The Korteweg–De Vries Equation: Does the Equation Satisfy the Criterion of Truth?

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Abstract

The irrefutable proof of the incorrectness of the Korteweg–De Vries (KdV) equation is proposed. The correct methodological basis for the proof is the unity of formal logic and rational dialectics. The unity of formal logic and rational dialectics is the only correct criterion of truth. The proof leads to the following irrefutable statement: the Korteweg–De Vries (KdV) equation – a partial differential equation with respect to the dimensionless displacement as the function of the dimensionless coordinate and the dimensionless time – represents an incorrect equation in physics. The proof of this statement is based on the following irrefutable assertions:

- the Korteweg-de Vries equation does not describe a physical phenomenon (process) because the dimensionless displacement, the dimensionless coordinate and the dimensionless time are non-physical quantities;
- in the formal logical point of view, the physical (or geometric) interpretation of the equation and solutions of the equation are inadmissible;
- comparison with experimental data (which have dimensions) is inadmissible (impossible).

Consequently, the Korteweg-de Vries equation is a gross error in physics.

Introduction

As is known, the Korteweg–De Vries (KdV) equation is a nonlinear partial differential equation of the third order. It plays an important role in the theory of nonlinear waves, mainly of a hydrodynamic nature. Therefore, it is worth paying attention to the history of the KdV equation.

"The history of the KdV equation started with experiments by John Scott Russell in 1834, followed by theoretical investigations by Lord Rayleigh and Joseph Boussinesq around 1870 and, finally, Korteweg and De Vries in 1895. The KdV equation was not studied much after this until Zabusky & Kruskal (1965) discovered numerically that its solutions seemed to decompose at large times into a collection of "solitons": well separated solitary waves. Moreover, the solitons seem to be almost unaffected in shape by passing through each other (though this could cause a change in their position). They also made the connection to earlier numerical experiments by Fermi, Pasta, Ulam, and Tsingou by showing that the KdV equation was the continuum limit of the FPUT system.

Development of the analytic solution by means of the inverse scattering transform was done in 1967 by Gardner, Greene, Kruskal and Miura. The KdV equation is now seen to be closely connected to Huygens' principle.

The KdV equation has several connections to physical problems. In addition to being the governing equation of the string in the Fermi–Pasta–Ulam–Tsingou problem in the continuum limit, it approximately describes the evolution of long, one-dimensional waves in many physical settings, including: shallow-water waves with weakly non-linear restoring forces; long internal waves in a density-stratified ocean; ion acoustic waves in a plasma, acoustic waves on a crystal lattice.

The KdV equation can also be solved using the inverse scattering transform such as those applied to the non-linear Schrödinger equation" (Wikipedia).

But, in my opinion, the long history of the KdV equation and the efforts of physicists and mathematicians may lose scientific significance, because the KdV equation was not formulated within the framework of the correct methodological basis of science: the unity of formal logic and rational (materialistic) dialectics [1– 6].

The purpose of this work is to propose an irrefutable proof that the KdV equation does not satisfy the correct criterion of truth: the unity of formal logic and rational dialectics.

On the Correct Methodological Basis of Science: The Unity of Formal Logic and Rational Dialectics
- "Dialectics is a theory and method of cognition of the phenomena of reality in their connection, development and self-movement, the science of the most general laws of development of nature, society and thinking. Dialectics operates with concepts expressed in verbal form. Dialectics was created by outstanding philosophers: Heraclitus, Plotinus, Proclus, Bruno, Kant, Fichte, Schelling, Hegel and others.
- Materialistic dialectics – as a synthesis and generalization of the history of the development of philosophy, science and social practice – is the science of the most general laws of the development of nature, society and thinking, the general methodology of cognition of the objective world. The most important principles of materialistic dialectics were formulated by Karl Marx, Friedrich Engels and Vladimir Lenin in the 19th–20th centuries on the basis of criticism of old philosophy" (Russian Wikipedia). Materialistic dialectics operates with concepts called categories. Categories are concepts of the most general nature. Categories express the most general and essential connections, properties, features, relationships of the developing material world in verbal form.
- Rational dialectics is a materialistic dialectics, corrected and developed by me on the basis of formal logic. One of the fundamental concepts of rational dialectics is the philosophical category of measure. Measure denotes the dialectical unity, the interdependence of the qualitative determinacy and quantitative determinacy of an object or phenomenon. Qualitative determinacy does not exist without quantitative determinacy; quantitative determinacy does not exist without qualitative determinacy. Measure expresses the boundaries within which objects or phenomena exist and are identical to themselves. Qualitative determinacy (property) is a holistic (integral, all-in-one) and special characteristic of an object or phenomenon; qualitative determinacy (property) is one of the aspects of an object or phenomenon; qualitative determinacy (property) determines and expresses the specificity of an object or phenomenon.

