(x)\, dl, \qquad n_{\text{em}} = \frac{\mu}{\tau} = 1 + \frac{2U}{c^2} + \frac{3}{2}\frac{U^2}{c^4} + \cdots . \tag{125}$$

For GW propagation we have

$$t_{\text{GW}} = \frac{1}{c}\int_{\text{LOS}} n_{\text{gw}}(x)\, dl = \frac{1}{c}\int_{\text{LOS}}\left[1 + \xi_{\text{gw}}\frac{2U}{c^2} + \cdots\right] dl. \tag{126}$$

The geometrical (propagation) difference is then

$$\Delta t_{\text{geom}} \equiv t_{\text{EM}} - t_{\text{GW}} = \frac{1}{c}\int_{\text{LOS}}\left[(1-\xi_{\text{gw}})\frac{2U}{c^2} + \frac{3}{2}\frac{U^2}{c^4} + \cdots\right] dl. \tag{127}$$

The first term ($\propto U/c^2$) is proportional to the deviation from universality ($1 - \xi_{\text{gw}}$), while the second is the common 2PN correction $\sim (GM/bc^2)^2$. For an isolated spherical potential with impact parameter b:

$$\Delta t_{\text{geom}} \simeq (1-\xi_{\text{gw}})\,\frac{2GM}{c^3}\,\ln\frac{4r_E r_R}{b^2} + \mathcal{O}\!\left(\frac{G^2M^2}{bc^5}\right), \tag{128}$$

where r_E, r_R are the distances from the mass to the entry/exit points of the trajectory.

Cosmological expansion. For a source at redshift z_s it is convenient to introduce a sky-averaged potential of large-scale structure $\overline{U}(z)$, so that

$$\Delta t_{\text{geom}} \simeq (1-\xi_{\text{gw}})\int_0^{z_s}\frac{dz}{H_{\text{obs}}(z)}\,\frac{2\,\overline{U}(z)}{c^3}\,(1+z), \tag{129}$$

with the dominant contribution from the vicinities of massive halos of the source and the receiver; LSS gives an additional contribution with the same sign but smaller magnitude.

GW170817-like scenarios: contribution of the host and receiver halos

For nearby events (standard sirens at $D \sim 10$–100 Mpc) the difference can be decomposed as

$$\Delta t_{\text{geom}} \simeq (1-\xi_{\text{gw}})\,[\,\Delta t_{\text{host}} + \Delta t_{\text{LSS}} + \Delta t_{\text{MW}}\,], \tag{130}$$

with Shapiro-type estimates for each halo,

$$\Delta t_{\text{halo}} \simeq \frac{2GM_{\text{halo}}}{c^3}\ln\frac{4\,r_{\text{in}}\,r_{\text{out}}}{b^2}\,. \tag{131}$$

For the Milky Way halo ($M_{\text{MW}} \sim 10^{12} M^{\odot}, b \sim 8$–$50$ kpc) the absolute Shapiro delay of the EM signal is of order 10^7–10^8 s (weeks–years), but in GR it is identical for EM and GW and thus cancels out. In the $\mu\tau$ approach the cancellation fails only in the fraction ($1 - \xi_{\text{gw}}$):

$$\Delta t^{\text{MW}}_{\text{geom}} \sim (1-\xi_{\text{gw}}) \times 10^{7\text{–}8}\ \text{s}. \tag{132}$$

To be compatible with the observed second-scale delay (as in GW170817), one needs

$$|1-\xi_{\text{gw}}| \quad 10^{-7}\text{–}10^{-8}, \tag{133}$$

i.e. the universality of gravitational "optics" for GW and EM is confirmed with high precision: $\xi_{\text{gw}} = 1 \pm \mathcal{O}(10^{-7})$. A similar order-of-magnitude bound is obtained from the host-galaxy halo ($M_{\text{host}} \sim 10^{11\text{–}12} M_{\odot}$), while LSS adds a subdominant contribution (a fraction of the galactic terms at small D).

Sky maps of expected delays

The dominance of the receiver halo contribution leads to an anisotropic map $\Delta t_{\text{geom}}(\hat{n})$ over the sky, depending on the impact parameter $b(\hat{n})$ of the line of sight through the Milky Way potential:

$$\Delta t_{\rm geom}(\hat{n}) \;\approx\; (1-\xi_{\rm gw})\,\frac{2GM_{\rm MW}}{c^3}\,\ln\frac{4\,r_{\rm out}(\hat{n})\,r_{\rm in}}{b^2(\hat{n})} \;+\; (1-\xi_{\rm gw})\sum_{\rm LSS}\frac{2GM_i}{c^3}\,\ln\frac{4\,r_i r_i'}{b_i^2}. \tag{134}$$

Characteristics: maxima occur along the Galactic plane/center (minimal b); minima at high Galactic latitudes; LSS adds large-angle variations (directions towards nearby clusters).

Construction procedure.
(i) Fix ξ_{gw} (e.g. $\xi_{gw} = 1$ and $\xi_{gw} = 1 \pm 10^{-7}$ for sensitivity tests);
(ii) use a Milky Way potential model (disk+bulge+halo) to compute $b(\hat{n})$ and the logarithmic factor;
(iii) add a catalog of LSS (Local Group, nearby clusters) as a discrete sum;
(iv) tabulate $\Delta t_{geom}(\hat{n})$ on a HEALPix sphere.

Expected amplitudes. For $|1 - \xi_{gw}| = 10^{-7}$ one finds

$$\Delta t_{\rm geom}(\hat{n}) \sim 0.1\text{–}5\ {\rm s}, \tag{135}$$

with maxima along the Galactic plane and moderate additional enhancements in the directions of nearby clusters.

Summary. In the $\mu\tau$ approach, multimessenger delays are described by a line integral over the potential along the trajectory; the leading term is $\propto (1-\xi_{gw})\,U/c^2$. GW170817-like events require $|1 - \xi_{gw}|10^{-7}$–10^{-8}, i.e. the universality of gravitational optics for GW and EM is confirmed to high accuracy. The maps $\Delta t_{geom}(\hat{n})$ are anisotropic due to the geometry of the Milky Way and nearby structures and are suitable for planning and analyzing future observations.

S-stars near Sgr A*: strong-field tests

We use the $\mu\tau$ formalism to perform a rigorous fit of the orbits of the "S-stars" around Sgr A*, and to extract constraints on (i) the 2PN corrections to GR induced by μ, τ (coefficients a_2, b_2 from §5.3), and (ii) the additional Newtonian contribution from the BH "shroud" and the Galactic halo. Below we specify the data/methodology, the analytic form of the corrections, and a scheme for joint inference together with the BH mass.

Data (S2, S62, S4714, S4716) and fitting methodology

Observational set. Astrometry $(\alpha(t), \delta(t))$ with typical precision $\sim$ 10–100 μas; line-of-sight velocities $v_{los}(t)$ from spectroscopy (pericenter passages); inter-instrument cross-calibration of zero points, scales, and orientations.

List of stars. S2 (the reference for the 1PN pericenter shift); S62, S4714, S4716 (smaller pericenters in units of R_s, crucial for 2PN and shroud effects).

Model parameters. BH core: $(M_\bullet, r_0, v_0)$; for star j: $(a, e, i, \Omega, \omega, T_0)_j$; $\mu\tau$ strong field: $\delta a_2 \equiv a_2 - 0,\ \delta b_2 \equiv b_2 - \frac{1}{2}$ (see §5.3); dark mass: local shroud $\rho_{shroud}(r) = Ar^{-4}$ with cutoff r_c; smooth halo: convolution with kernel $K(r)$, parametrized by $(\kappa, r_{c,\rm halo})$—inside the central 0.05 pc this is practically negligible (we impose tight priors). Fitted systematics include astrometric offsets, plate scale, field rotation, and possible additional noise terms.

Inference. We use maximum likelihood plus MCMC/nested sampling. The likelihood

$$\mathcal{L} \propto \exp\left[-\frac{1}{2}\sum_k \begin{pmatrix}\Delta\alpha_k & \Delta\delta_k & \Delta v_k\end{pmatrix} \mathbf{C}_k^{-1} \begin{pmatrix}\Delta\alpha_k \\ \Delta\delta_k \\ \Delta v_k\end{pmatrix}\right], \tag{136}$$

where the model predictions (α, δ, v) are obtained numerically from the equations of motion in the "observed metric", including GR 1PN + $\mu\tau$ 2PN + Newtonian mass ρ_{DM}.

$\mu\tau$ corrections to precessions and pericenters; forecasts for upcoming pericenters

(A) Orbital shifts: analytic expressions. For a quasi-Keplerian orbit with semi-major axis a, eccentricity e, and semi-latus rectum $p = a(1 - e^2)$ we introduce

$$\epsilon \equiv \frac{GM_\bullet}{pc^2} = \frac{R_s}{2p}, \qquad R_s = \frac{2GM_\bullet}{c^2}. \tag{137}$$

The standard 1PN pericenter shift coincides with GR:

$$\Delta\omega_{1\mathrm{PN}} = 6\pi\epsilon = \frac{3\pi R_s}{p}. \tag{138}$$

The 2PN correction is parameterized by δa_2, δb_2 as

$$\Delta\omega_{2\mathrm{PN}} = \kappa^{(\mathrm{GR})}_{2\mathrm{PN}}(e)\,\epsilon^2 + \underbrace{\left(c_\mu\,\delta a_2 + c_\tau\,\delta b_2\right)}_{\equiv\,\delta\kappa_{2\mathrm{PN}}}\epsilon^2, \tag{139}$$

where $\kappa^{(\mathrm{GR})}_{2\mathrm{PN}}(e)$ is the known GR function of e, and $c_\mu, c_\tau = \mathcal{O}(1)$ are the weights of the $\mu\tau$ contribution. Newtonian "dark mass". A profile $\rho \propto r^{-4}$ yields a retrograde correction

$$\Delta\omega_{\mathrm{DM}} \;\approx\; -\,\pi\,\frac{d}{d\ln r}\left(\frac{M_{\mathrm{DM}}(<r)}{M_\bullet}\right)\Bigg|_{r\sim a} \;\sim\; -\,\pi\,\frac{4\pi A}{M_\bullet}\,\frac{r_c}{a}, \tag{140}$$

i.e. rapidly decaying and typically ≪ 2PN on the scales of S-star orbits.
Total per-orbit shift:

$$\Delta\omega \simeq \underbrace{6\pi\epsilon}_{\text{GR 1PN}} + \underbrace{\left[\kappa^{(\mathrm{GR})}_{2\mathrm{PN}} + \delta\kappa_{2\mathrm{PN}}\right]\epsilon^2}_{\mu\tau\ \text{2PN}} + \underbrace{\Delta\omega_{\mathrm{DM}}}_{\text{Newtonian}}\,. \tag{141}$$

(B) Pericenter timing markers and redshifts. The shift of the argument of pericenter induces a shift in the epoch of pericenter passage,

$$\Delta T_{\mathrm{peri}} \;\simeq\; \frac{\Delta\omega}{2\pi}\,P, \tag{142}$$

where P is the orbital period. The gravitational component of the spectroscopic redshift at pericenter remains GR-equivalent at 1PN; 2PN terms introduce $\mathcal{O}(\epsilon^2)$ deviations in the shape of $v_{\mathrm{los}}(t)$.

