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The Superposition Law and Its Impact on Energy Conservation

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Abstract
The principle of energy conservation is a cornerstone of classical physics, yet its inconsistency with the linear superposition remains conceptually subtle: The Superposition Law (SL) states that the field-components of Electro Magnetic (EM) waves combine linearly when they overlap in space. In contrast, the Energy Conservation Law (ECL) requires that energy be summed quadratically. To explore this long-standing issue, we first specify Levine's model by aligning two radiation dipoles along a straight line to simplify the physical mechanism, whereby we confirm his observation of energy-doubling at the wave level. Then, the applicability of SL is illustrated at the sources and the analytic solution is provided to clarify the true origin of the energy-doubling phenomenon. The findings of this study are consistent with the results of the previous publication with the theoretical analysis and experimental validation.

Keywords: Electromagnetic Radiation, Energy-Doubling and Energy Conservation

Introduction
The Superposition Law (SL) governs the behavior of waves when two or more waves meet in space, and its linear additivity has been verified experimentally. In contrast, the Energy Conservation Law (ECL) can be viewed as a nonlinear additive law - more precisely, one based on the squared form of superposition - and is, to some extent, an empirical principle.

To clarify the distinction between these two laws, we examine the joint radiation of two electromagnetic (EM) waves of identical frequency under the far-field assumption. The resultant field of the superposition can be written as

$$\vec{E}(x,y,z,\omega t) = \vec{E}_1(x,y,z,\omega t) + \vec{E}_2(x,y,z,\omega t) \quad (1)$$

where ω is the angular frequency, and x,y,z are the Cartesian coordinates, $\vec{E}_1(x,y,z)$ and $\vec{E}_2(x,y,z)$ represent the field strengths of the radiative waves 1 and 2 from source 1 and 2 , and $\vec{E}(x,y,z)$ is the superposed field, respectively.

The radiation power of the superposed waves is calculated using the Poynting theorem, as follows

$$P = \frac{1}{\eta}\oiint_{\mathfrak{S}} < E^2(x,y,z) > \vec{k}\cdot d\vec{s} = P_1 + P_2 + \Gamma_{12} \quad (2)$$

with

$$P_i = \frac{1}{\eta}\oiint_{\mathfrak{S}} < E_i^2(x,y,z) > \vec{k}\cdot d\vec{s} \text{ for } i = 1,2 \quad (3)$$

and

$$\Gamma_{12} = \frac{1}{\eta} \oiint_{\mathfrak{S}} < \vec{E}_1 \cdot \vec{E_2} > \vec{k} \cdot d\vec{s} \qquad (4)$$

where P is the total radiation power, P_1 and P_2 are the powers radiated from source 1 and 2, Γ_{12} is the interference power between the two waves, $\langle \cdot \rangle$ represents the operator of time averaging, $\eta = \sqrt{\mu_0/\epsilon_0}$, μ_0 and ϵ_0 are the magnetic and electric permittivities of free space, $\vec{k}$ is unit vector in direction of power flow and $\mathfrak{S}$ is the close surface bounding the two sources, respectively.

Assuming temporarily that there is no electromagnetic (EM) coupling between the two sources, the ECL requires the interference power to vanish as

$$\Gamma_{12} \equiv 0 \qquad (5)$$

However, Levine's model reveals that when two sources are placed in close proximity, the phenomenon of $\Gamma_{12} \neq 0$ occurs [1]. This suggests that the Energy Conservation Law (ECL) does not hold at the wave level after the waves have departed from their sources. Then, the energy anomaly was incorrectly attributed to sources' coupling, with no analytical solution available to date.

The scientific contribution of this paper is to uncover the superposition behavior at the radiation sources, thereby revealing the inconsistency between the SL and ECL in terms of the radiation powers throughout the space.

Preliminaries

To provide an intuitive foundation, we recall the conventional radiation dipole model in the form of

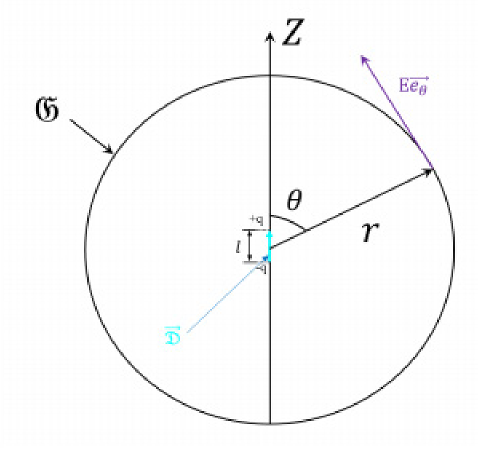
$$\vec{\mathfrak{D}} \equiv q l_0 \cos(\omega t) \vec{e}_z \qquad (6)$$

where $\vec{\mathfrak{D}}$ is the radiating dipole, ql_0 is defined as the dipole moment, q and l_0 are the magnitude of each electric charge and that of the distance between the two electric charges in each dipole, $\vec{\mathfrak{D}}$ is the radiating dipole, ω is the angular frequency and $\vec{e}_z$ is a unit vector along the direction of the dipole.