Objects and phenomena have not only a qualitative aspect, but also a quantitative aspect. Quantitative determinacy is the determinacy of objects and phenomena expressed by number. Number characterizes and expresses the quantitative aspects of an object and phenomenon. In essence and ultimately, mathematics is the science of operations on numbers. Numbers are the result of measurements. The results of measurements are real neutral numbers, because zero is a real neutral number. Zero is the origin. Numbers can have dimensions or not. The numbers as the results of measurements are concrete numbers (denominate number).
- Quantities in correct mathematics are designated by letters and take on numerical values. If quantities did not take on numerical values, then these quantities would not exist in correct mathematics. Letter designations of numbers are connected (linked, united) by symbols of correct mathematical (quantitative) operations: $"+", "-", "\cdot", ":", "\sqrt{\ }", "\lg"$. The resulting connection (union) is called a mathematical expression (relationship). Symbols (for example, $"i = \sqrt{-1}", "|\ |", "\sin", "\cos", "\frac{d}{dx}", "\int"$) that are not symbols of correct mathematical operations cannot be used (exist) in correct mathematical expressions [7-56].
- A mathematical expression (relationship, equation) is correct within the concept of measure if each member of the expression (relationship, equation) has the same qualitative determinacy (i.e. dimension). In other words, all members of the expression must have identical qualitative determinacy (i.e. dimension) and quantitatively describe the same material object or phenomenon. If the members of a mathematical expression do not have dimension, then this expression does not have qualitative determinacy and does not describe a material object or phenomenon. Such a mathematical expression is a gross error in physics.
- Formal logic is the science of correct thinking expressed in verbal form. Correct thinking is, firstly, thinking that is based on the following basic laws of formal logic: the law of identity (i.e., "**A** is **A**", where **A** is a statement); the law of lack (absence) of contradiction (i.e., "**A** is not **not-A**"; the law of the excluded middle (i.e., "**A** is either **B** or **not-B**", where **B** is a statement); the law of sufficient reason (i.e., "**A** is because **B** is"). The laws of formal logic form a unity. Correct thinking is, secondly, thinking that is based on concepts, operates with concepts. A concept is a form of thought that characterizes, fixes and expresses (in verbal form) the essential features of objects and

phenomena. Connected concepts form a proposition. The connection of propositions leading to the formation of a new proposition is called inference. A theory is a system of concepts.

- The solution to the problem of the relation between formal logic and rational dialectics is as follows. Both rational dialectics and formal logic are common means, methods of cognition of reality in verbal form. But the method of rational dialectics and the method of formal logic are different. Formal logic and rational dialectics are mutually complementary, mutually conditioned, mutually related methods of cognition that form a unity. The unity of formal logic and rational dialectics is the correct methodological basis of science. Therefore, the unity of formal logic and rational dialectics is also the correct criterion of truth.

Methodological Analysis of the Korteweg–De Vries (KdV) Equation

As is known, there are many different variations of the KdV equation. The simplest equation is as follows:

$$\frac{\partial \psi}{\partial t} + \frac{\partial^3 \psi}{\partial x^3} - 6\psi \frac{\partial \psi}{\partial x} = 0$$

where ψ, x, t are the height displacement of the water surface from its equilibrium height, the coordinate of the displacement point and the point of time (the instant of time) of the displacement, respectively. The essence (essential feature) of the equation and the solution of the equation is that ψ, x, t are dimensionless variables. The numerical values of dimensionless quantities are abstract numbers (absolute numbers, non-concrete numbers).

- As is known, dimensionless quantities are not physical or geometric quantities. Dimensionless quantities cannot be expressed (defined) using concepts and names. Therefore, such dimensionless quantities as "displacement", "coordinate" and "time" are meaningless terms. The dimensionless quantities "displacement", "coordinate" and "time" cannot be compared with the dimensional quantities "displacement (meter)", "coordinate (meter)" and "time (second)". This means that comparison of the solution of the KdV equation with experimental data is formal-logical and dialectical errors of the type **1 = 1 meter, 1 = 1 second**.
- The physical (or geometric) interpretation of the solution of the KdV equation is a formal-logical error of the type **A ≠ B = A**. Really, any interpretation is the following erroneous proposition: "If **A** is not **B**, then **A** is **B**". In the case of the KdV equation, the error has the following form:

("*not-meter*" is "*meter*").
("*not-second*" is "*second*").

- If the quantities had the dimensions "meter", "meter", and "second", respectively, then substitution these dimensional quantities into the KdV equation would lead to physical (or geometric) nonsense. The nonsense would be in that the members of the KdV equation would have different dimensions.

Discussion

The existence of gross errors in physics can be explained as follows:
- scientists have not learned (have not mastered) the correct methodological basis of science: the unity of formal logic and rational dialectics. Therefore, scientific works do not satisfy the correct criterion of truth the unity of formal logic and rational dialectics [7-56];
- correct formal-logical definitions of physical concepts are lack (absent) in physics. Therefore, scientific works do not satisfy the correct criterion of truth the unity of formal logic and rational dialectics [7-56];
- the use of pure mathematics (in particular, differential calculus) to describe the physical and geometric properties of material objects is meaningless. Therefore, scientific works do not satisfy the correct criterion of truth the unity of formal logic and rational dialectics [7-56].

In this point of view, the KdV equation and its solution are a gross error in physics. In addition, scientists have not understood that the correct definition of a wave is the following:

A wave is a moving locally disturbed state (physical or chemical state) of a macroscopic material object. Wave motion is the motion of a wave.

The KdV equation and its solution ignore the scientific fact that the state (property) of a material object is characterized by a dimensional value. Therefore, the KdV equation and its solution do not satisfy the correct criterion of truth.

Conclusion

Thus, the essence of the Korteweg-de Vries equation – a partial differential equation with respect to the function ψ, x, t, where ψ is the displacement, x is the coordinate, t is the time –is that ψ, x, t are dimensionless (non-physical) quantities. This means that the equation does not describe a physical phenomenon, process. In the formal logical point of view, the physical (or geometric) interpretations of the equation and solutions of the equation are inadmissible. Comparison with experimental data (which have dimensions) is inadmissible (impossible). The equation could have a physical (or geometric) meaning if only the quantities ψ, x, t had the dimensions "meter", "meter" and "second", respectively. But substitution of dimensional quantities into the equation leads to the following result: the

equation turns into physical (or geometric) nonsense.

Consequently, the Korteweg-de Vries equation is a gross error in physics.

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