(C) Forecasts (qualitative). Posteriors for $\{\Delta T_{\mathrm{peri}}, \Delta\omega\}$ on the next orbit are obtained by marginalizing over $(M_\cdot, \delta a_2, \delta b_2, A, r_c)$. Sensitivity estimates:
• S2: $\epsilon \sim 10^{-3} \Rightarrow 2\mathrm{PN} \sim 10^{-6}$–$10^{-5}$ rad/orbit—at the boundary of current astrometric precision; the shroud effect is even smaller.
• S62/S4714/S4716: smaller p/R_s so that 2PN $\propto \epsilon^2$ increases; for pericenters 10^2–10^3 R_s the $\mu\tau$ 2PN contribution becomes measurable jointly with $v_{\mathrm{los}}(t)$.

Joint inference with BH mass and halo profile; goodness-of-fit

(A) Joint fitting of "mass–2PN–dark mass". We use a single parameter vector

$$\Theta = \{\, M_\bullet,\ \delta a_2,\ \delta b_2,\ A,\ r_c,\ \kappa,\ r_{c,\mathrm{halo}},\ \text{systematics}\,\}. \tag{143}$$

Degeneracies: $M_\cdot$ and δb_2 are partially correlated via $g^{(2\mathrm{PN})}{}_{00}$; the shroud parameters (A, r_c) are anticorrelated with δa_2 in the apsidal dynamics. Resolution: multi-orbit fitting with different e, p plus pericenter v_{los} measurements.

(B) Consistency parameters and model tests.
• PPN matching: priors $|\delta a_2|, |\delta b_2| \ll 1$ (§5.3); we test whether the posterior prefers values close to zero.
• Absence of excess mass: $M_{\mathrm{DM}}(< 10^3 Rs) \ll 10{-}3M\cdot$ (a consequence of $\rho \propto r^{-4}$).
• Goodness-of-fit: χ^2/dof, posterior predictive checks, Bayes factor against "pure GR" ($\delta a_2 = \delta b_2 = 0, A = 0$).

(C) Expected bounds. With current precision (S2) and anticipated pericenters of more compact orbits, we expect

$$|\delta b_2|,\ |\delta a_2|\ \ 10^{-2}\text{–}10^{-1}\ \ \text{(individually)}, \qquad \Sigma_{\text{stars}}\ \Rightarrow\ \text{improvement by a factor of 2–3}. \tag{144}$$

For the shroud we obtain

$$\frac{4\pi A\, r_c}{M_\bullet}\ \ 10^{-3} \quad\Rightarrow\quad \Delta\omega_{\mathrm{DM}}\ \ 10^{-3}\,\Delta\omega_{1\mathrm{PN}}. \tag{145}$$

Summary. We have specified a reproducible procedure that combines astrometry and spectroscopy of S-stars in the $\mu\tau$ "observed metric", separates GR 1PN from $\mu\tau$ 2PN and Newtonian "dark mass" contributions, and yields predictive markers (shifts in the pericenter epoch, small 2PN distortions in v_{los}) for S62/S4714/S4716. A joint fit of multiple stars minimizes degeneracies and provides optimal constraints on (δa_2, δb_2), confirming GR equivalence at 1PN and opening the way to detecting subtle $\mu\tau$ effects in the strong-field regime.

Model parameters and statistical identifiability

We summarize the set of free parameters in the $\mu\tau$ approach, introduce natural priors, and discuss degeneracies. We then show how a joint analysis (PPN tests, S-stars, GW, lensing, cosmology) identifies combinations of parameters and yields forecasts for the precision of future measurements.

Parameter set: $\Lambda_\mu, \Lambda_\tau, m_\mu, m_\tau, \lambda, \tilde{g}, \ell_\star, r_c, \rho_0$

Fields and "micro" parameters (Lagrangian). Scaling fields:

$$\mu(x) = 1 + \phi_R/\Lambda_\mu, \qquad \tau(x) = 1 + \sigma/\Lambda_\tau.$$

Masses and self-interactions: m_μ, m_τ (quadratic terms $m^2\varphi^2/2$), and $\lambda = \{\lambda_\mu, \lambda_\tau, \lambda_\times\}$ (quartic terms, with $\lambda_\times$ mixing). Boundary coupling to the imaginary sector: $\tilde{g}$ is the dimensionless "permeability" of the $R \leftrightarrow I$ interfaces. Interface thickness: $\ell_\star$ is the physical thickness of the boundary layer (the EFT UV cutoff).

Pseudo-metric coefficients in the weak field (§5.2, §5.3). In the baseline calibration that reproduces the PPN limit of GR:

$$a_1 = +1, \quad a_2 = 0, \qquad b_1 = -1, \quad b_2 = \tfrac{1}{2},$$

and deviations $\delta a_2 \equiv a_2 - 0,\ \delta b_2 \equiv b_2 - \frac{1}{2}$ parametrize 2PN strong-field physics.

Wave sector (§10). $\kappa_b \ll 1$ is the effective "strength" of the scalar breathing mode (a function of $\Lambda_{\mu,\tau}$ and $\tilde{g}$). $\xi_{\rm gw}$ is the universality parameter for "gravitational optics" of GWs (in the GR-matching calibration $\xi_{\rm gw} = 1$).

"Dark matter" as feedback (§11). *Local BH shroud:* amplitude A in $\rho_{\rm shroud}(r) = A/r^4$ with cutoff $r_c^{\rm sh} \sim \max(\ell_\star, \varepsilon R_s)$. *Galactic halos:* convolution kernel $K(r) = \dfrac{\kappa}{4\pi}\dfrac{1}{r^2 + r_c^2} \Rightarrow$ quasi-isothermal profile $\rho_{\rm halo} \simeq \rho_0/(1 + (r/r_c)^2)$ with $\rho_0 r_c^2 = v_{\rm flat}^2/(4\pi G)$. Halo parameters: (κ, r_c) or equivalently (ρ_0, r_c).

Cosmology (§12). Effective background drifts $\varepsilon_\mu \equiv d\ln\mu/d\ln a,\ \varepsilon_\tau \equiv d\ln\tau/d\ln a$ (typically 10^{-2}), and parameters describing the growth of space quanta in $\rho_{\rm growth}(a)$: $(w_0 \approx -1,\ w_a \approx 0)$.

Physically motivated constraints (stability/causality). $m_{\mu,\tau}^2 \geq 0$; $\lambda_{\mu,\tau} > 0$; $\lambda_\times > -\lambda_\mu\lambda_\tau$ (potential bounded from below); $\ell_\star > 0$ and $\ell_\star/R_s \ll 1$ for astrophysical BHs; $\tau > 0$ (hyperbolicity).

Priors, degeneracies, and inference methods

Priors (broad but physically motivated).

- **PPN matching:** $|a_1 - 1|, |b_1 + 1| 10^{-5},\ |b_2 - \frac{1}{2}| 10^{-4}$ (Gaussian priors).
- **GW universality:** $|1 - \xi_{\rm gw}| 10^{-7}$ (multimessenger constraints).
- **Scalar polarization:** $\kappa_b \geq 0$ with a log-uniform prior on $[10^{-6}, 10^{-1}]$.
- **Interfaces:** $\tilde{g} \in [0, 1)$; $\ell_\star/R_s \in [10^{-12}, 10^{-3}]$.
- **Halos:** $r_c \in [1, 30]$ kpc; a narrow Gaussian prior on $\rho_0 r_c^2 = v_{\rm flat}^2/(4\pi G)$.
- **Cosmology:** $|\varepsilon_{\mu,\tau}| 10^{-2},\ |w_0 + 1|, |w_a| 0.1$.

Key degeneracies (and how to break them).

- *S-stars:* $M_\bullet \leftrightarrow \delta b_2$ (both enter $g_{00}^{(2PN)}$); $\delta a_2 \leftrightarrow (A/r_c)$ (both affect apsidal dynamics). **Breaking:** multi-orbit fits with different e, p + pericenter $v_{\rm los}(t)$ (§14).
- *Halo:* $\rho_0 \leftrightarrow r_c$ (fixed by the combination $\rho_0 r_c^2$ from $v_{\rm flat}$); baryonic M/L $\leftrightarrow \rho_0$. **Breaking:** rotation curves + weak lensing + baryonic profiles.
- *Cosmology:* $H_0 \leftrightarrow (\varepsilon_\mu - \varepsilon_\tau)$ in $H_{\rm obs}$; $w(a) \leftrightarrow \varepsilon_{\mu,\tau}$ in $w_{\rm eff}(a)$. **Breaking:** CMB θ_* + BAO (distances), RSD/WL (growth).
- *GW:* $(\ell_\star/R_s) \leftrightarrow \tilde{g}$ (both control echoes/QNM shifts). **Breaking:** multi-frequency ringdown (different ℓ, n) and stacking of events.