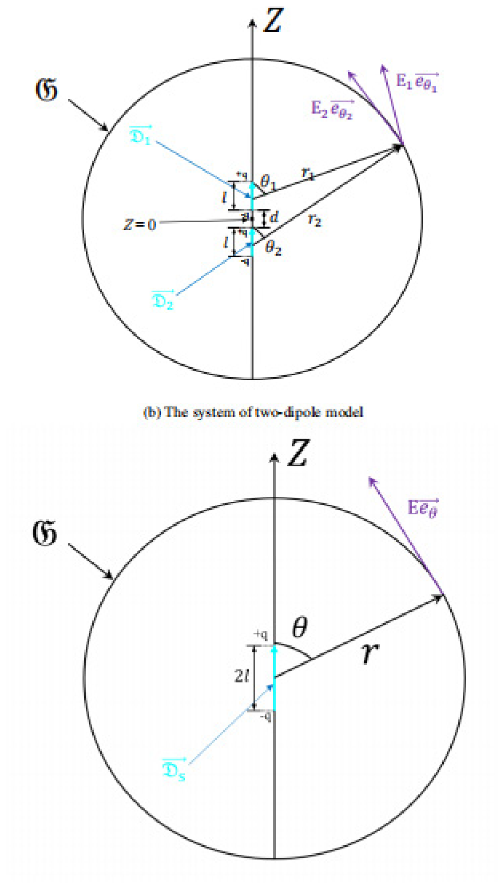
Since near fields do not contribute to radiation power, we focus solely on the radiative field and express its field in spherical coordinates as [2].

$$\vec{E} = \frac{\mu_0 (l_0 q) \omega^2 \sin(\theta)}{4\pi r} \cos(\omega t - \kappa r) \vec{e}_\theta, \qquad (7)$$

where κ=2π/λ is the wave number, c is the speed of light and r is the distance from the dipole to the observation point



(a) The single dipole model.



(b) The system of two-dipole model

(c) The effective dipole model.

Figure 1: Three Dipole Models

Integrating the Poynting vector over the closed surface yields the total radiation power,

$$P = \oiint_{\mathcal{S}} \langle \vec{S} \rangle \cdot d\vec{s} = \frac{\mu_0 \omega^4}{12\pi c^3} (l_0 q)^2 \qquad (8)$$

with

$$\vec{S} = \frac{1}{\mu_0 c} E_\theta^2 \vec{e}_r \qquad (9)$$

where P is the radiation power, c is the speed of light and $\vec{S}$ is the Poynting vector.

Before proceeding, we note the following two key points.

- The Amplitude of the radiative field $\vec{E}$ is proportional to the dipole moment ql_0.
- The Total radiation power is proportional to the square of that moment.

System Model

Based on the frame work recalled in the last section, we propose a radiation system that specifies Levine's mode by arranging two identical dipoles along a straight line to radiate powers in a co-phase manner into free space, as shown in Figure.1(b). The spatial symmetry observed in the geometric arrangement of the two dipoles is consistent with that described in [3].

First, it is conceptually crucial to emphasize that in the joint radiation of the two dipoles, each of them can still be described by the form given in (6), even when the EM coupling presents. The assertion is explained by the following reasoning:

- The spatial symmetry of system allows the two co-phase radiation dipoles to exist.
- The EM coupling does not change the frequency of the dipoles, in the regime of linear approximation.

Hence, we can express the dipoles in Figure.1(b) and adopt the formulation

$$\vec{\mathfrak{D}}_i \equiv ql(t)\vec{k}, \text{ for } i = 1,2 \qquad (10)$$

which $\vec{\mathfrak{D}}_i$ represents dipole i (for i=1,2).

It should be noted that although (10) and (6) are in the identical expression, when one dipole is removed from the proposed system, the power of the remaining dipole does not remain unchanged in the case nonzero EM coupling. Nevertheless, the form of (10) states that the relationship between each radiating dipole and its radiation power is true in the reality.

Secondly, the total radiation power is calculated under the far-field assumption as

$$|2l_0 + d| \ll \lambda \text{ and } \lambda \ll r_i \text{ for } l, d \neq 0 \text{ and } i = 1,2 \qquad (11)$$

where r_i is the distances between dipole i to the surface of the Poynting integral.

Consistency of Linear Superposition

Let us now analyze how the superposed wave makes its contributions to increase the total power in the proposed system.