Fisher/MCMC/nested sampling. For a data set d with observation vector $\mathbf{y}_d$ and model $\mathbf{m}(\Theta)$, the Fisher matrix is

$$F_{ij}^{(d)} = \sum_{\alpha,\beta} \frac{\partial m_\alpha}{\partial \theta_i} (C^{-1})_{\alpha\beta} \frac{\partial m_\beta}{\partial \theta_j}, \qquad \sigma(\theta_i) \simeq \sqrt{(F^{-1})_{ii}}.$$

The total log-likelihood is

$$\ln \mathcal{L}_{\rm tot} = \ln \mathcal{L}_{\rm PPN} + \ln \mathcal{L}_{\text{S-stars}} + \ln \mathcal{L}_{\rm GW} + \ln \mathcal{L}_{\rm RC/WL} + \ln \mathcal{L}_{\rm CMB/BAO/SN/RSD}.$$

We use MCMC for posteriors and nested sampling for Bayes factors B (comparison to "pure GR" with $\delta a_2 = \delta b_2 = \kappa_b = 0,\ \tilde{g} = 0$).

Identifiable combinations. $(\rho_0 r_c^2)$ — from $v_{\rm flat}$; $(\tilde{g}^2/\ell_\star)$ — from echo amplitudes; $(\delta a_2, \delta b_2)$ — from 2PN shifts of S-star orbits; $(\varepsilon_\mu - \varepsilon_\tau)$ — from ISW/WL; $1 - \xi_{\rm gw}$ — from multimessenger delays.

Joint contours and predictive power of experiments

Sensitivity matrix (qualitative).

Data class	Main parameters	Typical bounds
PPN (Cassini, LLR, VLBI, GP-B/LAGEOS)	a_1, b_1, b_2 (γ, β)	fixes 1PN at 10^{-5}–10^{-4}
S-stars (GRAVITY/ELT)	$\delta a_2, \delta b_2$; A/r_c; $M_\bullet$	$\sigma(\delta a_2), \sigma(\delta b_2) \sim 10^{-2}$ (3–4 orbits)
GW (LIGO/Virgo/KAGRA/ET/LISA)	$\ell_\star/R_s,\ \tilde{g},\ \kappa_b,\ \xi_{\rm gw}$	$\sigma(\ell_\star/R_s) \sim 10^{-2}$–$10^{-1}$; $\lvert 1-\xi_{\rm gw}\rvert 10^{-7}$
RC+WL (Roman/Euclid)	$\rho_0,\ r_c$ (or $\kappa,\ r_c$)	$\sigma(\rho_0 r_c^2)/(\rho_0 r_c^2) \sim 5\%$
Cosmology (CMB/BAO/SN/RSD/WL)	$\varepsilon_{\mu,\tau}$; w_0, w_a	$\lvert\varepsilon_{\mu,\tau}\rvert 10^{-2}$; $\lvert w_0+1\rvert, \lvert w_a\rvert 0.1$

Joint contours. PPN fixes the 1PN calibration $\Rightarrow$ it narrows the prior space of $(\Lambda_\mu, \Lambda_\tau)$. S-stars+GW isolate the strong-field subset $(\delta a_2, \delta b_2, \ell_\star/R_s, \tilde{g})$ almost independently of cosmology. RC/WL+cosmology determine (ρ_0, r_c) and $(\varepsilon_\mu, \varepsilon_\tau)$ with minimal correlation to the S-star block.

Forecast for future instruments.

- **ELT/GRAVITY+:** $\sim 3\times$ better astrometry $\Rightarrow \sigma(\delta a_2), \sigma(\delta b_2) \to$ few $\times 10^{-3}$; sensitivity to $A/r_c 10^{-3}\, M_\bullet$ within $< 10^3 R_s$.
- **ET/LISA:** separated QNM overtones $\Rightarrow \sigma(\ell_\star/R_s) \sim 10^{-2}$; stacked echoes $\Rightarrow \tilde{g}^2/\ell_\star \sim 10^{-3}/R_s$.
- **Roman/Euclid + Rubin:** $\sigma(\rho_0 r_c^2)/(\rho_0 r_c^2) \to 2\%$; WL maps test the shape of $K(r)$.
- **PTA/SKA:** bounds on breathing polarization $\kappa_b 10^{-2}$.
- **3G sirens:** $\lvert 1 - \xi_{\rm gw}\rvert \to 10^{-8}$–$10^{-9}$ from ensembles of low-z events.

Identifiability summary. The model $(\Lambda_\mu, \Lambda_\tau, m_{\mu,\tau}, \lambda, \tilde{g}, \ell_\star, r_c, \rho_0)$ is statistically separable into four weakly correlated blocks:

1. *PPN anchor* (1PN calibration);
2. *Strong-field block* $(\delta a_2, \delta b_2, \ell_\star, \tilde{g})$ (S-stars+GW);
3. *Halo block* (ρ_0, r_c) (RC+WL);
4. *Background block* $(\varepsilon_\mu, \varepsilon_\tau, w_0, w_a)$ (CMB/BAO/SN/RSD/WL).

Gluing these blocks together yields an overconstrained system with cross-checks (for example, $\xi_{\rm gw}$ is controlled simultaneously by multimessengers and PPN optics), which provides internal consistency tests for the theory and a clear route to its potential falsification.

Quantum consistency: renormalizability and EFT status

In this section we clarify the UV status of the $\mu\tau$ approach. We show that: (i) at the linear level (small fluctuations ϕ_R, σ, b_i around $\mu = \tau = 1,\ B_i = 0$) the couplings to standard matter are power-counting renormalizable; (ii) the full theory is an effective field theory (EFT) with a physical cutoff $\Lambda \sim 1/\ell_\star$ (the interface thickness Σ); (iii) radiative stability is ensured by the universality of couplings, correct signs of kinetic terms, and small dimensionless constants; the "fifth force" is suppressed by finite masses (m_μ, m_τ), the UV cutoff, and boundary screening.

Power counting and counterterms; strictly renormalizable regime (linear μ, τ)

We expand the fields as

$$\mu = 1 + \frac{\phi_R}{\Lambda_\mu}, \qquad \tau = 1 + \frac{\sigma}{\Lambda_\tau}, \qquad B_i = b_i, \tag{146}$$

with canonical mass dimensions in $D = 4$: $[\phi_R] = [\sigma] = [b_i] = 1,\ [\Lambda_{\mu,\tau}] = 1$. The "minimal substitutions" yield universal couplings

$$\mathcal{L}_{\rm int}(\mu) \supset \frac{\phi_R}{\Lambda_\mu}\, m\, \mathcal{O}_m, \qquad \mathcal{L}_{\rm int}(\tau) \supset \frac{\sigma}{\Lambda_\tau}\, \mathcal{O}_{\partial_t}, \qquad \mathcal{L}_{\rm int}(B) \supset b_i\, \mathcal{O}_{0i}, \tag{147}$$

where $\mathcal{O}_m \sim \bar{\psi}\psi,\ X^2,\ A_\alpha A^\alpha$ (mass densities), $\mathcal{O}_{\partial_t}$ are temporal currents (parts of kinetic terms), and $\mathcal{O}_{0i} \sim T_{0i}$ are momentum fluxes. These operators have dimension 2–3, so the vertices have net dimension 4:

$$[\phi_R]+[m]+[\bar{\psi}\psi] = 1+1+3 = 5 \;\&\; 1/\Lambda_\mu \Rightarrow 4, \quad [\sigma]+[\bar{\psi}\partial_t\psi] = 1+4 = 5 \;\&\; 1/\Lambda_\tau \Rightarrow 4, \quad [b_i T_{0i}] = 1+3 = 4.$$

Thus, at the linear level all couplings are marginal (power-counting renormalizable).

SM loops generate counterterms of the same structure:

- redefinitions of field Z-factors for ϕ_R, σ, b_i;
- renormalization of $\Lambda_{\mu,\tau}$ (logarithmic running);
- local potential terms $V(\phi_R, \sigma) = \frac{1}{2}m_\mu^2\phi_R^2 + \frac{1}{2}m_\tau^2\sigma^2 + \lambda$-interactions (dimension 4).

Summary of §16.1. For $|\phi_R|/\Lambda_\mu \ll 1,\ |\sigma|/\Lambda_\tau \ll 1,\ |b_i| \ll 1$ the $\mu\tau$ interactions with the SM are strictly power-counting renormalizable; UV divergences are absorbed into a finite set of constants $(Z,\ m_{\mu,\tau},\ \lambda,\ \Lambda_{\mu,\tau})$.

Effective theory with physical cutoff Λ $\sim 1/\ell\star$ (boundary thickness)

Nonlinearities (quadratic/cubic in ϕ_R, σ) and the boundary sector $\mathcal{L}_{\rm bdry}$ generate a tower of higher-dimension operators, e.g.

$$\frac{c_1}{\Lambda^2}(\partial\phi_R)^2\,\mathcal{O} \;+\; \frac{c_2}{\Lambda^2}(\partial\sigma)^2\,F_{\mu\nu}F^{\mu\nu} \;+\; \frac{c_3}{\Lambda^2}(\partial b)^2\,\bar{\psi}\psi \;+\; \cdots, \qquad \Lambda \sim \frac{1}{\ell_\star}. \tag{148}$$

At energies $E \ll \Lambda$ such operators are suppressed and a finite number of coefficients suffices (EFT logic).

Practical regimes.

- **Solar System / S-stars:** $E \sim p r_s^{-1} \ll \Lambda \Rightarrow$ contributions $\mathcal{O}(E^2/\Lambda^2)$ are negligible; PPN agreement is stable.
- **GW ringdown of BHs:** $f \sim c/r_s \Rightarrow$ relative corrections $\delta f/f \sim \mathcal{O}(\ell_\star/r_s)$; this is the window on UV interface physics.
- **Cosmology:** $E \sim H_0 \ll \Lambda \Rightarrow$ the EFT is fully controlled; background drifts $\varepsilon_{\mu,\tau}$ are IR parameters (§12).

Summary of §16.2. The full $\mu\tau$ theory is an EFT with cutoff $\Lambda \sim 1/\ell_\star$. The observable phenomenology away from QNMs/echoes is controlled by a finite set of constants; UV sensitivity is concentrated in small but measurable corrections to ringdown.

Radiative stability, RG flows; limits on the "fifth force" and screening

Radiative stability and RG running. The dimensionless couplings $g_\mu \sim m/\Lambda_\mu$, $g_\tau \sim 1/\Lambda_\tau$ acquire logarithmic running from standard SM anomalous dimensions; universality of the couplings (mass proportionality) protects against composition-dependent effects. Radiative corrections to $V(\phi_R, \sigma)$ shift $m^2_{\mu,\tau}$ and λ within ranges compatible with small $|\varepsilon_{\mu,\tau}|$ (§12). Small $m_{\mu,\tau}$ are technically natural for $Z > 0$ near the potential minimum. Renormalization in the vector sector is equivalent to local redefinitions of the observed metric $g^{\rm obs}_{\mu\nu}$ (variations of μ, τ, B) and is encoded in dimension-4 counterterms.

The "fifth force" and its suppression. Exchange of ϕ_R, σ between massive sources yields Yukawa tails

$$V(r) \simeq -\frac{G m_1 m_2}{r}\Big[1 + \alpha_\mu\, e^{-m_\mu r} + \alpha_\tau\, e^{-m_\tau r}\Big], \qquad \alpha_{\mu,\tau} \sim \mathcal{O}\Big(\frac{\bar m}{\Lambda_{\mu,\tau}}\Big)^2, \tag{149}$$

where $\bar m$ is a characteristic mass in $\mathcal{O}_m$. Tests of the $1/r^2$ law and the equivalence principle require either $\lambda_{\mu,\tau} = m^{-1}_{\mu,\tau}$ mm–cm, or $\alpha_{\mu,\tau} 10^{-5} - 10^{-6}$ on meter–astronomical scales. Both regimes are natural: $m_{\mu,\tau} \neq 0$ (minimum of V) $\Rightarrow$ short range; large $\Lambda_{\mu,\tau}$ (from PPN) $\Rightarrow$ small α.

Screening in strong fields. Near BHs the boundary channel switches on: when $\mu \geq \mu_{\rm th}$ the excess scale is dumped into the imaginary sector, reducing the effective local "charge" in ϕ_R, σ. This is a nonlinear, topological screening mechanism that: (i) saturates $T^{(R)}_{00}$ and $\mu(r)$ profiles (no blow-up), (ii) prevents accumulation of a large "fifth force" in potential wells, (iii) reconciles strong-field S-star phenomenology with GW data (§14, §10).

Summary of §16.3. Universal and weak $\mu\tau$ couplings are radiatively stable; the "fifth force" is suppressed either by short range ($m^{-1}_{\mu,\tau}$) or by small α, and in strong fields by boundary screening. The theory is causal, unitary, and radiatively robust within the EFT domain of validity ($E \ll \Lambda \sim 1/\ell_\star$).

Section summary. Locally (linear regime), $\mu\tau$ interactions are power-counting renormalizable; the number of counterterms is finite. Globally, $\mu\tau$ is an EFT with physical cutoff $\Lambda \sim 1/\ell_\star$; weak-field deviations from GR are suppressed by $\mathcal{O}(E^2/\Lambda^2)$. Radiative and phenomenological stability are ensured: small running, no ghosts, suppression of any "fifth force", and nonlinear screening. This secures the quantum consistency of the approach at accessible energies and delineates where to look for UV traces (QNM shifts, echoes, subtle cosmological drifts).

Consistency and causality

In this section we formalize the mathematical soundness of the $\mu\tau$ approach: hyperbolicity of the equations and a well-posed Cauchy problem; energy conditions and the absence of pathologies (ghosts, gradient instabilities); local Lorentz invariance in tangent frames. The key objects are the "observed" line element

$$ds^2_{\rm obs} = -\tau^2 c^2\, dt^2 + 2\,\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij}\, dx^i dx^j, \tag{150}$$

and the universal time-flow operator

$$D_t = \tau\big(\partial_t + B_i \partial_i\big). \tag{151}$$

Hyperbolicity and the Cauchy problem; absence of superluminal information transfer

Principal part (principal symbol). For all linear $\mu\tau$ equations, the principal part has a lightlike form with respect to ds^2_{obs}:

$$\text{Scalar } X: \quad D_t^2 X - \nabla^2 X + \cdots = 0, \tag{152}$$

$$\text{Dirac/Weyl/Majorana:} \quad (i\gamma^0 D_t + i\gamma^i \partial_i)\psi + \cdots = 0, \tag{153}$$

$$\text{Maxwell/Proca:} \quad \text{wave equations for transverse modes } \sim D_t^2 - \nabla^2, \tag{154}$$

$$\text{Rarita–Schwinger:} \quad \gamma^{\mu\nu\rho} D_\nu \psi_\rho + \cdots = 0, \tag{155}$$

so that the characteristic surfaces coincide with the null cone of ds^2_{obs}.

Characteristics. For radial propagation with $B_i = 0$, the characteristic speed is

$$v_{\text{char}} = \frac{dr}{dt} = \frac{c\,\tau}{\mu}, \tag{156}$$

and for $B_i \neq 0$ there is an additional small drift $\propto B$. None of the modes has characteristics outside the ds^2_{obs} cone $\Rightarrow$ no superluminal information transfer occurs.

Strong hyperbolicity and Cauchy well-posedness. Introducing the vector of first derivatives $Y = (\Phi,\ D_t\Phi,\ \nabla\Phi)$ for any field Φ, the system can be cast in symmetric-hyperbolic form

$$\partial_t Y = A_i\,\partial_i Y + B\,Y, \tag{157}$$

under the conditions

$$\tau > 0, \qquad \|\nabla\mu\|,\ \|\nabla\tau\|,\ \|B\| \text{ small on the Cauchy scale.} \tag{158}$$

These conditions hold in the weak-field regime and locally in the strong-field regime, away from the level $\tau = 0$ itself. At the operational horizon $\tau = 0$ (see §9), only the coordinate time t degenerates; the proper time $d\lambda^2 = -(ds^2_{\text{obs}})/c^2$ and the evolution along D_t remain well-defined on both sides of the horizon. The boundary sector $R \leftrightarrow I$ is implemented as well-behaved (dissipative/semi-transparent) boundary conditions on a layer of thickness $\ell_\star$, which preserves well-posedness.

Energy conditions; absence of ghosts and gradient instabilities

Positivity of kinetic terms. The Lagrangian for the scaling fields is chosen with signs

$$\mathcal{L}_{\mu\tau} \supset +\frac{1}{2}(\partial\phi_R)^2 + \frac{1}{2}(\partial\sigma)^2 - V(\phi_R, \sigma), \tag{159}$$

which exclude ghosts. For the vector shift B_i we use a transverse kinetic term with a positive coefficient (in the gauge $\partial_i B_i = 0$).

Propagation speeds. The principal part gives $c_s^2 = 1$ (with respect to ds^2_{obs}) for all dynamical modes, up to suppressed corrections of order $\mathcal{O}(\Lambda^{-2})$ (§16).

Energy conditions (in the observed frame). The energy–momentum tensor of the total content "matter+$\mu\tau$" satisfies

$$\textbf{NEC:} \quad T^{\text{tot}}_{\mu\nu}\,k^\mu k^\nu \geq 0 \quad \text{for any null } k^\mu \text{ of the observed metric,} \tag{160}$$

$$\textbf{WEC/Dominant:} \quad \rho \geq 0, \quad \rho \geq |S_i| \text{ for local observers,} \tag{161}$$

provided the potential obeys $V \geq V_{\min}$ and the gradients of μ, τ remain moderate. The boundary flux at the interface Σ is constructed such that $j_0^{\text{surf}} \geq 0$ (see §9), i.e. the energy in the real sector R does not increase due to exchange with I, which prevents runaway profiles.

Local Lorentz invariance in tangent frames

Tangent frames. At each point we introduce tetrads orthonormal with respect to ds^2_{obs}:

$$\hat{e}^0 = \tau\, c\, dt, \qquad \hat{e}^i = \mu\, dx^i \quad (\text{for } B_i \neq 0: \;\; \hat{e}^0 = \tau(c\, dt + B_i\, dx^i)). \tag{162}$$

In these frames, the local equations reduce to their SR forms with the flat metric $\eta_{\hat{\alpha}\hat{\beta}}$; gauge and spinor couplings coincide with those of special relativity.

Equivalence and absence of preferred frames. The universal replacements $m \to m\,\mu$ and $\partial_t \to D_t$ ensure that locally all microphysics depends only on ds^2_{obs}:

- the equivalence principle is realized in a scaling formulation (all fields "see" the same causal cone);
- there are no preferred-frame effects: in PPN language $\alpha_{1,2,3} = \xi = \zeta_i = 0$;
- GW and light propagate with the same speed c in tangent frames (coincidence of arrival times when $\xi_{\text{gw}} = 1$, §13).

Microcausality in QFT. Commutators/anticommutators of $\mu\tau$+SM fields vanish outside the observed light cone, because propagators are built from the same principal symbol. In the EFT regime $E \ll \Lambda \sim 1/\ell_\star$, any $\mathcal{O}(\Lambda^{-2})$ nonlocalities are suppressed and do not violate microcausality.

Section summary. The $\mu\tau$ formalism defines a well-posed hyperbolic dynamics with causality governed by the observed metric; it is free of ghosts and gradient instabilities, preserves local Lorentz invariance, and does not introduce preferred frames. Together with EFT quantum consistency (§16), this ensures the mathematical and physical coherence of the theory in its domain of applicability for energies and fields.

Comparison with alternatives and the GR limit

We compare the $\mu\tau$ approach with the main classes of alternative theories of gravity, and specify where it matches GR, where it deviates, and why. We then list observational signatures by which the theory can be tested.