Considering Eq. (10) and (7), the superposed field can be expressed as

$$\vec{E}_s = \vec{E}_1 + \vec{E}_2 \qquad (12)$$

with

$$\vec{E}_i = \frac{\mu_0 \omega^2 (l_0 q) \sin(\theta_i)}{4\pi r_i} \cos(\omega t - \kappa r_i) \vec{e}_{\theta_i} \text{ for } i = 1,2 \qquad (13)$$

where $\vec{E}_i$ denotes the radiative field by dipole i, as shown in Figure.1(b).

Under the far-field assumption in (11), employing the first order approximation of (13) yields

$$\vec{E}_1 = \vec{E}_2 \approx \frac{\mu_0 \omega^2 (l_0 q) \sin(\theta)}{4\pi r} \cos(\omega t - \kappa r) \vec{e}_\theta \qquad (14)$$

making the integrand in (5) remain strictly greater than zero, i.e.,

$$\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = \langle E_1^2 \rangle > 0 \qquad (15)$$

which rules out the possibility of its self-consistency required by the ECL, indicating that the ECL does not hold at the wave level as presented by Levine in 1980 [1].

Further, proceeding from (14), the superposed field can be expressed as

$$\vec{E}_s = 2 \frac{\mu_0 \omega^2 (l_0 q) \sin(\theta)}{4\pi r} \cos(\omega t - \kappa r) \vec{e}_\theta \qquad (16)$$

which is of critical importance explained as follows.

On the one hand, equation (16) indicates that the amplitude of radiative field doubles on the surface of the Poynting integration. On the other hand, we can attribute the reason to a source in the expression of

$$\vec{E}_s = \frac{\mu_0 \omega^2 (2 l_0 q) \sin(\theta_i)}{4\pi r} \cos(\omega t - \kappa r) \vec{e}_{\theta_i} \qquad (17)$$

where ($2l_0$ q) is an equivalent dipole moment.

Hence, from the observation point of view at the far-field, the radiative fields are the same as that from a dipole in the form of

$$\vec{\mathfrak{D}}_s \equiv 2 q l_0 \cos(\omega t) \vec{k} \qquad (18)$$

which inspires a concept of the superposed dipole by

$$\vec{\mathfrak{D}}_s = \vec{\mathfrak{D}}_1 + \vec{\mathfrak{D}}_2 \qquad (19)$$

where $\vec{\mathfrak{D}}_s$ is the resultant dipole of the superposition.

It is obvious that the derivation either from the superposed wave (16) or from the superposed dipole in (18), we can find the energy-doubling by

$$P_s = \oint\!\!\oint_{\mathcal{S}} \langle \vec{S} \rangle \cdot d\vec{s} = 4 \times \frac{\mu_0 \omega^4}{12\pi c^3} (l_0 q)^2 = 2(P_1 + P_2) \qquad (20)$$

Through the above derivations, we can agree that it is the SL that doubles the energy due to its effects either in the wave or at the dipoles.

Since the two dipoles illustrated in Figure. 1(b) are not physically connected to each other, from point of radiation, we define the superposed dipole as the effective dipole and depict it in Figure. 1(c), because of its radiation role that replaces the two dipoles, observed from the far-field.

Finally, we address the EM problem of near-fields as follows. As has been known, the near-fields do not make any contributions to the radiation power and, thus, consume zero power from the sources after the system reaches its radiation stability. In the proposed system, the radiative fields do not couple with each other, because each dipole radiates zero power to the other one along its poles.

This has been confirmed by the experimental result of η_{ij} = Re{Z_{ij} }/Re{Z_{ii} }≈0.1 (for i=1,2), where η_{ij} and Z_{ij} are the EM coupling factor and the complex radiation impedances as stated in [3]. Hence each of the two dipoles works theoretically as it radiates alone in terms of its power.

It is noted that the above derivations are consistent with Noether's theorem for measuring the energy in the time domain.

Discussion and Conclusion

The following two points are presented for making the concepts of this work clearer as follows.

• Energy-doubling in Waves

The proposed system enables the co-phase interference of the two waves to occur at every point in space and, hence, doubles the total radiative power.

•Effective Dipole

The linear superposition of the two dipoles results in an effective dipole which makes the consistency in the power radiation above.

Actually, energy-abnormal phenomena have been reported not only in the EM waves, but also in quantum mechanical systems, both of which are fundamentally associated with the principle of wave superposition [2,6-9]. In the present study, we conclude that the observed energy-doubling arises as a special case of applying SL simultaneously to both the sources and the waves, which is consistent with our previous results reported in [3].

Finally, we make a supplementary statement as follows: The energy-doubling is observed only in the range of $d/\lambda \ll 1$. When the distanced increases, an energy fluctuation is observed. Until $d/\lambda > 2.5$, the ECL dominates [3,4].

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