Brans–Dicke/$f(R)$, TeVeS, teleparallel, emergent models: a correspondence map

(i) Scalar–tensor theories (Brans–Dicke, $f(R)$). **Common features:** scalar degrees of freedom that modify the effective "strength" of gravity. **Key difference:** in $\mu\tau$, the scalars μ, τ *operationally* deform masses and clock rates in the matter equations (replacements $m \to m\mu$, $\partial_t \to \tau(\partial_t + B_i\partial_i)$), and the observed metric is

$$ds^2_{\text{obs}} = -\tau^2 c^2 dt^2 + 2\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij}\, dx^i dx^j. \tag{163}$$

This is not a varying G (as in Brans–Dicke) and not a purely geometric $f(R)$ redefinition of curvature; it is a scale-and-clock deformation of matter on a flat background, followed by a geometric readout. **PPN:** the calibration of (a_1, b_1, b_2) is such that $\gamma = \beta = 1$, $\alpha_i = \xi = \zeta_i = 0$; in Brans–Dicke this is possible only in the limit $\omega_{\text{BD}} \to \infty$.

(ii) TeVeS (Bekenstein). **Common features:** the presence of one scalar and one vector beyond the tensor sector; an attempt to address halo phenomenology. **Difference:** our B_i is the gravito-magnetic time drift (frame dragging), not an independent metric vector; the scalars do not generate MOND-like transitions but instead lead to a convolution response via the $R \leftrightarrow I$ interfaces (§11). This preserves PPN agreement and avoids preferred-frame effects.

(iii) Teleparallel theories (TEGR, $f(T)$). **Common features:** alternative geometric formulations (through torsion). **Difference:** we work with a flat background, and the "geometry of observations" arises from (μ, τ, B_i); no torsion tensor is introduced, and all dynamics is read through tangent frames to ds^2_{obs}.

(iv) Emergent/entropic gravity. Common features: gravitational effects as macroscopic manifestations of microstructure. **Difference:** we write a local EFT action with observable fields/parameters $(\Lambda_{\mu,\tau}, \tilde{g}, \ell_\star)$; "dark matter" arises as a convolution with kernel $K(r)$, not as an entropic force law.

Map summary. The $\mu\tau$ construction combines metric phenomenology of GR (PPN, lensing, GW) with an operational scale-and-clock deformation of matter and a testable strong-field microphysics (interfaces Σ, parameters $\ell_\star, \tilde{g}$).

Where $\mu\tau$ agrees with GR, where it deviates, and why

Agreements (by construction).

- **Solar System, 1PN:** $\gamma = \beta = 1$, $\alpha_i = \xi = \zeta_i = 0 \Rightarrow$ light deflection, Shapiro delay, Mercury precession, frame-dragging are as in GR.
- **Speed of light and GW:** $c_{\rm em} = c_{\rm gw} = c$ in tangent frames; multimessenger universality $\xi_{\rm gw} = 1$ (experimentally to 10^{-7}; §13).
- **Binary radiation:** the quadrupole law and leading GW phase evolution coincide.

Deviations (physical origin).

- **2PN in strong fields:** depend on $(a_2, b_2) \Rightarrow$ small shifts of pericenter precessions and higher-order time delays.
- **Scalar "breathing" GW mode:** suppressed by $\kappa_b \ll 1$, absent in GR.
- **Horizon boundary:** parameters $\ell_\star, \tilde{g} \Rightarrow$ tiny shifts in QNM frequencies/damping and possible echoes.
- **Dark component:** instead of CDM particles, a convolution response $K(r) \Rightarrow$ quasi-isothermal halos with cores rather than NFW cusps (§11).
- **Cosmological background:** slow drifts $\varepsilon_{\mu,\tau} \Leftrightarrow$ a mildly evolving $w(a) \neq -1$.

Physical reason. In $\mu\tau$, gravity is a universal deformation of the scale and internal time of matter. In the weak field this reproduces the GR metric up to 1PN; in the strong field the dynamical degrees μ, τ and the boundary topology $(\Sigma, \ell_\star, \tilde{g})$ come into play, which have no analogue in purely geometric GR.

Observational differences and how to measure them

(A) Strong-field tests near BHs. QNM ringdown after mergers:

$$\frac{\delta f}{f},\ \frac{\delta\tau}{\tau} \sim \mathcal{O}\left(\frac{\ell_\star}{r_s}\right) + \mathcal{O}(\tilde{g}^2), \tag{164}$$

measurements: multi-frequency ringdown spectrum (LIGO A+/ET/LISA), stacked searches for echoes. **S-stars around Sgr A*:** 2PN shifts $\propto \epsilon^2$ and a small retrograde contribution from the "shroud" $\rho \sim r^{-4}$; measurements: GRAVITY/ELT, joint fits of S2/S62/S4714/S4716.

(B) GW polarizations and multimessengers. Breathing polarization: amplitude set by κ_b; measurements: network of interferometers + PTA/SKA, polarization correlations. **EM–GW arrival-time differences:** parameter $|1 - \xi_{\rm gw}|$; measurements: standard sirens, with priority to directions with small impact parameter in the Milky Way potential.

(C) Galactic halos and lensing. Cored, quasi-isothermal halos and flat rotation curves from $K(r)$; measurements: joint fits of rotation curves + weak lensing + baryonic profiles (Roman/Euclid/Rubin). Consistency check:

$$\rho_0 r_c^2 \simeq \frac{v_{\rm flat}^2}{4\pi G}. \tag{165}$$

(D) Late-time cosmology. Slow drifts $\varepsilon_{\mu,\tau}$: ISW, growth $f\sigma_8$, cosmic shear; measurements: CMB (Planck/Simons), BAO/SN, RSD/WL — a global Bayesian fit.

(E) GR limit.

$$\mu \to 1, \quad \tau \to 1, \quad B_i \to 0, \quad a_2 \to 0, \quad b_2 \to \tfrac{1}{2}, \quad \kappa_b \to 0, \quad \xi_{\rm gw} \to 1, \quad \tilde{g} \to 0, \quad \ell_\star/r_s \to 0. \qquad (166)$$

In this limit, all distinctive signatures vanish and the theory becomes indistinguishable from GR+ΛCDM.

(F) Bullet Cluster. The model predicts: (i) the lensing mass peak follows the BH carriers of a subcluster, (ii) the lensing–X-ray offset grows with the BH mass fraction and the relative velocity, (iii) the morphology of "tails" is consistent with ballistic behaviour of the compact component. This distinguishes the $\mu\tau$ picture from purely modified gravity without explicit compact anchoring, and allows one to estimate the BH mass fraction from the geometry of the offset.

Section summary. The $\mu\tau$ theory agrees with GR in all tested 1PN regimes and at the quadrupole level for GW emission, but predicts small and specific deviations: 2PN shifts in strong fields, a weak scalar polarization, QNM shifts/echoes, quasi-isothermal halos, and tiny cosmological drifts. These signatures define an observational programme sufficient to distinguish $\mu\tau$ gravity experimentally as a self-consistent, operationally formulated alternative to the geometric description of GR.

Discussion

In this section we summarize the status of the $\mu\tau$ approach, highlight its strengths and limitations, formulate conceptual implications, and outline open problems that move the model from a consistent EFT to a programme of testable investigations.

Strengths and limitations of the $\mu\tau$ approach

Strengths.

- **Operational formulation.** Gravity is defined through *measurable* deformations of scale and "clocks" (μ, τ) in all field equations (minimal substitutions). The observed line element

$$ds^2_{\rm obs} = -\tau^2 c^2 dt^2 + 2\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij} dx^i dx^j$$

  is directly tied to data (lensing, Shapiro delay, PPN).

- **Compatibility with local SR.** In tangent frames, microphysics reduces to the standard relativistic one; a single causal cone for all fields (§5, §17).

- **PPN anchor and agreement with GR at tested levels.** By construction, $\gamma = \beta = 1$, $\alpha_i = \xi = \zeta_i = 0$; quadrupolar GW emission coincides with GR (§§7, 10, 18).

- **Controlled EFT.** A physical cutoff $\Lambda \sim 1/\ell_\star$ (interface thickness), linear couplings that are power-counting renormalizable; UV sensitivity is concentrated in small QNM shifts / possible echoes (§16).

- **Strong-field regularity.** Singularities are replaced by boundary dynamics at $R \leftrightarrow I$ interfaces, ensuring $\mu\tau$-completeness of trajectories and finiteness of observable quantities (§9).

- **Unified description of "dark" phenomena.** Dark matter as a convolution with kernel $K(r) \to$ quasi-isothermal halos (§11); dark energy as $\Lambda_{\rm eff} = V_0 + \rho_{\rm growth}$ with slowly varying $w(a)$ (§12); multimessenger universality $c_{\rm em} = c_{\rm gw}$ to $\sim 10^{-7}$ (§13).

- **Testability.** Clear signatures: 2PN shifts for S-stars, a weak "breathing" GW polarization, QNM shifts/echoes, EM–GW delay sky maps, cosmological drifts of $w_{\rm eff}(a)$ (§§10–18, 15).

Limitations.

- **Microscopics of interfaces.** The topology and dynamics of the imaginary sector and of the boundaries Σ are still phenomenological $(\ell_\star, \tilde{g})$; a microscopic model is needed.
- **Non-uniqueness of the Lagrangian.** Variants equivalent at the data level are admissible (field redefinitions, dimension-4 terms); a principled selection (symmetries/minimality) is required.
- **Parametrization.** The set $(\Lambda_{\mu,\tau}, m_{\mu,\tau}, \lambda, \tilde{g}, \ell_\star, \kappa, \xi_{\rm gw}, \varepsilon_{\mu,\tau})$ requires joint calibration; degeneracies are possible (see §15).
- **Early-Universe tests.** A full verification for BBN/CMB perturbations is not yet completed; the early fraction of $\rho_{\Lambda\rm eff}$ and structure growth must be controlled.
- **Relation to Kerr geometry.** For rotating BHs, the role of B_i and the QNM structure in $\mu\tau$ variables require extended analytical control.
- **EFT domain of validity.** Beyond $E \ll \Lambda$ a UV completion or robust error estimates for the EFT are needed.

Conceptual implications

- **Ontology of gravity.** Gravity is a universal deformation of matter properties (scale μ and clock rate τ), and geometry *emerges* operationally through $ds^2_{\rm obs}$.
- **Quanta of space and the imaginary sector.** Space consists of quanta with real and imaginary parts; the interfaces Σ are channels for exchanging scale/energy: they stabilize strong fields, generate the effective dark kernel $K(r)$ on galactic scales, and contribute to $\Lambda_{\rm eff}$ in cosmology.
- **Clocks and causality.** There is a single causal cone for all fields; the apparent "time reversal" at the horizon is a property of $D_t = \tau(\partial_t + B^i\partial_i)$ without violating microcausality.
- **Graviton and polarizations.** A gravitational wave is a joint oscillation of $(\delta\mu, \delta\tau, \delta B)$ with a GR-like tensor core and a weak breathing admixture; the speed in tangent frames equals c.
- **"Dark" sectors as a geometric projection.** Dark matter and dark energy are not new particles but consequences of topology, quantum-space growth, and boundary feedback.

Open problems and priorities

1. **Microscopic theory of interfaces.** Derive $\ell_\star$, $\tilde{g}$ and the form of $L_{\rm bdry}$ from first principles (discrete/topological models, matrix/spin-network analogues) under unitarity and causality.
2. **Exact solutions and rotating BHs.** Construct $\mu\tau$ analogues of Schwarzschild/Kerr in closed form; derive QNM spectra and echo conditions analytically with their dependence on $\ell_\star/r_s$, $\tilde{g}^2$.
3. **Numerical $\mu\tau$ relativity.** 3+1 integrators with explicit D_t and finite interface thickness; simulations of collapse/merger; waveform catalogues.
4. **Early Universe.** Match the background to BBN/CMB: bounds on early $\rho_{\Lambda\rm eff}$, the sign and magnitude of $\varepsilon_{\mu,\tau}(a)$ at $z10^3$, predictions for primordial GW and ISW; a quasi-geometric baryogenesis scheme.
5. **Halo physics from first principles.** Derive $K(r)$ from the statistics of Σ and the baryonic distribution; predict the diversity of profiles and their relation to morphology/environment.
6. **Experimental "killer tests".** S-stars (2PN/pericentre timing; ELT/GRAVITY+), multi-frequency QNM and echo stacking (ET/LISA), polarizations (κ_b via PTA/SKA + ground-based network), EM–GW delay maps and pushing $|1-\xi_{\rm gw}| \to 10^{-8}$–$10^{-9}$, cosmological joint fit of $w_{\rm eff}(a)$ and $\varepsilon_{\mu,\tau}$ (WL/RSD/BAO/SN).

7. **Relation to diffeomorphisms.** Formalize how GR solution classes are realized as calibrations of (μ, τ, B); prove the "GR limit" as a theorem (conditions on parameters $\Rightarrow$ local equivalence to all observations).

8. **Quantization of the $\mu\tau$ fields.** Canonical/path-integral quantization for (ϕ_R, σ, b_i) with boundary degrees of freedom; microcausality and unitarity of propagators in the presence of Σ.

9. **Laboratory tests of "clocks and rods".** Atomic clocks/interferometry to search for tiny drifts of μ, τ in controlled potentials (equivalence principle in the new formulation).

Summary. The $\mu\tau$ approach combines strict phenomenological agreement with GR in tested regimes with a minimal set of new, concretely testable predictions in strong fields, the GW sector, halos and cosmology. Its conceptual strength lies in shifting "curvature" from the background geometry to matter properties, which makes it natural to embed quantum ingredients (EFT with a physical cutoff, boundary dynamics) without losing causality or local Lorentz invariance. The key to further progress is the microscopics of interfaces, exact strong-field solutions, and consistent multi-domain tests that turn the theory into a programme of reproducible and falsifiable research.

Conclusion
Main conclusions

We have proposed an operational $\mu\tau$ approach to gravity, in which the fundamental quantities are the scale of elementary degrees of freedom $\mu(x)$ and the rate of internal clocks $\tau(x)$ (plus the drift B_i), and the "observed" geometry emerges as

$$ds^2_{\text{obs}} = -\tau^2 c^2 dt^2 + 2\,\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij}\, dx^i dx^j. \tag{167}$$

This formulation:

- **Agrees with GR** in all 1PN tests (PPN anchor), at the quadrupole level of GW emission, and for the EM/GW propagation speed in tangent frames.
- **Defines a controlled EFT** with physical cutoff $\Lambda \sim 1/\ell_\star$, linearly power-counting renormalizable, free of ghosts and gradient instabilities, with a well-posed hyperbolic Cauchy problem.
- **Provides strong-field regularization** via a boundary layer $R \leftrightarrow I$ of finite thickness $\ell_\star$, and gives a unified scheme for three "dark" phenomena:
  - dark matter as a convolution with kernel $K(r) \rightarrow$ quasi-isothermal halos and flat rotation curves;
  - dark energy as $\Lambda_{\text{eff}} = V_0 + \rho_{\text{growth}}$ (vacuum + quantum-space growth);
  - multimessenger delays as a line integral over the potential with $\xi_{\text{gw}} \approx 1$.

The key departures from GR are localized at the 2PN level (parameters $\delta a_2, \delta b_2$), in a weak scalar breathing GW polarization ($\kappa_b \ll 1$), and in the horizon microphysics ($\ell_\star, \tilde{g}$) — all of which are observationally testable.

Key predictions for near-term tests

Strong fields / black holes.

- **Ringdown (QNM).** Shifts in the frequencies and damping times of the dominant $(2,2,0)$ mode:

  $$\frac{\delta f}{f},\ \frac{\delta\tau}{\tau} \ \sim\ \mathcal{O}\!\left(\frac{\ell_\star}{r_s}\right) + \mathcal{O}(\tilde{g}^2),$$

  plus possible echoes with spacing

  $$\Delta t_{\text{echo}} \ \sim\ 2\, r_s \ln\!\left(\frac{r_s}{\ell_\star}\right).$$

Target level: $|\delta f/f| \sim 10^{-2}$ (ET/LISA), sensitive to $\ell_\star/r_s$.

- **S-stars around Sgr A*.** 2PN pericentre shifts $\propto \epsilon^2$ with $\epsilon = R_s/(2p)$ and a subdominant retrograde contribution from the "shroud" $\rho \propto r^{-4}$. Expected bounds: $\sigma(\delta a_2)$, $\sigma(\delta b_2) \sim \text{few} \times 10^{-3}$ with ELT/GRAVITY+ and a joint fit of S2/S62/S4714/S4716.

Gravitational waves and multimessengers.

1. Universality of optics: $|1 - \xi_{\rm gw}|10^{-7}$ (already); target 10^{-8}–10^{-9} (3G sirens).
2. Scalar polarization (breathing): $\kappa_b 10^{-2}$ (PTA/SKA + ground-based network).
3. Anisotropic EM–GW delay sky maps, dominated by the Milky Way potential; maxima along the Galactic plane.

Galaxy dynamics and lensing.

- Cored halos: $\rho_{\rm halo} \simeq \rho_0/(1 + (r/r_c)^2)$ and flat rotation curves with $\rho_0 r_c^2 = v_{\rm flat}^2/(4\pi G)$.
- Target: joint RC+WL fits with $\sim 2\%$ precision on $\rho_0 r_c^2$.

Late-time cosmology.

- Slow drifts of clocks and rods: $\varepsilon_\mu, \varepsilon_\tau \equiv d\ln\mu/d\ln a,\ d\ln\tau/d\ln a \sim 10^{-3}$–$10^{-2}$, leading to a mild $w_{\rm eff}(a) \neq -1$ and small corrections to ISW, WL and $f\sigma_8$.
- Target: a joint CMB/BAO/SN/RSD/WL fit with $|\varepsilon_{\mu,\tau}|3 \times 10^{-3}$.

Falsification criteria.

- Absence of 2PN shifts for tight S-stars at sensitivity $\sigma \sim 10^{-3}$.
- Non-detection of QNM shifts/echoes with $|\delta f/f| < 10^{-2}$ across many events.
- Strong bounds $|1 - \xi_{\rm gw}| < 10^{-9}$ and $\kappa_b < 10^{-3}$ with no accompanying $\mu\tau$ signatures.
- Systematic mismatch between $\rho_0 r_c^2$ and $v_{\rm flat}^2/(4\pi G)$ across a population of galaxies.

Plan for further work (GW polarizations, QNM, cosmology)

(A) GW polarization sector. Bayesian analyses of the interferometer network with explicit $\mu\tau$ parameterization $(\kappa_b, \xi_{\rm gw})$; stacked searches for breathing modes and angular patterns (together with PTA/SKA); publication of open waveform templates (public code repository for reproducibility).

(B) Ringdown and echoes. A $\mu\tau$ analogue of the Regge–Wheeler/Zerilli potential with finite-thickness $\ell_\star$ boundary conditions; catalogues of QNM spectra and echo transfer functions on a grid $(\ell_\star/r_s, \tilde{g})$; multi-event stacking analyses for LIGO/Virgo/KAGRA and a strategy for ET/LISA.

(C) Cosmology and structure growth. Implementation of $H_{\rm obs}(a)$, $w_{\rm eff}(a)$, $\Xi(a) = d\ln(\mu/\tau)/d\ln a$ in CLASS/CAMB; a joint Planck+BAO+SN+RSD+WL fit with PPN/GW/S-star priors; forecast ISW/WL maps for Euclid/Roman/Rubin.

(D) S-stars and strong-field geodesy. A numerical integrator for "observed" geodesics with $\mu\tau$ 2PN and subtle light effects (lensing, delays); cooperation with GRAVITY/ELT for campaigns around the pericentres of S62/S4714/S4716.

(E) Microscopics of interfaces. $R \leftrightarrow I$ models based on topological/discrete schemes; derivation of $(\ell_\star, \tilde{g})$ from first principles; checks of unitarity/microcausality at the propagator level with boundary degrees of freedom.

Overall summary. The μτ approach offers a minimally extended, quantum-consistent, and observationally testable scheme of gravity in which "curvature" is shifted from the background to matter properties. The theory reproduces GR where it has been tested and formulates a small set of clear, achievable predictions by which near-future observations can either strengthen it as a working alternative or decisively falsify it [1-42].

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Conflict of Interest

The authors declare that they have no financial, professional, or personal conflicts of interest that could influence the results, interpretation, or presentation of the materials in this work. Funding sources and computing resources had no impact on the design of the study, the choice of analysis methods, the interpretation of the data, or the decision to publish.Any potential future relationships (grants, consulting, code commercialization) will be disclosed in due course in updates to the repository and in the supplementary materials accompanying the article.

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Appendices: technical details and full derivations

Throughout we use the "observed" line element

$$ds^2_{\rm obs} = -\tau^2 c^2 dt^2 + 2\,\tau^2 B_i\, dx^i dt + \mu^2 \delta_{ij}\, dx^i dx^j, \tag{168}$$

the minimal substitutions in matter equations $m \to m\mu$, $\partial_t \to D_t \equiv \tau(\partial_t + B_i\partial_i)$, and the weak-field expansions

$$\mu = 1 + \frac{\phi_R}{\Lambda_\mu}, \qquad \tau = 1 + \frac{\sigma}{\Lambda_\tau}, \qquad B_i = b_i, \tag{169}$$

where ϕ_R, σ, b_i are small dynamical excitations.

Full form of the Lagrangian and variation
Dynamical part of the action

$$S = \int d^4x\, \Big(L_{\rm SM}(\mu,\tau,B) + L_{\mu\tau} + L_B + L_{\rm bdry} \Big). \tag{170}$$

(i) Matter / SM fields (minimal substitutions).

$$L_{\rm SM}(\mu,\tau,B) = L_{\rm kin}[\Phi;\ \partial_t \to D_t,\ \nabla] - \sum_a m_a \mu\, \bar\psi_a \psi_a - \sum_i \frac{1}{2} m_i^2 \mu^2 X_i^2, \tag{171}$$

where $\Phi = \{\psi_a, X_i, A_\alpha, \dots\}$, $D_t = \tau(\partial_t + B_i\partial_i)$.

(ii) Scale fields μ, τ.

$$L_{\mu\tau} = \frac{1}{2}(\partial\phi_R)^2 + \frac{1}{2}(\partial\sigma)^2 - V(\phi_R,\sigma), \tag{172}$$

$$V = \frac{1}{2} m_\mu^2 \phi_R^2 + \frac{1}{2} m_\tau^2 \sigma^2 + \lambda_\mu \phi_R^4 + \lambda_\tau \sigma^4 + \lambda_\times \phi_R^2 \sigma^2. \tag{173}$$

(iii) Vector drift B_i (gauge $\partial_i B_i = 0$).

$$L_B = \frac{1}{2}\Big[(\partial_t B_i)^2 - c_B^2 (\partial_j B_i)^2\Big] + \zeta\, B_i\, T^{0i}_{\rm SM}. \tag{174}$$

(iv) Boundary layer $R \leftrightarrow I$ (thickness ℓ_*).

$$L_{\rm bdry} = \frac{\tilde g}{\ell_*}\Big(\phi_R J_R(\Sigma) + \sigma J_T(\Sigma)\Big) - \frac{\chi}{2\ell_*^2}\Big(\phi_R^2 + \eta\,\sigma^2\Big) \quad \text{on } \Sigma. \tag{175}$$

Euler–Lagrange equations and sources

In the bulk (away from Σ):

$$\phi_R + V_\phi = \frac{1}{\Lambda_\mu} S_\mu, \qquad\qquad \sigma + V_\sigma = \frac{1}{\Lambda_\tau} S_\tau, \tag{176}$$

$$\partial_t^2 B_i - c_B^2 \nabla^2 B_i = -\,\zeta\, T^{\rm SM}_{0i}, \tag{177}$$

where

$$S_\mu \equiv \sum_a m_a \bar\psi_a \psi_a + \sum_i m_i^2 X_i^2, \qquad S_\tau \equiv \frac{\delta L_{\rm SM}}{\delta(\partial_t)} \cdot (\partial_t + B_i\partial_i). \tag{178}$$

On the interface Σ (effective jump conditions):

$$\big[n^\alpha \partial_\alpha \phi_R\big]_\Sigma = -\frac{\tilde g}{\ell_*} J_R + \frac{\chi}{\ell_*^2}\phi_R, \tag{179}$$

$$\big[n^\alpha \partial_\alpha \sigma\big]_\Sigma = -\frac{\tilde g}{\ell_*} J_T + \frac{\chi\eta}{\ell_*^2}\sigma. \tag{180}$$

Energy–momentum tensor

$$T^\alpha{}_\beta = T^\alpha{}_\beta(\mathrm{SM})(\mu,\tau,B) + \partial^\alpha\phi_R\,\partial_\beta\phi_R + \partial^\alpha\sigma\,\partial_\beta\sigma + \partial^\alpha B_i\,\partial_\beta B_i - \delta^\alpha{}_\beta\,L, \tag{181}$$

with $\partial_\alpha T^\alpha{}_\beta = 0$ in the bulk and balance with surface fluxes $\propto \tilde{g}/\ell_*$ on Σ.

PPN derivation from the Lagrangian

Expansions in the Newtonian potential U and standard PPN potentials:

$$\mu = 1 + a_1\frac{U}{c^2} + a_2\frac{U^2}{c^4} + \cdots\,, \tag{182}$$

$$\tau = 1 + b_1\frac{U}{c^2} + b_2\frac{U^2}{c^4} + \cdots\,, \tag{183}$$

$$B_i = \beta_V\frac{V_i}{c^3} + \beta_W\frac{W_i}{c^3} + \cdots\,. \tag{184}$$

Then

$$g_{00} = -\tau^2 = -1 - 2b_1\frac{U}{c^2} - \Big(2b_2 + b_1^2\Big)\frac{U^2}{c^4} + \cdots\,, \tag{185}$$

$$g_{ij} = \mu^2\delta_{ij} = \Big(1 + 2a_1\frac{U}{c^2} + (2a_2 + a_1^2)\frac{U^2}{c^4} + \cdots\Big)\delta_{ij}, \tag{186}$$

$$g_{0i} = \tau^2 B_i = \Big(1 + \mathcal{O}(c^{-2})\Big)\Big(\beta_V\frac{V_i}{c^3} + \beta_W\frac{W_i}{c^3}\Big). \tag{187}$$

GR calibration:

$$a_1 = +1, \quad a_2 = 0, \quad b_1 = -1, \quad b_2 = \tfrac{1}{2}, \quad \beta_V = -4, \quad \beta_W = 0, \tag{188}$$

which gives the standard PPN parameters $\gamma = \beta = 1$, $\alpha_{1,2,3} = \xi = \zeta_{1,2,3,4} = 0$. Deviations at 2PN are encoded by $\delta a_2 \equiv a_2$, $\delta b_2 \equiv b_2 - \frac{1}{2}$.

"Gravitational optics"

The refractive index of light:

$$n(x) = \frac{\mu}{\tau} = 1 + \frac{2U}{c^2} + \frac{3}{2}\frac{U^2}{c^4} + \cdots\,. \tag{189}$$

For spherical symmetry (impact parameter b) the deflection angle and Shapiro delay:

$$\hat{\alpha} \simeq 2\int_{-\infty}^{+\infty}\frac{\partial_\perp n}{n}\,dz = \frac{4GM}{bc^2} + \mathcal{O}\left(\frac{G^2M^2}{b^2c^4}\right), \tag{190}$$

$$\Delta t = \frac{1}{c}\int_{\mathrm{LOS}}(n-1)\,dl = \frac{2GM}{c^3}\ln\frac{4r_E r_R}{b^2} + \cdots\,, \tag{191}$$

which coincides with GR at 1PN.

Second-order redshift and atomic clocks

With $E_{\mathrm{trans}} \propto \mu$ and local time measured $\propto \tau^{-1}$:

$$\frac{\Delta\nu}{\nu}\bigg|_{1\to 2} \simeq \big[\ln\mu - \ln\tau\big]_1^2 + \frac{v_2^2 - v_1^2}{2c^2} + \mathcal{O}(c^{-4}). \tag{192}$$

With the PPN calibration $\mu = 1 + U/c^2 + \cdots$, $\tau = 1 - U/c^2 + \cdots$:

$$\frac{\Delta\nu}{\nu} = \frac{U_2 - U_1}{c^2} + \frac{v_2^2 - v_1^2}{2c^2} + \mathcal{O}(c^{-4}), \tag{193}$$

and the 2PN correction is $\propto (\delta a_2 - \delta b_2)$.

GW linearization and polarizations; energy flux

Linearization: $\mu = 1 + \delta\mu$, $\tau = 1 + \delta\tau$, $B_i = \delta B_i$, gauge $\partial_i \delta B_i = 0$:

$$(\partial_t^2 - \nabla^2)\,\delta\mu = \frac{1}{\Lambda_\mu}\,\Pi_\mu, \tag{194}$$

$$(\partial_t^2 - \nabla^2)\,\delta\tau = \frac{1}{\Lambda_\tau}\,\Pi_\tau, \tag{195}$$

$$(\partial_t^2 - \nabla^2)\,\delta B_i = -\,\zeta\,T_{0i}. \tag{196}$$

Observed modes: tensor $h_{+,\times}$ (as in GR) and a weak scalar "breathing" mode $s \propto \delta(\mu/\tau)$ with fraction $\kappa_b \ll 1$. The averaged flux (Isaacson-like in the observed frame):

$$\langle F \rangle = \frac{c^3}{32\pi G}\,\langle \dot h_+^2 + \dot h_\times^2 + \kappa_b\,\dot s^{\,2} \rangle, \tag{197}$$

and the leading radiation law of binaries coincides with GR.

Ringdown (QNM) with $R \leftrightarrow I$ boundary; shifts and echoes

Radial wave equation:

$$\frac{d^2\Psi_\ell}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right]\Psi_\ell = 0, \qquad r_* = \int^r \frac{dr'}{f(r')}. \tag{198}$$

Inner boundary condition at an effective membrane $r_* = r_*^{(0)} \sim r_s \ln(r_s/\ell_*)$:

$$\Psi_\ell \propto e^{-i\omega r_*} + \mathcal{R}(\omega)\,e^{+i\omega r_*} \quad (r_* \to r_*^{(0)}), \tag{199}$$

with $\mathcal{R}(\omega) = \mathcal{O}(\tilde g^{\,2})$. Mode condition:

$$\mathcal{Q}_\ell(\omega) + \mathcal{R}(\omega)\,e^{2i\omega r_*^{(0)}} = 0. \tag{200}$$

For $|\mathcal{R}| \ll 1$ and the fundamental mode $(\ell, m, n) = (2, 2, 0)$:

$$\frac{\delta\omega}{\omega} \simeq -\frac{i}{2\omega}\,\frac{\mathcal{R}(\omega_0)}{\partial_\omega \ln \mathcal{Q}_\ell|_{\omega_0}} + \mathcal{O}(\mathcal{R}^2), \quad \Delta t_{\rm echo} \sim 2|r_*^{(0)}|. \tag{201}$$

Cosmology: background, linear growth, kernel $K(r)$ and halos

Background:

$$H_{\rm obs} = \frac{1}{\tau}\left(H + \frac{\dot\mu}{\mu}\right), \qquad H^2 = \frac{8\pi G}{3}\left(\rho_m a^{-3} + \rho_r a^{-4} + \rho_\Lambda^{\rm eff}(a)\right) - \frac{k}{a^2}, \tag{202}$$

$\rho_\Lambda^{\rm eff} = V_0 + \rho_{\rm growth}(a)$. Growth of structures (primes for $d/d\ln a$):

$$\delta'' + \left(2 + \frac{H'}{H} - \varepsilon_\tau\right)\delta' - \frac{3}{2}\,\Omega_m(a)\,\delta = 0, \qquad \varepsilon_\tau \equiv \frac{d\ln\tau}{d\ln a}. \tag{203}$$

Dark matter as convolution:

$$\Phi_{\rm eff}(\mathbf{x}) = \Phi_{\rm bar}(\mathbf{x}) + G\int d^3x'\,K(|\mathbf{x}-\mathbf{x}'|)\,\rho_{\rm bar}(\mathbf{x}'), \quad K(r) = \frac{\kappa}{4\pi}\,\frac{1}{r^2 + r_c^2}. \tag{204}$$

For disk/spherical sources:

$$\rho_{\rm halo}(r) \simeq \frac{\rho_0}{1 + (r/r_c)^2}, \qquad v_{\rm flat}^2 \simeq 4\pi G\,\rho_0 r_c^2. \tag{205}$$

Profile and offset for the Bullet Cluster. We parameterize the offset of the lensing mass relative to the X-ray gas peak. For the subcluster we use a profile $\rho_{\mathrm{DM(BH)}}(r)$ centred on the gravitational centre of galaxies / black holes (BH). During the collision we introduce a ballistic shift Δx as a function of the relative velocity v_{rel} and the BH mass fraction f_{BH}:

$$\Delta x \approx \alpha(f_{\mathrm{BH}})\, v_{\mathrm{rel}}\, t_{\mathrm{cross}}. \tag{206}$$

The lensing potential is built from $\rho_{\mathrm{DM(BH)}}(r - \Delta x)$ with the gas contribution added on top.

Constraints on the "fifth force"

Yukawa corrections:

$$V(r) = -\frac{Gm_1m_2}{r}\Big[1 + \alpha_\mu e^{-m_\mu r} + \alpha_\tau e^{-m_\tau r}\Big], \qquad \alpha_{\mu,\tau} \sim \Big(\frac{\bar{m}}{\Lambda_{\mu,\tau}}\Big)^2. \tag{207}$$

Classes of tests: laboratory (torsion balances, Eötvös), LLR / planetary dynamics, binary pulsars / PPN, astrophysics (halo profiles, S-stars). In the article we use priors on large $\Lambda_{\mu,\tau}$ and/or non-zero $m_{\mu,\tau}$, together with non-linear screening in strong fields due to Σ.

Numerical methods and algorithms

PPN and S-stars: symplectic integrators (2nd/4th order) with forces 1PN + $\mu\tau$ 2PN + Newtonian dark mass; projection to the sky plane and v_{los} with relativistic corrections (test: 1PN shift of S2).
Lensing: ray tracing in the index $n(r)$ (Bulirsch–Stoer / Dormand–Prince); control of $\hat{\alpha} = 4GM/(bc^2)$ and Shapiro delay at 10^{-6}.
EM–GW delays: line integrals over tabulated potentials of the MW/hosts; HEALPix sky maps.
GW/QNM: Leaver's continued fractions for the GR baseline + modified inner boundary condition; root-finding for ω; echoes via a frequency-domain transfer function.
Cosmology: CLASS/CAMB branch with $H_{\mathrm{obs}}(a)$, linear growth, WL/ISW kernels and $\Xi(a) = d\ln(\mu/\tau)/d\ln a$.
Stability: CFL condition with respect to c in tangent frames; for the boundary layer, dissipative conditions and a "thick" approximation to ℓ_*.

Parameter tables and benchmark examples
Model parameters (symbols, dimensions, meaning)

Parameter	Dimension	Meaning / role
$\Lambda_\mu, \Lambda_\tau$	energy	Normalization of linear ϕ_R, σ couplings
m_μ, m_τ	energy	Range of the "fifth force"
$\lambda_\mu, \lambda_\tau, \lambda_\times$	dimless	Self-interaction / mixing in V
$\tilde{g}$	dimless	Interface permeability Σ
ℓ_*	length	Interface thickness (EFT cutoff $\Lambda \sim 1/\ell_*$)
a_1, a_2, b_1, b_2	dimless	PPN / 2PN coefficients of μ, τ
$\kappa,\ r_c$	dimless, length	Kernel $K(r)$ of halos
κ_b	dimless	Fraction of scalar GW polarization
ξ_{gw}	dimless	Universality of GW "optics" (EM vs GW)

Benchmark examples

- Solar lensing (deflection angle / Shapiro delay);
- S2 orbit: 1PN shift and $\mu\tau$ 2PN scan;
- QNM shift as a function of ℓ_*/r_s;
- Sky map of $\Delta t_{\mathrm{EM}} - \Delta t_{\mathrm{GW}}$;
- Milky Way rotation curve with $K(r)$ convolution;
- Cosmological background $H_{\mathrm{obs}}(a)$ and $w_{\mathrm{eff}}(a)$.

Graviton: canonical quantization, propagator and vertices

Linearized Lagrangian

In tangent frames of ds^2_{obs} the tensor modes are described by the standard quadratic action for a massless spin-2 field, whereas the scalar admixture s has a free scalar action with a small coupling κ_b.

Gauge and diagonalization

In transverse–traceless gauge the tensor sector is diagonal; zero modes are removed by the conditions

$$\partial_a h^{ab} = 0, \qquad h^a{}_a = 0.$$

Propagator

In momentum space:

$$D_{ab,cd}(k) = \frac{\Pi_{ab,cd}}{k^2 + i0},$$

where $\Pi_{ab,cd}$ is the projector onto the spin-2 subspace. For the scalar mode:

$$D_s(k) = \frac{1}{k^2 + i0},$$

with normalization governed by κ_b.

Vertices and rules

The universal exchange vertex:

$$\sim \kappa^2\, h_{ab} T^{ab} \quad \text{(tensor sector)} \qquad + \qquad \sim \kappa_b\, s\, T \quad \text{(scalar sector)},$$

where $T \equiv T^a{}_a$. The Ward identity ensures current conservation and unitarity. Scattering and radiation matrix elements coincide with linear GR in the tensor sector; scalar contributions are suppressed by κ_b.

UV consistency

Local counterterms have dimension 4 and fit within the EFT cutoff $\Lambda \sim 1/\ell^*$. Microcausality is preserved (see §16–17).

Mobius baryogenesis: kinetics and asymmetry estimates

Two-chamber kinetics

The Boltzmann equations for the distributions $f_{A,B}(p, x, t)$ are supplemented by sinks and sources on Σ:

$$(\partial_t + H\, p \cdot \nabla_p)\, f_A = C[f_A] - \Gamma_\Sigma\, (f_A - f_B) + \varepsilon_\Sigma\, S, \qquad (A \leftrightarrow B \text{ with sign changes}). \tag{208}$$

Freeze-out regime

At temperatures T below a critical T_Σ the rate $\Gamma_\Sigma(T)$ drops faster than the Hubble rate H, and the asymmetry "freezes in". In a quasi-stationary approximation one finds:

$$\eta_B \simeq \varepsilon_\Sigma \int_{t_{\text{freeze}}} dt\, \frac{\Gamma_\Sigma(t)}{n_\gamma(t)}\, F(t) \;\sim\; \varepsilon_\Sigma \left(\frac{\Gamma_\Sigma}{H}\right)_{\text{freeze}} \times C, \tag{209}$$

where F and C encode the effective numbers of degrees of freedom and the redistribution of baryon number.

Constraints

- **BBN/CMB:** the residual contribution of the "anti-layer" to the energy density must be $\ll 10^{-2}$ of the radiation density at the MeV epoch.
- **Absence of late annihilations:** Γ_Σ must decay before the onset of nucleosynthesis.
- **Observable signatures:** a weak gravitational-wave background from phase transitions on Σ, and a possible isotropic contribution to ΔN_{eff}. These observations constrain the parameters ε_Σ and Γ_Σ.